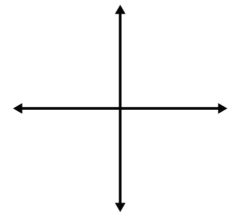
$210^\circ \times \frac{\pi}{180^\circ} = \frac{7\pi}{6}$

Trigonometry
LESSON ONE - *Degrees and Radians*
Lesson Notes

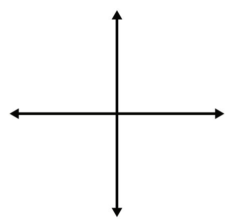
Example 1 Define each term or phrase and draw a sample angle. Angle Definitions

a) angle in standard position:

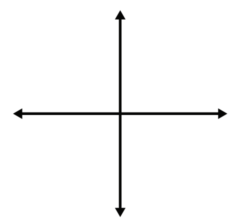


Draw a standard position angle, θ.

b) positive and negative angles:

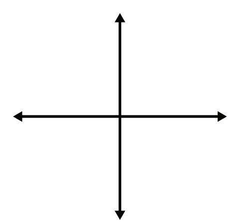


Draw θ = 120°

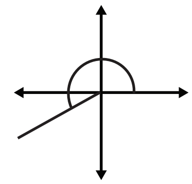


Draw θ = -120°

c) reference angle:

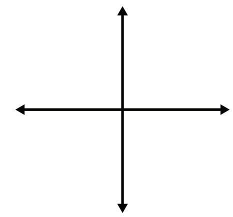


Find the reference angle of θ = 150°.



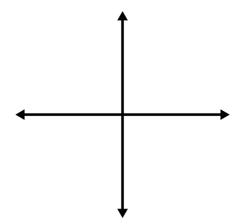
$$210^\circ \times \frac{\pi}{180^\circ} = \frac{7\pi}{6}$$

d) co-terminal angles:



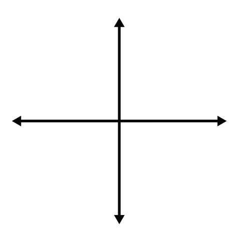
Draw the first positive co-terminal angle of 60°.

e) principal angle:

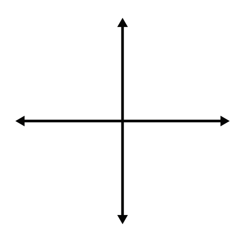


Find the principal angle of θ = 420°.

f) general form of co-terminal angles:



Find the first four ***positive*** *co-terminal angles of θ = 45°.*



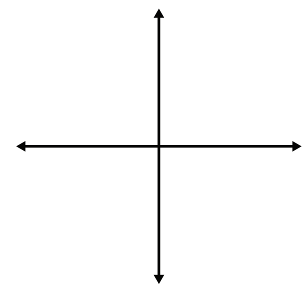
Find the first four ***negative*** *co-terminal angles of θ = 45°.*

Example 2 Three Angle Types:
***Degrees**, **Radians**, and **Revolutions**.*

Angle Types and Conversion Multipliers

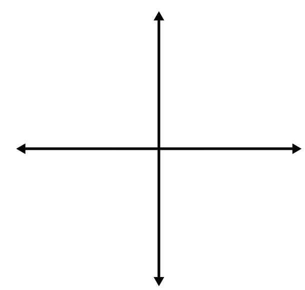
a) Define degrees, radians, and revolutions.

i) Degrees:



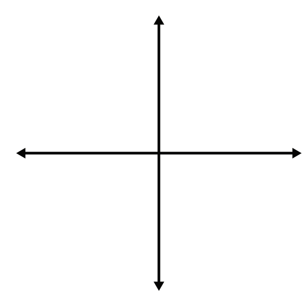
Draw θ = 1°

ii) Radians:

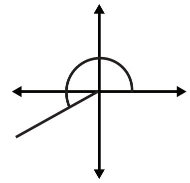


Draw θ = 1 rad

iii) Revolutions:



Draw θ = 1 rev



$$210^\circ \times \frac{\pi}{180^\circ} = \frac{7\pi}{6}$$

b) Use conversion multipliers to answer the questions and fill in the reference chart.
Round all decimals to the nearest hundredth.

Conversion Multiplier Reference Chart

	degree	radian	revolution
degree			
radian			
revolution			

i) $23^\circ \times \boxed{}$ = ________ rad

ii) $23^\circ \times \boxed{}$ = ________ rev

iii) $2.6 \times \boxed{}$ = ________ °

iv) $2.6 \times \boxed{}$ = ________ rev

v) 0.75 rev × $\boxed{}$ = ________ °

vi) 0.75 rev × $\boxed{}$ = ________ rad

c) Contrast the decimal approximation of a radian with the exact value of a radian.

i) Decimal Approximation: $45^\circ \times \boxed{}$ = ________ rad

ii) Exact Value: $45^\circ \times \boxed{}$ = ________ rad

Example 3 Convert each angle to the requested form.
Round all decimals to the nearest hundredth.

Angle Conversion Practice

a) convert 175° to an approximate radian decimal.

b) convert 210° to an exact-value radian.

c) convert 120° to an exact-value revolution.

d) convert 2.5 to degrees.

