Name ______________________ Class ______________ Date __________

13.1 Exponential Growth Functions

Essential Question: How is the graph of $g(x) = ab^{x-h} + k$ where $b > 1$ related to the graph of $f(x) = b^x$?

Resource Locker

Explore 1 Graphing and Analyzing $f(x) = 2^x$ and $f(x) = 10^x$

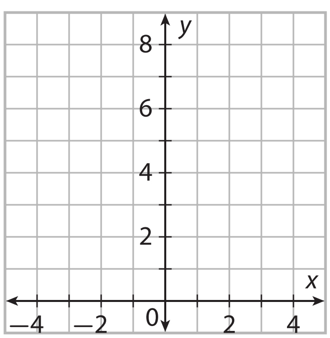
An **exponential function** is a function of the form $f(x) = b^x$, where the base b is a positive constant other than 1 and the exponent x is a variable. Notice that there is no single parent exponential function because each choice of the base b determines a different function.

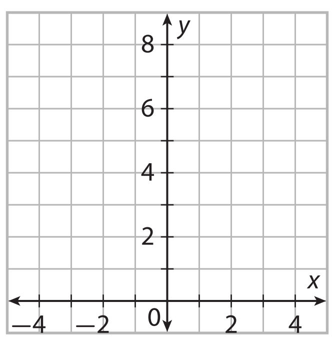
(A) Complete the input-output table for each of the parent exponential functions below.

x	$f(x) = 2^x$
−3	
−2	
−1	
0	
1	
2	
3	

x	$p(x) = 10^x$
−3	
−2	
−1	
0	
1	
2	
3	

(B) Graph the parent functions $f(x) = 2^x$ and $p(x) = 10^x$ by plotting points.





(C) What is the domain of each function?

Domain of $f(x) = 2^x$: $\{x|$ ______ $\}$

Domain of $p(x) = 10^x$: $\{x|$ ______ $\}$

(D) What is the range of each function?

Range of $f(x) = 2^x$: $\{y|$ ______ $\}$

Range of $p(x) = 10^x$: $\{y|$ ______ $\}$

(E) What is the y-intercept of each function?

y-intercept of $f(x) = 2^x$: $\boxed{}$

y-intercept of $p(x) = 10^x$: $\boxed{}$

(F) What is the trend of each function?

In both $f(x) = 2^x$ and $p(x) = 10^x$, as the value of x increases, the value of y increases/decreases.

Reflect

1. Will the domain be the same for every exponential function? Why or why not?

______

______

2. Will the range be the same for every exponential function in the form $f(x) = b^x$, where b is a positive constant? Why or why not?

______

______

______

3. Will the value of the y-intercept be the same for every exponential function? Why or why not?

______

______

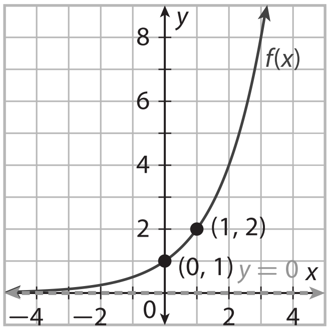
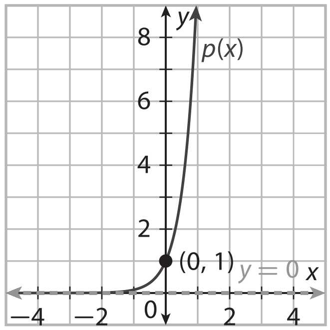
Explain 1 Graphing Combined Transformations of $f(x) = b^x$ Where $b > 1$

A given exponential function $g(x) = a\left(b^{x-h}\right) + k$ with base b can be graphed by recognizing the differences between the given function and its parent function, $f(x) = b^x$. These differences define the parameters of the transformation, where k represents the vertical translation, h is the horizontal translation, and a represents either the vertical stretch or compression of the exponential function and whether it is reflected across the x-axis.

You can use the parameters in $g(x) = a\left(b^x - h\right) + k$ to see what happens to two reference points during a transformation. Two points that are easily visualized on the parent exponential function are $(0, 1)$ and $(1, b)$.

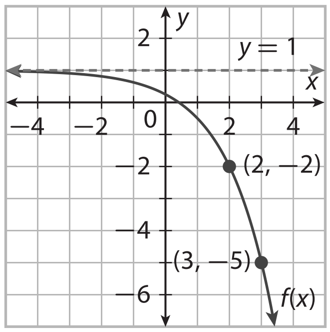
In a transformation, the point $(0, 1)$ becomes $(h, a + k)$ and $(1, b)$ becomes $(1 + h, ab + k)$. The asymptote $y = 0$ for the parent function becomes $y = k$.

