Name ______________________ Class ____________ Date ________

11.4 AA Similarity of Triangles

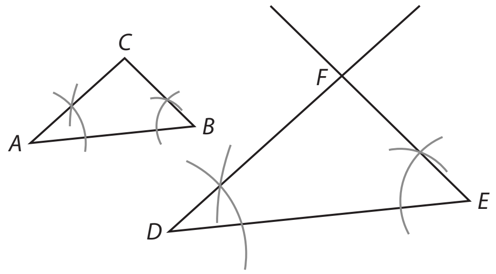
Essential Question: How can you show that two triangles are similar?

Resource Locker

Explore Exploring Angle-Angle Similarity for Triangles

Two triangles are similar when their corresponding sides are proportional and their corresponding angles are congruent. There are several shortcuts for proving triangles are similar.

(A) Draw a triangle and label it $\triangle ABC$. Elsewhere on your page, draw a segment longer than $\overline{AB}$ and label the endpoints D and E.



(B) Copy $\angle CAB$ and $\angle ABC$ to points D and E, respectively. Extend the rays of your copied angles, if necessary, and label their intersection point F. You have constructed $\triangle DEF$.

(C) You constructed angles D and E to be congruent to angles A and B, respectively. Therefore, angles C and F must also be __________ because of the __________ Theorem.

(D) Check the proportionality of the corresponding sides.

$$\frac{AB}{DE} = \frac{\square}{\square} = \square \qquad \frac{AC}{DF} = \frac{\square}{\square} = \square \qquad \frac{BC}{EF} = \frac{\square}{\square} = \square$$

Since the ratios are ________ the sides of the triangles are __________.

Reflect

1. **Discussion** Compare your results with your classmates. What conjecture can you make about two triangles that have two corresponding congruent angles?

Explain 1 Proving Angle-Angle Triangle Similarity

The Explore suggests the following theorem for determining whether two triangles are similar.

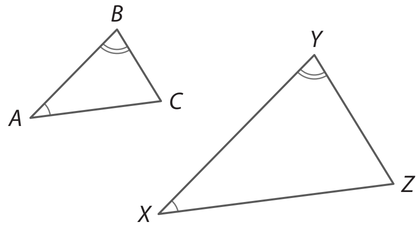
Angle-Angle (AA) Triangle Similarity Theorem

If two angles of one triangle are congruent to two angles of another triangle, then the two triangles are similar.

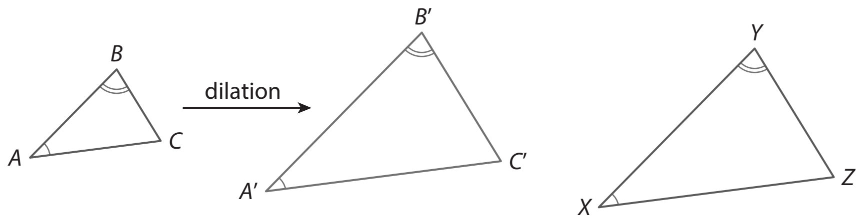
Example 1 **Prove the Angle-Angle Triangle Similarity Theorem.**

Given: $\angle A \cong \angle X$ and $\angle B \cong \angle Y$

Prove: $\triangle ABC \sim \triangle XYZ$



(1) Apply a dilation to $\triangle ABC$ with scale factor $k = \frac{XY}{AB}$. Let the image of $\triangle ABC$ be $\triangle A'B'C$.



$\triangle A'B'C$ is similar to $\triangle ABC$, and $\angle A' \cong$ ______ and $\angle B' \cong$ ______

because ______________________________.

Also, $A'B' = k \cdot AB =$ ______________.

(2) It is given that $\angle A \cong \angle X$ and $\angle B \cong \angle Y$

By the Transitive Property of Congruence, $\angle A' \cong$ ______ and $\angle B' \cong$ ______.

So, $\triangle A'B'C' \cong \triangle XYZ$ by ______________________________.