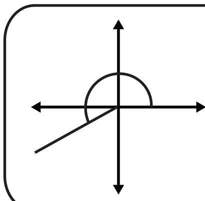
e) convert $\frac{3\pi}{2}$ to degrees.

f) write $\frac{3\pi}{2}$ as an approximate radian decimal.

g) convert $\frac{\pi}{2}$ to an exact-value revolution.

h) convert 0.5 rev to degrees.

i) convert 3 rev to radians.



$$210^\circ \times \frac{\pi}{180^\circ} = \frac{7\pi}{6}$$

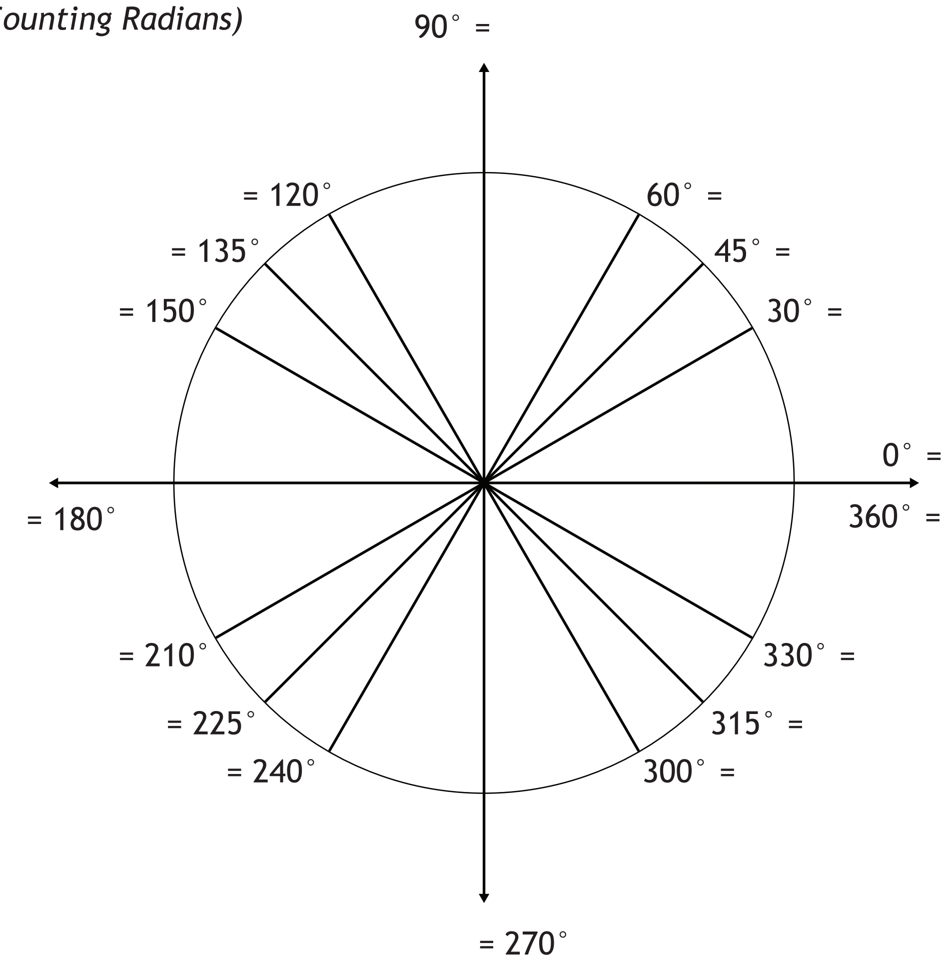
Example 4

The diagram shows commonly used degrees. Find exact-value radians that correspond to each degree. When complete, memorize the diagram.

Commonly Used Degrees and Radians

a) ***Method One:*** Find all exact-value radians using a conversion multiplier.

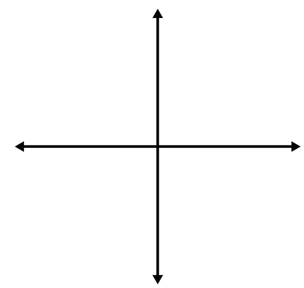
b) ***Method Two:*** Use a shortcut. *(Counting Radians)*



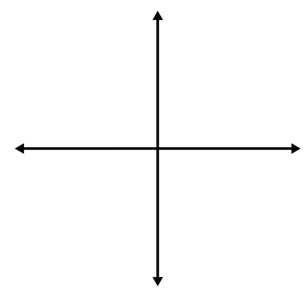
Example 5 Draw each of the following angles in standard position. State the reference angle.