The graphs of $f(x) = 2^x$ and $p(x) = 10^x$ are shown below with the reference points and asymptotes labeled.

$f(x) = 2^x$	$p(x) = 10^x$
	

Example 1 **State the domain and range of the given function. Then identify the new values of the reference points and the asymptote. Use these values to graph the function.**

(A) $g(x) = -3\left(2^{x-2}\right) + 1$



The domain of $g(x) = -3\left(2^{x-2}\right) + 1$ is $\left\{x \mid -\infty < x < \infty\right\}$.

The range of $g(x) = -3\left(2^{x-2}\right) + 1$ is $\left\{y \mid y < \mathbf{1}\right\}$.

Examine $g(x)$ and identify the parameters.

$a = -3$, which means that the function is reflected across the x-axis and vertically stretched by a factor of 3.

$h = 2$, so the function is translated 2 units to the right.

$k = 1$, so the function is translated 1 unit up.

The point $(0, 1)$ becomes $(h, a + k)$.

$$
\begin{aligned}
(h, a + k) &= (2, -3 + 1) \\
&= (2, -2)
\end{aligned}
$$

$(1, b)$ becomes $(1 + h, ab + k)$.

$$
\begin{aligned}
(1 + h, ab + k) &= \left(1 + 2, -3(2) + 1\right) \\
&= (3, -6 + 1) \\
&= (3, -5)
\end{aligned}
$$

The asymptote becomes $y = k$.

$y = k \quad \rightarrow \quad y = 1$

Plot the transformed points and asymptote and draw the curve.

(B) $q(x) = 1.5\left(10^{x-3}\right) - 5$

The domain of $q(x) = 1.5\left(10^{x-3}\right) - 5$ is $\left\{x \mid \underline{\qquad\qquad\qquad}\right\}$.

The range of $q(x) = 1.5\left(10^{x-3}\right) - 5$ is $\left\{y \mid \underline{\qquad\qquad}\right\}$.

Examine $q(x)$ and identify the parameters.

$a = \boxed{}$ so the function is stretched vertically by a factor of 1.5.

$h = \boxed{}$ so the function is translated 3 units to the right.

$k = \boxed{}$ so the function is translated 5 units down.

The point $(0,1)$ becomes $(h, a + k)$.

$(h, a + k) = (3, 1.5 - 5) = \underline{\qquad\qquad}$

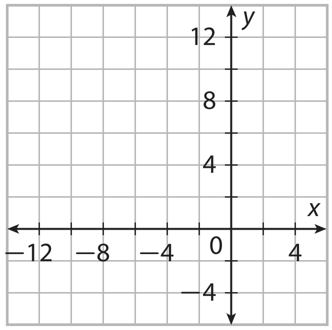
$(1, b)$ becomes $(1 + h, ab + k)$.

$(1 + h, ab + k) = (1 + 3, 1.5(10) - 5) = \underline{\qquad\qquad}$

The asymptote becomes $y = k$.

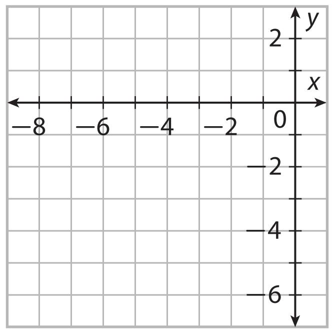
$y = k \quad \rightarrow \quad y = \boxed{}$

Plot the transformed points and asymptote and draw the curve.

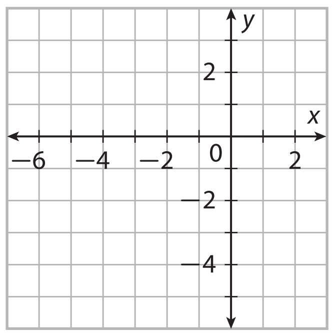


Your Turn

4. $g(x) = 4\left(2^{x+2}\right) - 6$



5. $q(x) = -\frac{3}{5}\left(10^{x+2}\right) + 3$



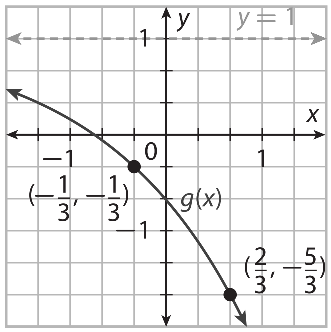
Explain 2 Writing Equations for Combined Transformations of $f(x) = b^x$ Where $b > 1$

Given the graph of an exponential function, you can use your knowledge of the transformation parameters to write the function rule for the graph. Recall that the asymptote will give the value of k and the x-coordinate of the first reference point is h. Then let y_1 be the y-coordinate of the first point and solve the equation $y_1 = a + k$ for a.