This means there is a sequence of rigid motions that maps $\triangle A'B'C'$ *to* $\triangle YYZ$.

The dilation followed by this sequence of rigid motions shows that there is a sequence of similarity transformations that maps $\triangle ABC$ to $\triangle XYZ$. Therefore, $\triangle ABC \sim \triangle XYZ$.

Reflect

2. Discussion Compare and contrast the AA Similarity Theorem with the ASA Congruence Theorem.

3. In $\triangle JKL$, $m\angle J = 40°$ and $m\angle K = 55°$. In $\triangle MNP$, $m\angle M = 40°$ and $m\angle P = 85°$. A student concludes that the triangles are not similar. Do you agree or disagree? Why?

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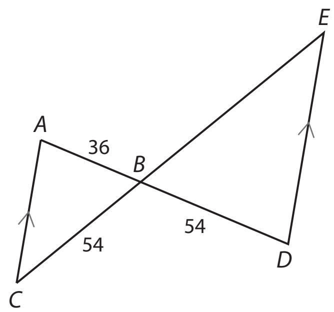
Explain 2 Applying Angle-Angle Similarity

Architects and contractors use the properties of similar figures to find any unknown dimensions, like the proper height of a triangular roof. They can use a bevel angle tool to check that the angles of construction are congruent to the angles in their plans.

Example 2 **Find the indicated length, if possible.**

(A) *BE*



First, determine whether $\triangle ABC \sim \triangle DBE$.

By the Alternate Interior Angles Theorem, $\angle A \cong \angle D$ and $\angle C \cong \angle E$, so $\triangle ABC \sim \triangle DBE$ by the AA Triangle Similarity Theorem.

Find *BE* by solving a proportion.

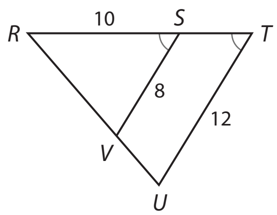
$$\frac{BD}{BA} = \frac{BE}{BC}$$

$$\frac{54}{36} = \frac{BE}{54}$$

$$\frac{54}{36} \cdot 54 = \frac{BE}{54} \cdot 54$$

$$BE = 81$$

(B) *RT*



Check whether $\triangle RSV \sim \triangle RTU$:

It is given in the diagram that $\angle$ ☐ $\cong \angle$ ☐. $\angle R$ is shared by both triangles,

so $\angle R \cong \angle R$ by the __________ Property of Congruence.

So, by the ______________________________, $\triangle RST \sim \triangle RTU$.

Find *RT* by solving a proportion.

$$\frac{RT}{RS} = \frac{TU}{SV}$$

$$\frac{RT}{\square} = \frac{\square}{\square}$$

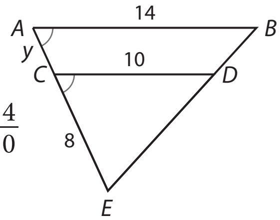
$$RT = \square$$

Reflect

4. In Example 2*A*, is there another way you can set up the proportion to solve for *BE*?

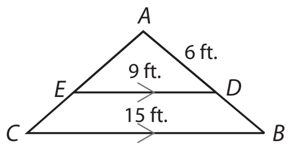
5. **Discussion** When asked to solve for *y*, a student sets up the proportion as shown. Explain why the proportion is wrong. How should you adjust the proportion so that it will give the correct result?

$$\frac{y}{8} = \frac{14}{10}$$

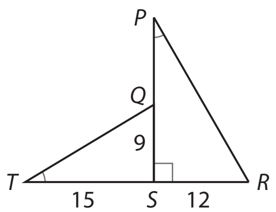


Your Turn

6. A builder was given a design plan for a triangular roof as shown. Explain how he knows that $\triangle AED \sim \triangle ACB$. Then find *AB*.



7. Find *PQ*, if possible.



Explain 3 Applying SSS and SAS Triangle Similarity

In addition to Angle-Angle Triangle Similarity, there are two additional shortcuts for proving two triangles are similar.