Reference Angles

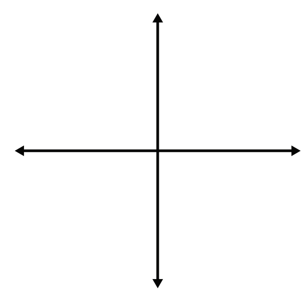
a) 210°



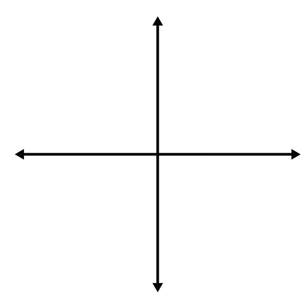
b) -260°



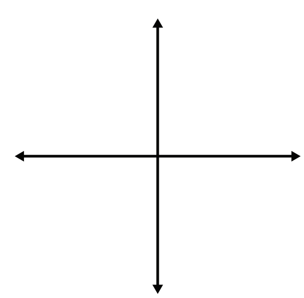
c) 5.3

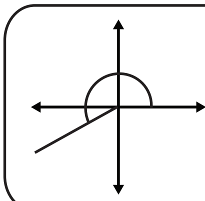


d) $-\frac{5\pi}{4}$



e) $\frac{12\pi}{7}$



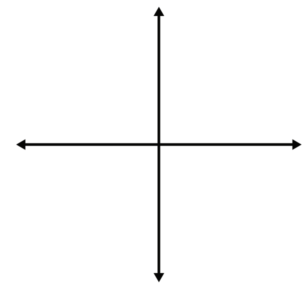


$210^\circ \times \frac{\pi}{180^\circ} = \frac{7\pi}{6}$

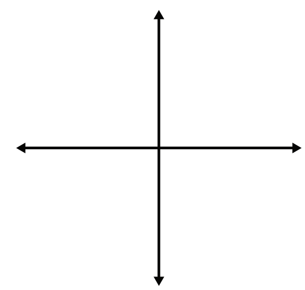
Example 6 Draw each of the following angles in standard position. State the principal and reference angles.

Principal and Reference Angles

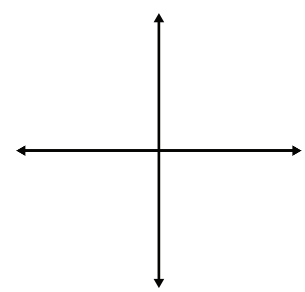
a) 930°



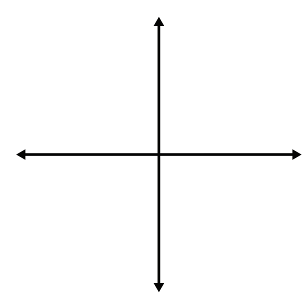
b) -855°



c) 9



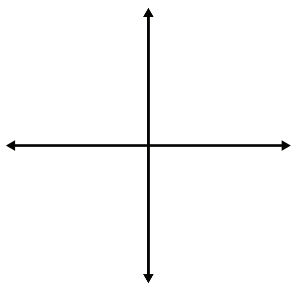
d) $-\frac{10\pi}{3}$



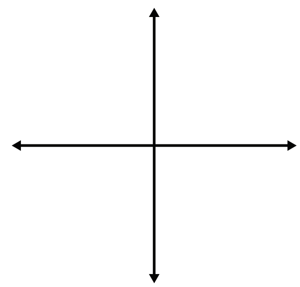
Example 7 For each angle, find all co-terminal angles within the stated domain.

Co-terminal Angles

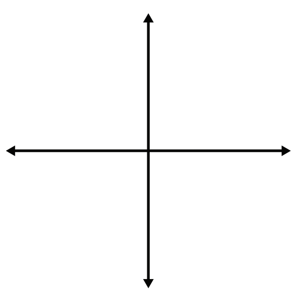
a) 60°, Domain: $-360^\circ \leq \theta < 1080^\circ$



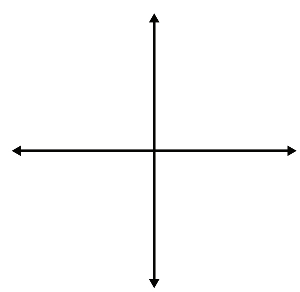
b) -495°, Domain: $-1080^\circ \leq \theta < 720^\circ$

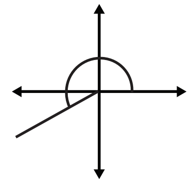


c) 11.78, Domain: $-2\pi \leq \theta < 4\pi$



d) $\frac{8\pi}{3}$, Domain: $-\frac{13\pi}{2} \leq \theta < \frac{37\pi}{5}$





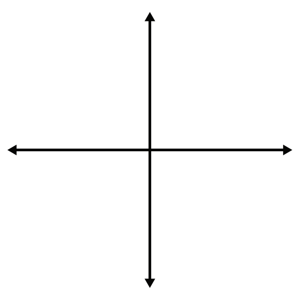
$210^\circ \times \frac{\pi}{180^\circ} = \frac{7\pi}{6}$

Example 8

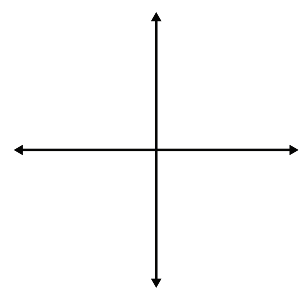
For each angle, use estimation to find the principal angle.