Finally, use a, h, and k to write the function in the form $g(x) = a\left(b^{x-h}\right) + k$.

Example 2 **Write the exponential function that will produce the given graph, using the specified value of *b*. Verify that the second reference point is on the graph of the function. Then state the domain and range of the function in set notation.**

Ⓐ Let $b = 2$.



The asymptote is $y = 1$, showing that $k = 1$.

The first reference point is $\left(-\frac{1}{3}, -\frac{1}{3}\right)$. This shows that $h = -\frac{1}{3}$ and that $a + k = -\frac{1}{3}$.

Substitute $k = 1$ and solve for a.

$$a + k = -\frac{1}{3}$$
$$a + 1 = -\frac{1}{3}$$
$$a = -\frac{4}{3}$$
$$h = -\frac{1}{3}$$
$$k = 1$$

Substitute these values into $g(x) = a\left(b^{x-h}\right) + k$ to find $g(x)$.

$$g(x) = a\left(b^{x-h}\right) + k$$
$$= -\frac{4}{3}\left(2^{x+\frac{1}{3}}\right) + 1$$

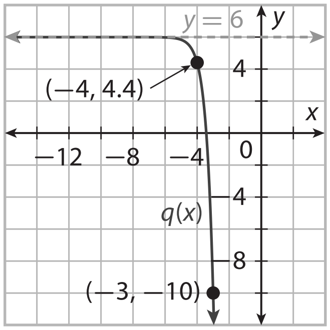
Verify that $g\left(\frac{2}{3}\right) = -\frac{5}{3}$.

$$g\left(\frac{2}{3}\right) = -\frac{4}{3}\left(2^{\frac{2}{3}+\frac{1}{3}}\right) + 1$$
$$= -\frac{4}{3}\left(2^1\right) + 1$$
$$= -\frac{4}{3}(2) + 1$$
$$= \frac{3}{3} - \frac{8}{3}$$
$$= -\frac{5}{3}$$

The domain of $g(x)$ is $\left\{x \mid -\infty < x < +\infty\right\}$.

The range of $g(x)$ is $\left\{y \mid y < 1\right\}$.

Ⓑ Let $b = 10$.



The asymptote is $y = \boxed{}$, showing that $k = \boxed{}$.

The first reference point is $(-4, 4.4)$. This shows that $h = \boxed{}$ and that $a + k = \boxed{}$. Substitute for k and solve for a.

$$a + k = \boxed{}$$

$$a + \boxed{} = \boxed{}$$

$$a = \boxed{} \qquad h = \boxed{} \qquad k = \boxed{}$$

Substitute these values into $q(x) = a\left(b^{x-h}\right) + k$ to find $q(x)$.

$$q(x) = a\left(b^{x-h}\right) + k = \boxed{}\left(10^{x-\boxed{}}\right) + \boxed{}$$

Verify that $q(-3) = -10$.

$$q(-3) = \boxed{}\left(10^{-3-\boxed{}}\right) + \boxed{}$$

$$= \boxed{}\left(10^{\boxed{}}\right) + \boxed{}$$

$$= \boxed{} + \boxed{}$$

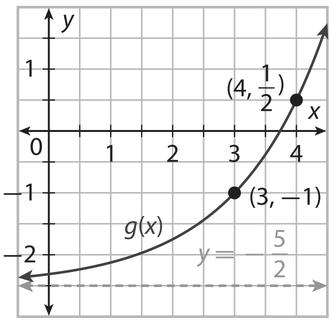
$$= \boxed{}$$

The domain of $q(x)$ is ______________________. The range of $q(x)$ is _________.

Your Turn

Write the exponential function that will produce the given graph, using the specified value of *b*. Verify that the second reference point is on the graph of the function. Then state the domain and range of the function in set notation.