Side-Side-Side (SSS) Triangle Similarity Theorem

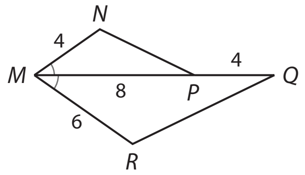
If the three sides of one triangle are proportional to the corresponding sides of another triangle, then the triangles are similar.

Side-Angle-Side (SAS) Triangle Similarity Theorem

If two sides of one triangle are proportional to the corresponding sides of another triangle and their included angles are congruent, then the triangles are similar.

Example 3 **Determine whether the given triangles are similar. Justify your answer.**

(A)



You are given two pairs of corresponding side lengths and one pair of congruent corresponding angles, so try using SAS.

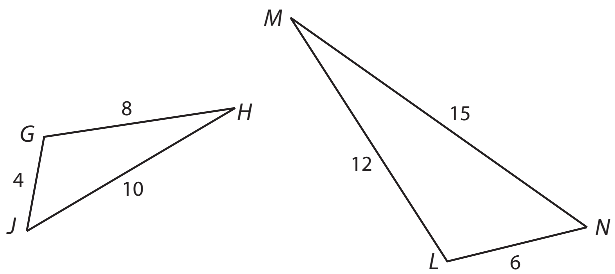
Check that the ratios of corresponding sides are equal.

$\frac{MN}{MR} = \frac{4}{6} = \frac{2}{3}$ $\qquad$ $\frac{MP}{MQ} = \frac{8}{8+4} = \frac{8}{12} = \frac{2}{3}$

Check that the included angles are congruent: $\angle NMP \cong \angle QMR$ is given in the diagram.

Therefore $\triangle NMP \sim \triangle RMQ$ by the SAS Triangle Similarity Theorem.

(B)



You are given ______ pairs of corresponding side lengths and ______ congruent corresponding angles, so try using ____________________.

Check that the ratios of corresponding sides are equal.

$\frac{LM}{GH} = \frac{\square}{\square} = \frac{\square}{\square}$ $\qquad$ $\frac{MN}{} = \frac{\square}{\square} = \frac{\square}{\square}$ $\qquad$ $\frac{}{} = \frac{\square}{\square} = \frac{\square}{\square}$

Therefore $\triangle$ $\square$ $\sim \triangle$ $\square$ by ____________________.

Since you are given all three pairs of sides, you don't need to check for congruent angles.

Reflect

8. Are all isosceles right triangles similar? Explain why or why not.

____

____

____

9. Why isn't Angle-Side-Angle (ASA) used to prove two triangles similar?

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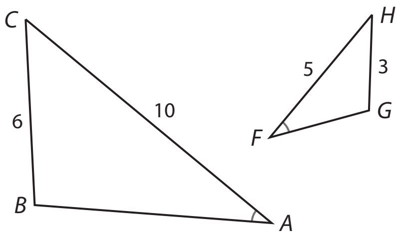
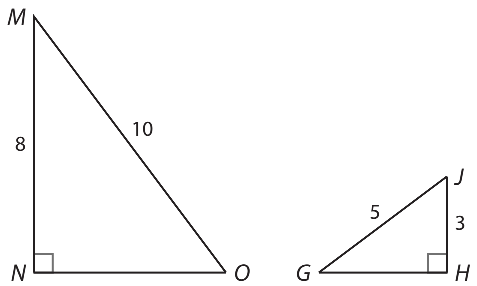
Your Turn

If possible, determine whether the given triangles are similar. Justify your answer.

10.



11.



Elaborate

12. Is triangle similarity transitive? If you know $\triangle ABC \sim \triangle DEF$ and $\triangle DEF \sim \triangle GHJ$, is $\triangle ABC \sim \triangle GHJ$? Explain.

13. The AA Similarity Theorem applies to triangles. Is there an AAA Similarity Theorem for quadrilaterals? Use your geometry software to test your conjecture or create a counterexample.