Principal Angle of a Large Angle

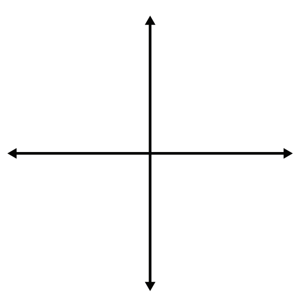
a) 1893°



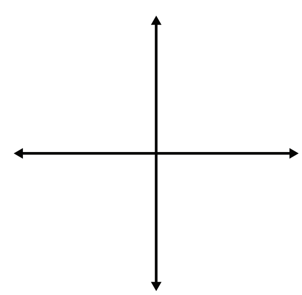
b) -437.24



c) $\frac{912\pi}{15}$



d) $\frac{95\pi}{6}$



Example 9

Use the general form of co-terminal angles to find the specified angle.

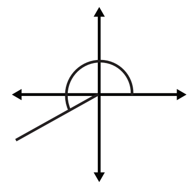
General Form of Co-terminal Angles

a) principal angle = 300°
(find co-terminal angle 3 rotations counter-clockwise)

b) principal angle = $\frac{2\pi}{5}$
(find co-terminal angle 14 rotations clockwise)

c) How many rotations are required to find the principal angle of -4300°? State the principal angle.

d) How many rotations are required to find the principal angle of $\frac{32\pi}{3}$?
State the principal angle.

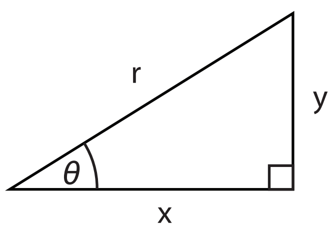


$210^\circ \times \frac{\pi}{180^\circ} = \frac{7\pi}{6}$

Example 10

Six Trigonometric Ratios

In addition to the three primary trigonometric ratios (sinθ, cosθ, and tanθ), there are three reciprocal ratios (cscθ, secθ, and cotθ). Given a triangle with side lengths of x and y, and a hypotenuse of length r, the six trigonometric ratios are as follows:

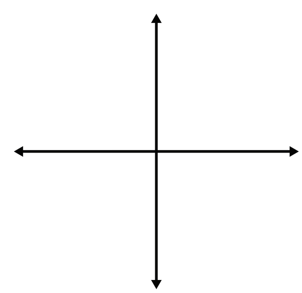


$\sin\theta = \frac{y}{r}$ $\qquad \csc\theta = \frac{1}{\sin\theta} = \frac{r}{y}$

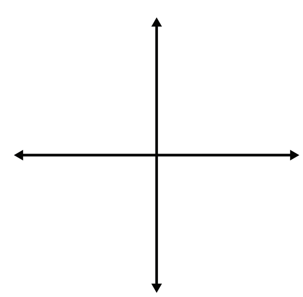
$\cos\theta = \frac{x}{r}$ $\qquad \sec\theta = \frac{1}{\cos\theta} = \frac{r}{x}$

$\tan\theta = \frac{y}{x}$ $\qquad \cot\theta = \frac{1}{\tan\theta} = \frac{x}{y}$

a) If the point P(-5, 12) exists on the terminal arm of an angle θ in standard position, determine the exact values of all six trigonometric ratios. State the reference angle and the standard position angle.



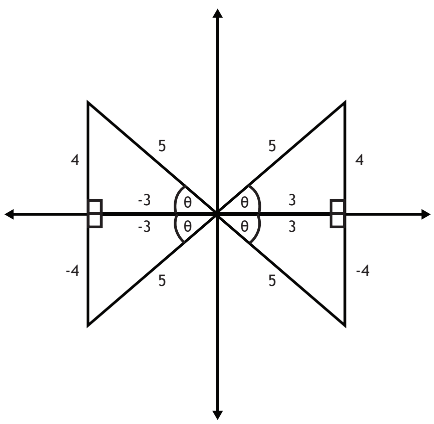
b) If the point P(2, -3) exists on the terminal arm of an angle θ in standard position, determine the exact values of all six trigonometric ratios. State the reference angle and the standard position angle.



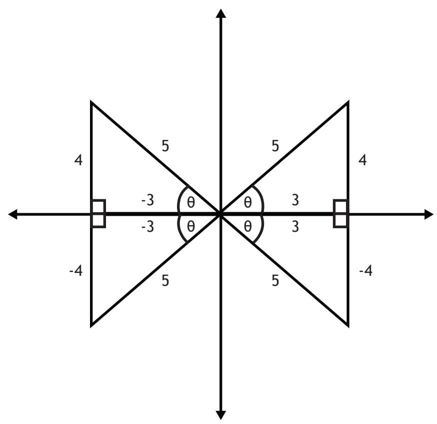
Example 11 Determine the sign of each trigonometric ratio in each quadrant.