6. $b = 2$

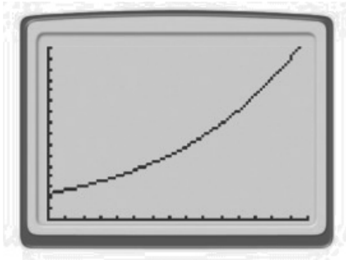


Explain 3 Modeling with Exponential Growth Functions

An **exponential growth function** has the form $f(t) = a(1 + r)^t$ where $a > 0$ and r is a constant percent increase (expressed as a decimal) for each unit increase in time t. That is, since $f(t + 1) = (1 + r) \cdot f(t) = f(t) + r \cdot f(t)$, the value of the function increases by $r \cdot f(t)$ on the interval $[t, t + 1]$. The base $1 + r$ of an exponential growth function is called the **growth factor**, and the constant percent increase r, in decimal form, is called the **growth rate**.

Example 3 **Find the function that corresponds with the given situation. Then use the graph of the function to make a prediction.**

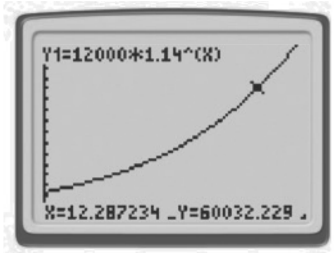
(A) Tony purchased a rare guitar in 2000 for \$12,000. Experts estimate that its value will increase by 14% per year. Use a graph to find the number of years it will take for the value of the guitar to be \$60,000.



Write a function to model the growth in value for the guitar.

$$f(t) = a(1 + r)^t$$
$$= 12,000(1 + 0.14)^t$$
$$= 12,000(1.14)^t$$

Use a graphing calculator to graph the function.

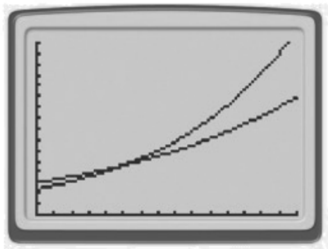


Use the graph to predict when the guitar will be worth \$60,000.

Use the TRACE feature to find the t-value where $f(t) \approx 60,000$.

So, the guitar will be worth \$60,000 approximately 12.29 years after it was purchased.

(B) At the same time that Tony bought the \$12,000 guitar, he also considered buying another rare guitar for \$15,000. Experts estimated that this guitar would increase in value by 9% per year. Determine after how many years the two guitars will be worth the same amount.



Write a function to model the growth in value for the second guitar.

$$g(t) = a(1 + r)^t$$
$$= \boxed{}\left(1 + \boxed{}\right)^t$$
$$= \boxed{}\left(\boxed{}\right)^t$$

Use a graphing calculator to graph the two functions.

Use the graph to predict when the two guitars will be worth the same amount.

Use the intersection feature to find the t-value where $g(t) = \boxed{}$.

So, the two guitars will be worth the same amount ________ years after 2000.

Reflect

7. In part A, find the average rates of change over the intervals $(0, 4)$, $(4, 8)$, and $(8, 12)$. Do the rates increase, decrease, or stay the same?

Your Turn

Find the function that corresponds with the given situation. Then graph the function on a calculator and use the graph to make a prediction.

8. John researches a baseball card and finds that it is currently worth \$3.25. However, it is supposed to increase in value 11% per year. In how many years will the card be worth \$26?

Elaborate

9. How are reference points helpful when graphing transformations of $f(x) = b^x$ or when writing equations for transformed graphs?

10. Give the general form of an exponential growth function and describe its parameters.

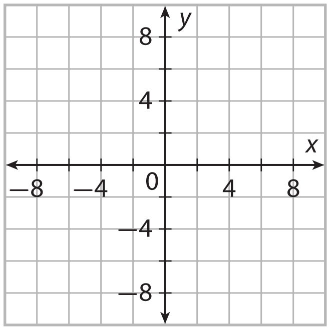
11. **Essential Question Check-In** Which transformations of $f(x) = b^x$ change the function's end behavior? Which transformations change the function's y-intercept?

Evaluate: Homework and Practice

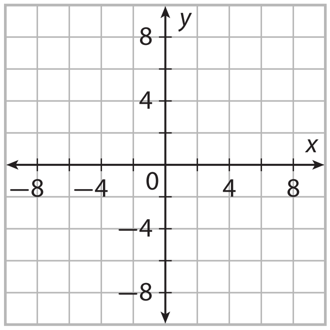
Personal Math Trainer
- Online Homework
- Hints and Help
- Extra Practice

Describe the effect of each transformation on the parent function. Graph the parent function and its transformation. Then determine the domain, range, and y-intercept of each function.