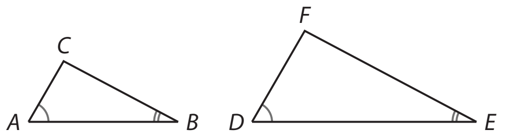
14. **Essential Question Check-In** How can you prove triangles are similar?

Evaluate: Homework and Practice

Personal Math Trainer
- Online Homework
- Hints and Help
- Extra Practice

Show that the triangles are similar by measuring the lengths of their sides and comparing the ratios of the corresponding sides.

1.

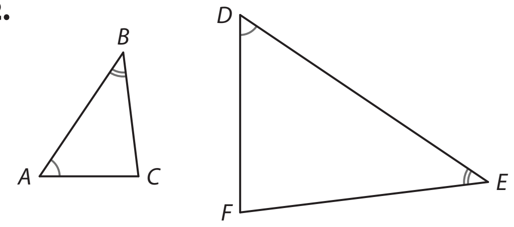


$\frac{DE}{AB} = \frac{\square}{\square} = \square$

$\frac{DF}{AC} = \frac{\square}{\square} = \square$

$\frac{EF}{BC} = \frac{\square}{\square} = \square$

2.



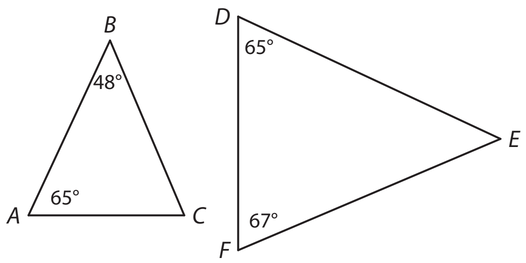
$\frac{AB}{DE} = \frac{\square}{\square} = \square$

$\frac{AC}{DF} = \frac{\square}{\square} = \square$

$\frac{BC}{EF} = \frac{\square}{\square} = \square$

Determine whether the two triangles are similar. If they are similar, write the similarity statement.

3.



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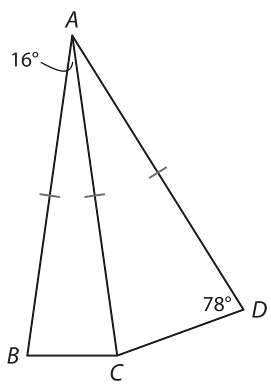
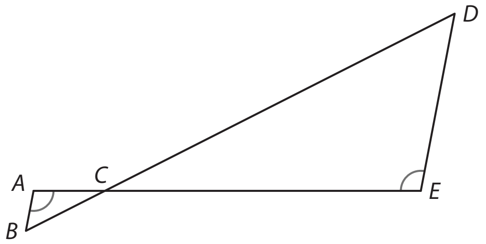
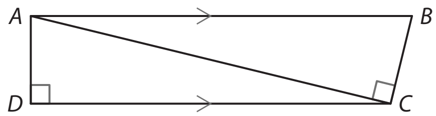


Determine whether the two triangles are similar. If they are similar, write the similarity statement.

5.

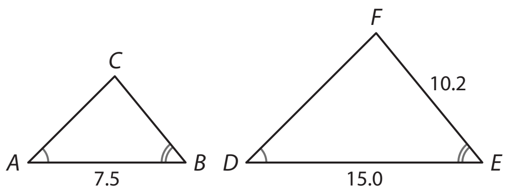


6.

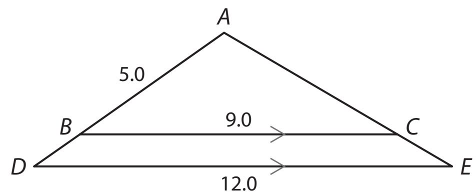


Explain how you know whether the triangles are similar. If possible, find the indicated length.