Signs of Trigonometric Ratios

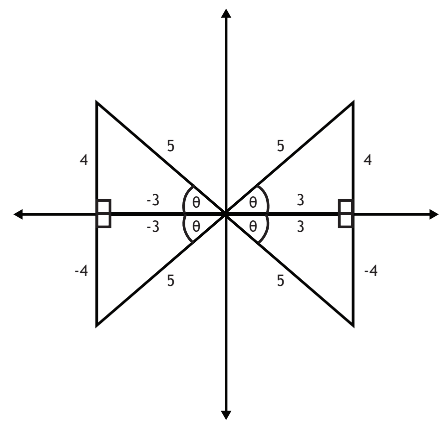
a) $\sin\theta$



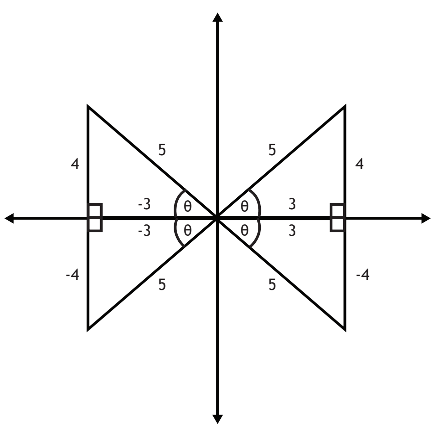
b) $\cos\theta$



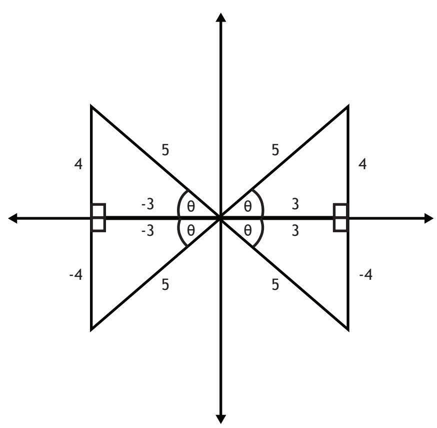
c) $\tan\theta$



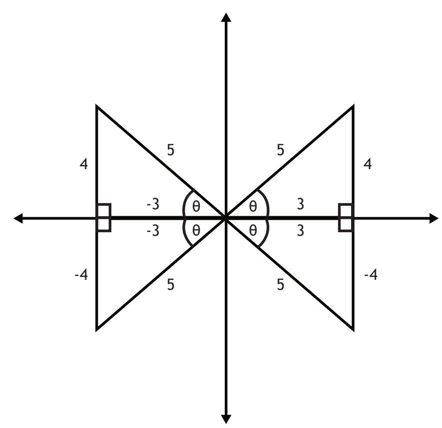
d) $\csc\theta$



e) $\sec\theta$



f) $\cot\theta$



g) How do the quadrant signs of the reciprocal trigonometric ratios ($\csc\theta$, $\sec\theta$, and $\cot\theta$) compare to the quadrant signs of the primary trigonometric ratios ($\sin\theta$, $\cos\theta$, and $\tan\theta$)?

Trigonometry
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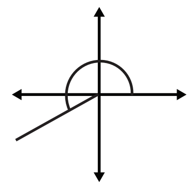


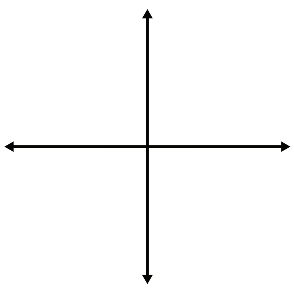
$$210^\circ \times \frac{\pi}{180^\circ} = \frac{7\pi}{6}$$

Example 12 Given the following conditions, find the quadrant(s) where the angle θ could potentially exist.

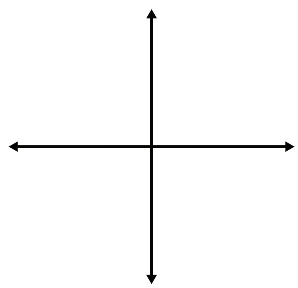
What Quadrant(s) is the Angle in?

a)

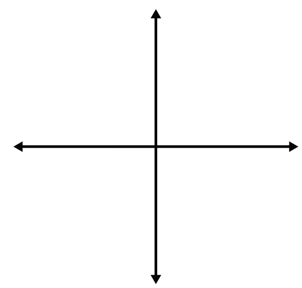
i) $\sin\theta < 0$



ii) $\cos\theta > 0$

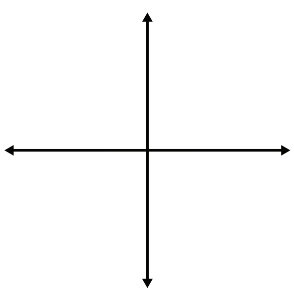


iii) $\tan\theta > 0$

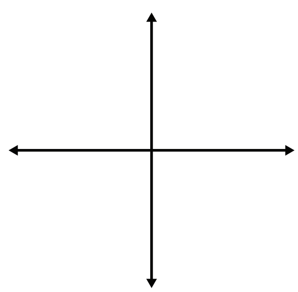


b)

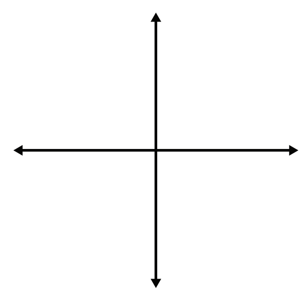
i) $\sin\theta > 0$ and $\cos\theta > 0$



ii) $\sec\theta > 0$ and $\tan\theta < 0$

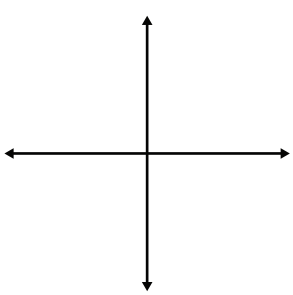


iii) $\csc\theta < 0$ and $\cot\theta > 0$

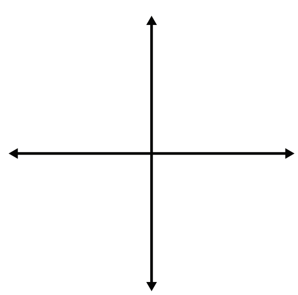


c)