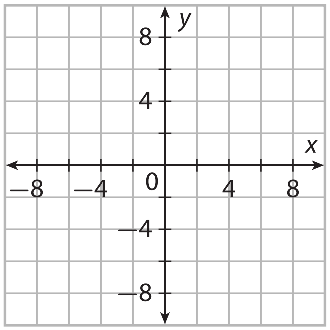
1. $f(x) = 2^x$ and $g(x) = 2(2^x)$



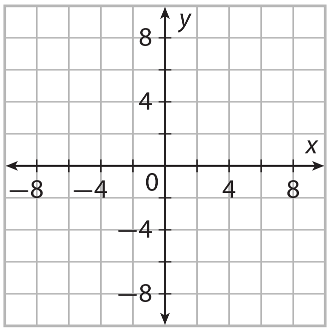
2. $f(x) = 2^x$ and $g(x) = -5(2^x)$



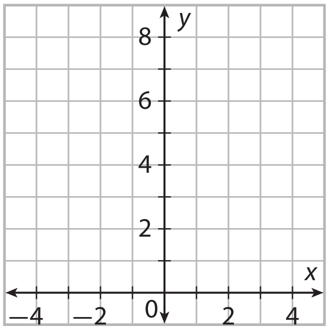
3. $f(x) = 2^x$ and $g(x) = 2^{x+2}$



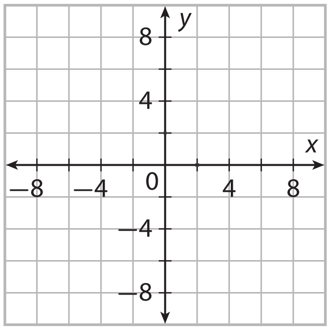
4. $f(x) = 2^x$ and $g(x) = 2^x + 5$



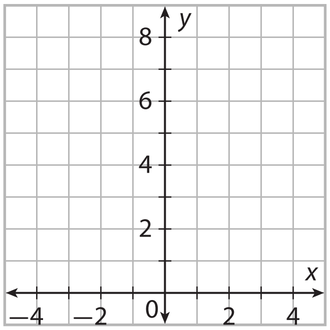
5. $f(x) = 10^x$ and $g(x) = 2(10^x)$



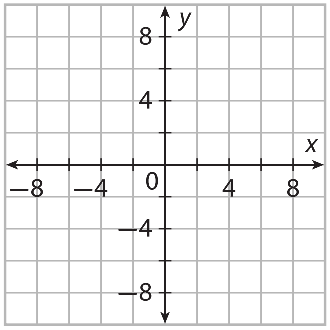
6. $f(x) = 10^x$ and $g(x) = -4(10^x)$



7. $f(x) = 10^x$ and $g(x) = 10^{x-2}$



8. $f(x) = 10^x$ and $g(x) = 10^x - 6$

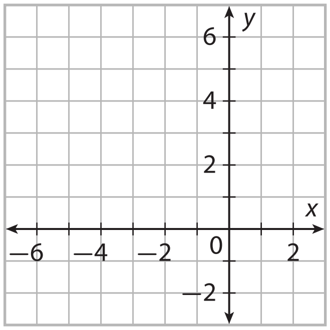


9. Describe the graph of $g(x) = 2^{(x-3)} - 4$ in terms of $f(x) = 2^x$.

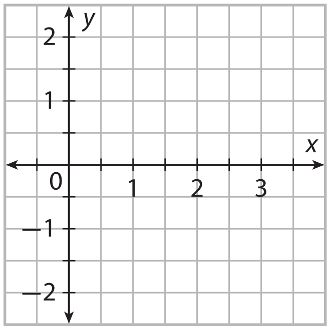
10. Describe the graph of $g(x) = 10^{(x+7)} + 6$ in terms of $f(x) = 10^x$.

State the domain and range of the given function. Then identify the new values of the reference points and the asymptote. Use these values to graph the function.

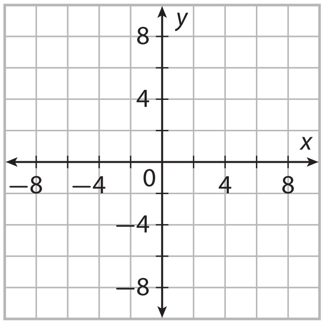
11. $h(x) = 2\left(3^{x+2}\right) - 1$



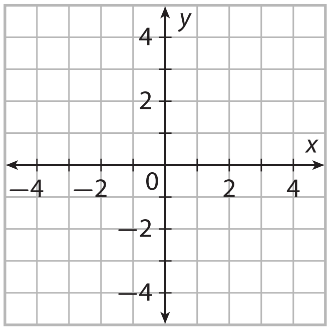
12. $k(x) = -0.5\left(4^{x-1}\right) + 2$