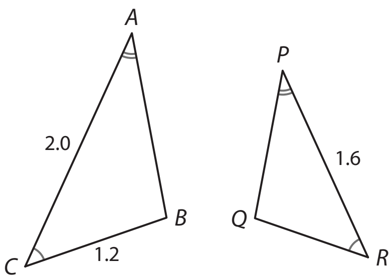
7. *AC*



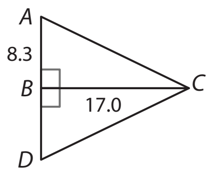
8. *AD*



9. *QR*

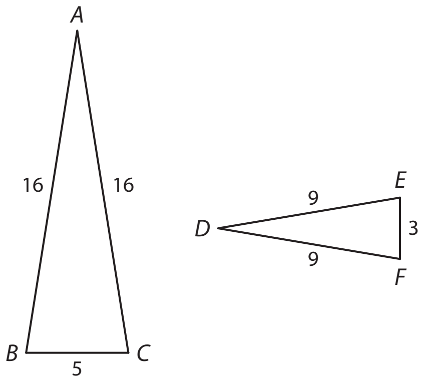


10. Find *BD*.

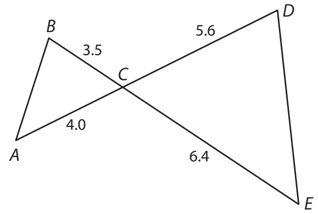


Show whether or not each pair of triangles are similar, if possible. Justify your answer, and write a similarity statement when the triangles are similar.

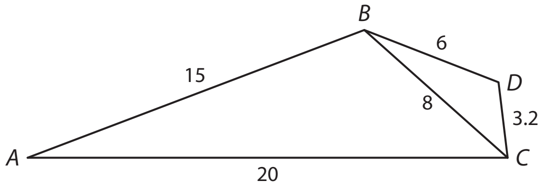
11.



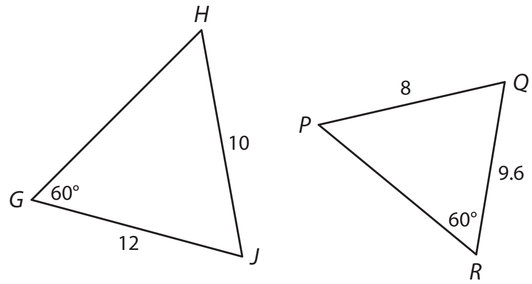
12.



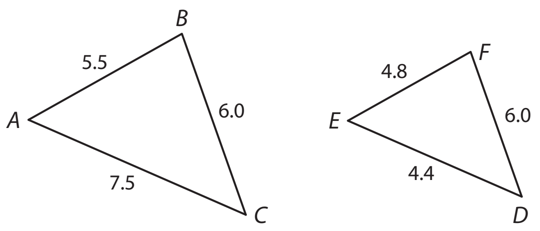
13.



14.



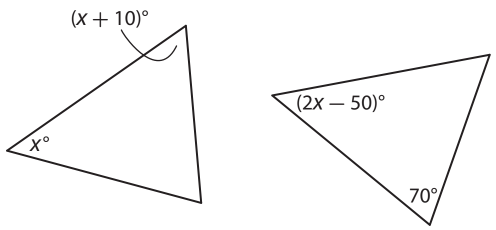
15. Explain the Error A student analyzes the two triangles shown below. Explain the error that the student makes.



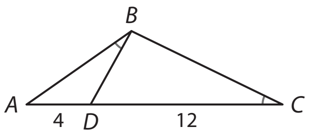
$\frac{AB}{EF} = \frac{5.5}{4.8} = 1.15$, and $\frac{BC}{DF} = \frac{6}{6} = 1$

Because the two ratios are not equal, the two triangles are not similar.

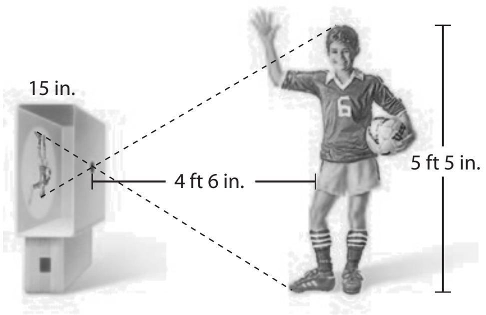
16. Algebra Find all possible values of x for which these two triangles are similar.