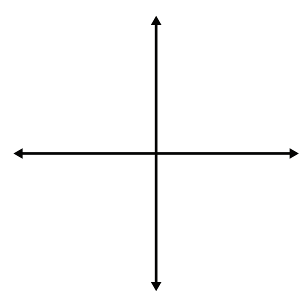
i) $\sin\theta < 0$ and $\csc\theta = \frac{1}{2}$



ii) $\cos\theta = -\frac{\sqrt{3}}{2}$ and $\csc\theta < 0$



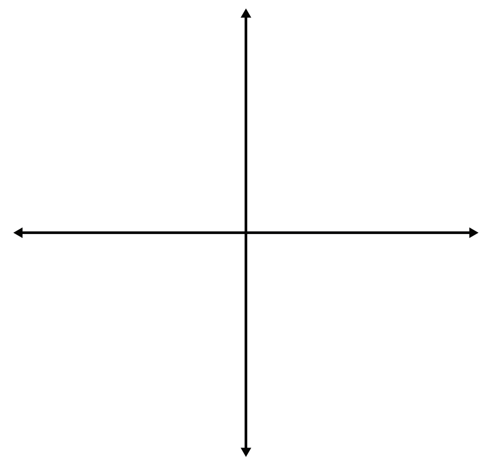
iii) $\sec\theta > 0$ and $\tan\theta = 1$



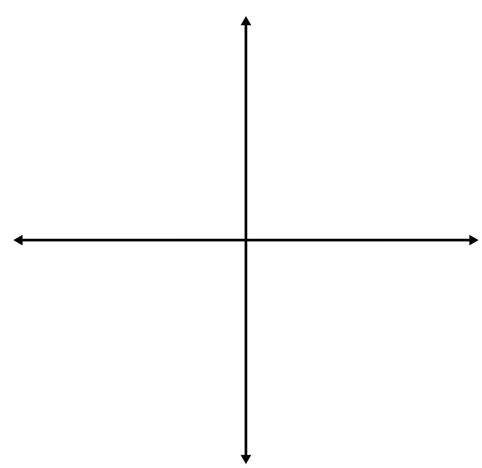
Example 13 Given one trigonometric ratio, find the exact values of the other five trigonometric ratios. State the reference angle and the standard position angle, to the nearest hundredth of a radian.

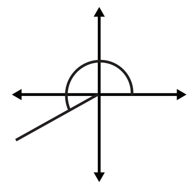
Exact Values of Trigonometric Ratios

a) $\cos\theta = -\frac{12}{13}, \quad \pi \le \theta < \frac{3\pi}{2}$



b) $\csc\theta = \frac{7}{3}, \quad \frac{\pi}{2} \le \theta < \pi$





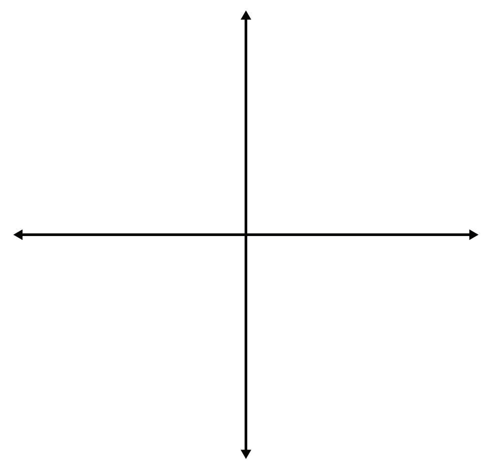
$210^\circ \times \frac{\pi}{180^\circ} = \frac{7\pi}{6}$

Example 14

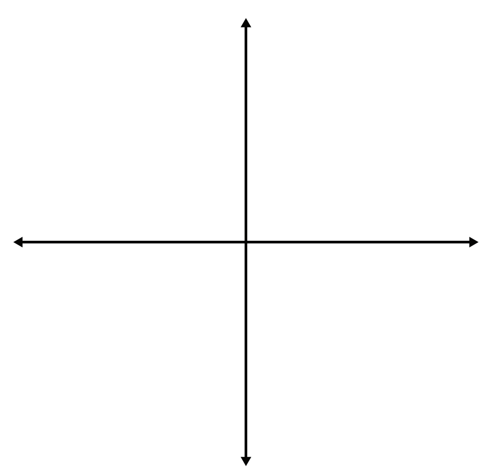
Given one trigonometric ratio, find the exact values of the other five trigonometric ratios. State the reference angle and the standard position angle, to the nearest hundredth of a degree.