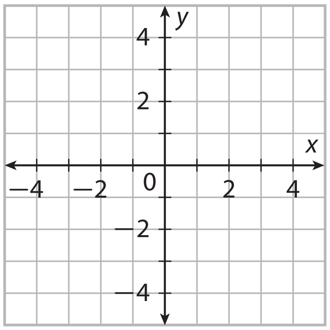
13. $f(x) = 3\left(6^{x-7}\right) - 8$



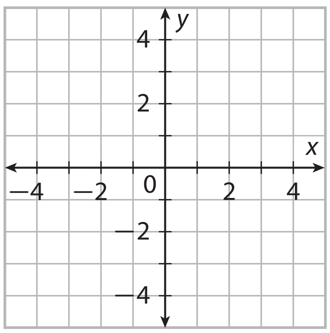
14. $f(x) = -3\left(2^{x+1}\right) + 3$



15. $h(x) = -\frac{1}{4}\left(5^{x+1}\right) - \frac{3}{4}$

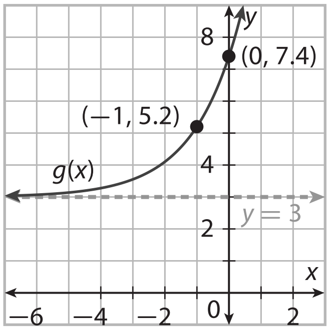


16. $p(x) = 2\left(4^{x-3}\right) - 5$

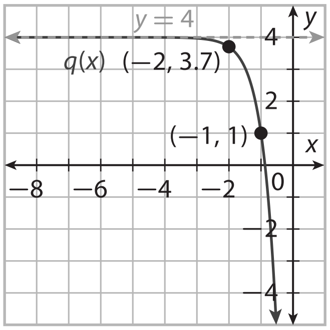


Write the exponential function that will produce the given graph, using the specified value of *b*. Verify that the second reference point is on the graph of the function. Then state the domain and range of the function in set notation.

17. $b = 2$



18. $b = 10$



Find the function that corresponds with the given situation. Then graph the function on a calculator and use the graph to make a prediction.

19. A certain stock opens with a price of \$0.59. Over the first three days, the value of the stock increases on average by 50% per day. If this trend continues, how many days will it take for the stock to be worth \$6?

20. Sue has a lamp from her great-grandmother. She has it appraised and finds it is worth \$1000. She wants to sell it, but the appraiser tells her that the value is appreciating by 8% per year. In how many years will the value of the lamp be \$2000?

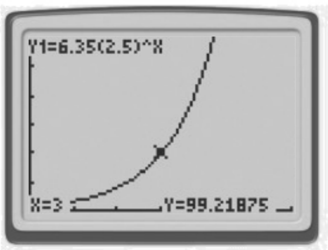
21. The population of a small town is 15,000. If the population is growing by 5% per year, how long will it take for the population to reach 25,000?

22. Bill invests \$3000 in a bond fund with an interest rate of 9% per year. If Bill does not withdraw any of the money, in how many years will his bond fund be worth \$5000?

H.O.T. Focus on Higher Order Thinking

23. Analyze Relationships Compare the end behavior of $g(x) = 2^x$ and $f(x) = x^2$. How are the graphs of the functions similar? How are they different?

24. Explain the Error A student has a baseball card that is worth \$6.35. He looks up the appreciation rate and finds it to be 2.5% per year. He wants to find how much it will be worth after 3 years. He writes the function $f(t) = 6.35(2.5)^t$ and uses the graph of that function to find the value of the card in 3 years.



According to his graph, his card will be worth about \$99.22 in 3 years. What did the student do wrong? What is the correct answer?

Lesson Performance Task

Like all collectables, the price of an item is determined by what the buyer is willing to pay and the seller is willing to accept. The estimated value of a 1948 Tucker 48 automobile in excellent condition has risen at an approximately exponential rate from about \$500,000 in December 2006 to about \$1,400,000 in December 2013.

a. Find an equation in the form $V(t) = V_0\left(1 + r\right)^t$, where V_0 is the value of the car in dollars in December 2006, r is the average annual growth rate, t is the time in years since December 2006, and $V(t)$ is the value of the car in dollars at time t. (Hint: Substitute the known values and solve for r.)

b. What is the meaning of the value of r?

c. If this trend continues, what would be the value of the car in December 2017?