17. Multi-Step Identify two similar triangles in the figure, and explain why they are similar. Then find AB.



18. The picture shows a person taking a pinhole photograph of himself. Light entering the opening reflects his image on the wall, forming similar triangles. What is the height of the image to the nearest inch?

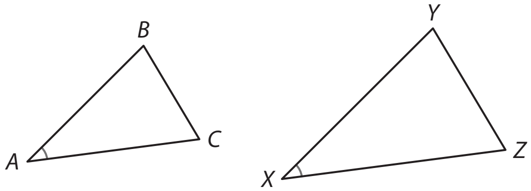


H.O.T. Focus on Higher Order Thinking

19. Analyze Relationships Prove the SAS Triangle Similarity Theorem.

Given: $\frac{XY}{AB} = \frac{XZ}{AC}$ **and** $\angle A \cong \angle X$

Prove: $\triangle ABC \sim \triangle XYZ$

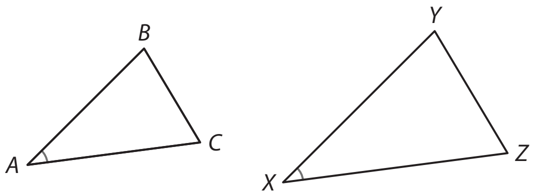


20. Analyze Relationships Prove the SSS Triangle Similarity Theorem.

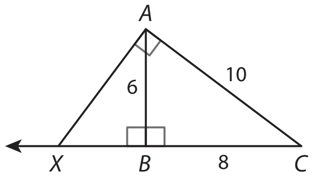
Given: $\frac{XY}{AB} = \frac{XZ}{AC} = \frac{YZ}{BC}$

Prove: $\triangle ABC \sim \triangle XYZ$

(Hint: The main steps of the proof are similar to those of the proof of the AA Triangle Similarity Theorem.)



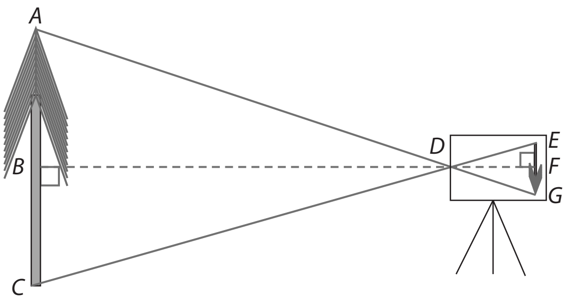
21. Communicate Mathematical Ideas A student is asked to find point X on $\overleftrightarrow{BC}$ such that $\triangle ABC \sim \triangle XBA$ and XB is as small as possible. The student does so by constructing a perpendicular line to $\overleftrightarrow{AC}$ at point A, and then labeling X as the intersection of the perpendicular line with $\overleftrightarrow{BC}$. Explain why this procedure generates the similar triangle that the student was requested to construct.



22. Make a Conjecture Builders and architects use scale models to help them design and build new buildings. An architecture student builds a model of an office building in which the height of the model is $\frac{1}{400}$ of the height of the actual building, while the width and length of the model are each $\frac{1}{200}$ of the corresponding dimensions of the actual building. The model includes several triangles. Describe how a triangle in this model could be similar to the corresponding triangle in the actual building, then describe how a triangle in this model might not be similar to the corresponding triangle in the actual building. Use a similarity theorem to support each answer.

Lesson Performance Task

The figure shows a camera obscura and the object being "photographed." Answer the following questions about the figure:



1. Explain how the image of the object would be affected if the camera were moved closer to the object. How would that limit the height of objects that could be photographed?

2. How do you know that $\triangle ADC$ is similar to $\triangle GDE$?

3. Write a proportion you could use to find the height of the pine tree.

4. $DF = 12$ in., $EG = 8$ in., $BD = 96$ ft. How tall is the pine tree?