Exact Values of Trigonometric Ratios

a) $\sec\theta = \frac{5}{4}$, $\sin\theta < 0$



b) $\tan\theta = -\frac{2}{3}$, $\sec\theta > 0$



Example 15

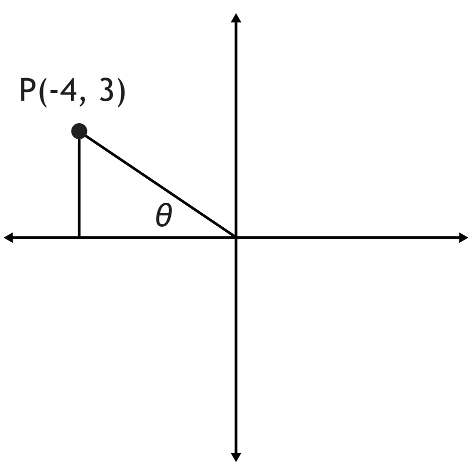
Calculating θ with a calculator.

Calculator Concerns

a) When you solve a trigonometric equation in your calculator, the answer you get for θ can seem unexpected. Complete the following chart to learn how the calculator processes your attempt to solve for θ.

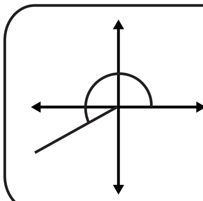
If the angle θ could exist in either quadrant ___ or ___ ...	The calculator always picks quadrant
I or II	
I or III	
I or IV	
II or III	
II or IV	
III or IV	

b) Given the point P(-4, 3), Mark tries to find the reference angle using a sine ratio, Jordan tries to find it using a cosine ratio, and Dylan tries to find it using a tangent ratio. Why does each person get a different result from their calculator?



Mark's Calculation of θ (using sine)	Jordan's Calculation of θ (using cosine)	Dylan's Calculation of θ (using tan)
$\sin\theta = \frac{3}{5}$	$\cos\theta = \frac{-4}{5}$	$\tan\theta = \frac{3}{-4}$
$\theta = 36.87^\circ$	$\theta = 143.13^\circ$	$\theta = -36.87^\circ$

Trigonometry
LESSON ONE - *Degrees and Radians*
Lesson Notes



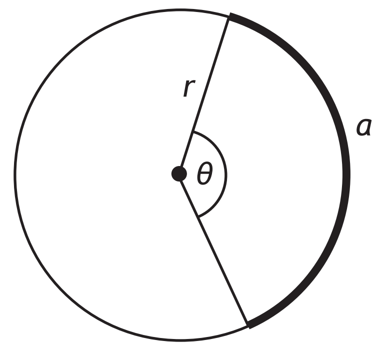
$210^\circ \times \frac{\pi}{180^\circ} = \frac{7\pi}{6}$

Example 16

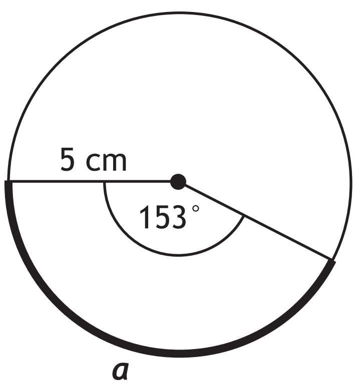
Arc Length

The formula for arc length is $a = r\theta$, where a is the arc length, θ is the central angle in radians, and r is the radius of the circle. The radius and arc length must have the same units.

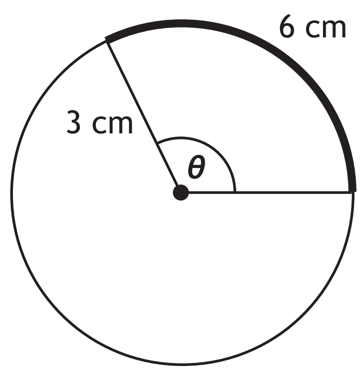
a) Derive the formula for arc length, $a = r\theta$.



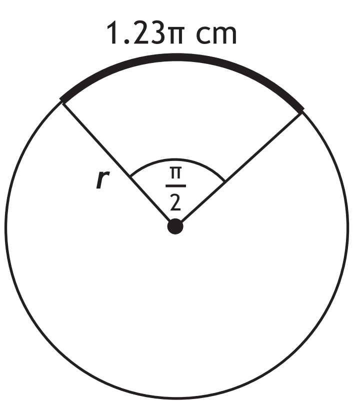
b) Solve for a, to the nearest hundredth.



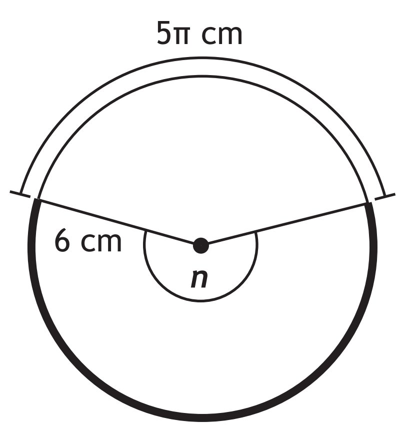
c) Solve for θ. *(express your answer as a degree, to the nearest hundredth.)*



d) Solve for r, to the nearest hundredth.



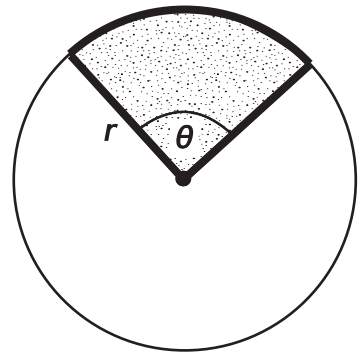
e) Solve for n. *(express your answer as an exact-value radian.)*



Example 17 Area of a circle sector.

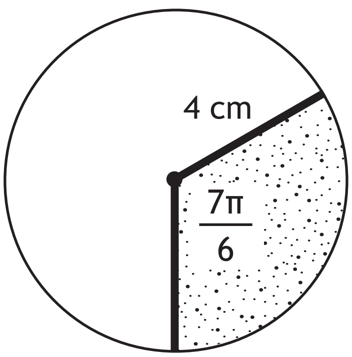
Sector Area

a) Derive the formula for the area of a circle sector, $A = \frac{r^2\theta}{2}$.

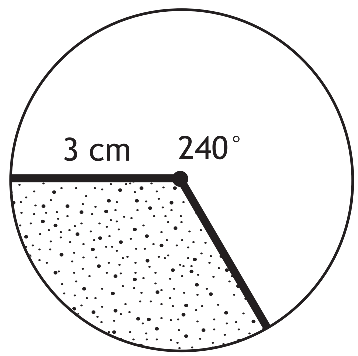


In parts (b - e), find the area of each shaded region.

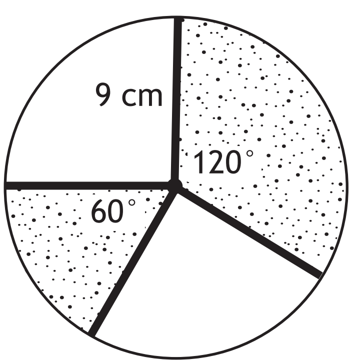
b)



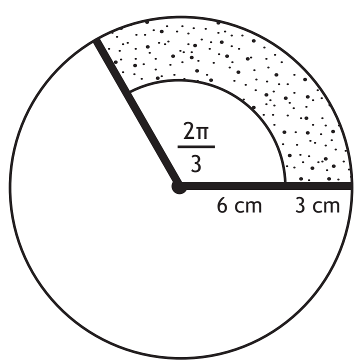
c)

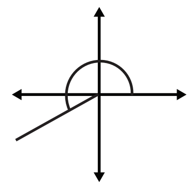


d)



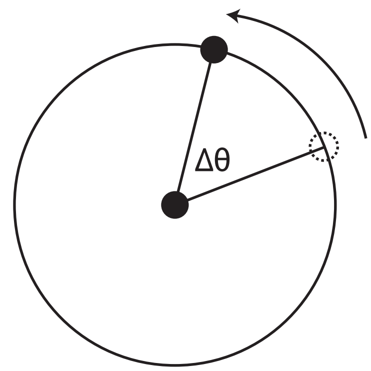
e)





$$210^\circ \times \frac{\pi}{180^\circ} = \frac{7\pi}{6}$$

Example 18



The formula for angular speed is $\omega = \frac{\Delta\theta}{\Delta T}$, where ω (Greek: Omega) is the angular speed, Δθ is the change in angle, and ΔT is the change in time. Calculate the requested quantity in each scenario. Round all decimals to the nearest hundredth.

a) A bicycle wheel makes 100 complete revolutions in 1 minute. Calculate the angular speed in degrees per second.

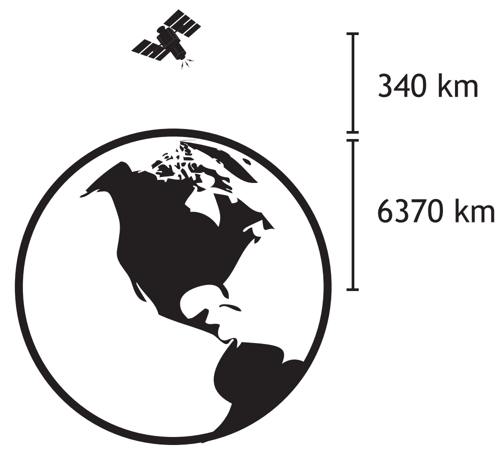
b) A Ferris wheel rotates 1020° in 4.5 minutes. Calculate the angular speed in radians per second.

c) The moon orbits Earth once every 27 days. Calculate the angular speed in revolutions per second. If the average distance from the Earth to the moon is 384 400 km, how far does the moon travel in one second?

d) A cooling fan rotates with an angular speed of 4200 rpm. What is the speed in rps?

e) A bike is ridden at a speed of 20 km/h, and each wheel has a diameter of 68 cm. Calculate the angular speed of one of the bicycle wheels and express the answer using revolutions per second.

Example 19



A satellite orbiting Earth 340 km above the surface makes one complete revolution every 90 minutes. The radius of Earth is approximately 6370 km.

a) Calculate the angular speed of the satellite. Express your answer as an exact value, in radians/second.

b) How many kilometres does the satellite travel in one minute? Round your answer to the nearest hundredth of a kilometre.