Name ______________________ Class ______________ Date ________

11.1 Dilations

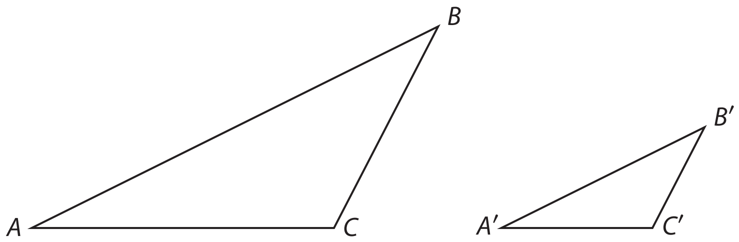
Essential Question: How does a dilation transform a figure?

Resource Locker

Explore 1 Investigating Properties of Dilations

A **dilation** is a transformation that can change the size of a polygon but leaves the shape unchanged. A dilation has a *center of dilation* and a *scale factor* which together determine the position and size of the image of a figure after the dilation.

Use $\triangle ABC$ and its image $\triangle A'B'C'$ after a dilation to answer the following questions.



(A) Use a ruler to measure the following lengths. Measure to the nearest tenth of a centimeter.

$AB =$ ☐ cm $\quad A'B' =$ ☐ cm

$AC =$ ☐ cm $\quad A'C' =$ ☐ cm

$BC =$ ☐ cm $\quad B'C' =$ ☐ cm

(B) Use a protractor to measure the corresponding angles.

$m\angle A =$ ☐ $\quad m\angle A' =$ ☐

$m\angle B =$ ☐ $\quad m\angle B' =$ ☐

$m\angle C =$ ☐ $\quad m\angle C' =$ ☐

(C) Complete the following ratios

$\frac{A'B'}{AB} = \frac{\square}{\square} = \square \qquad \frac{A'C'}{AC} = \frac{\square}{\square} = \square \qquad \frac{B'C'}{BC} = \frac{\square}{\square} = \square$

Reflect

1. What do you notice about the corresponding sides of the figures? What do you notice about the corresponding angles?

2. **Discussion** What similarities are there between reflections, translations, rotations, and dilations? What is the difference?

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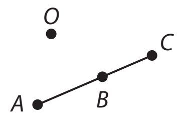
Explore 2 Dilating a Line Segment

The dilation of a line segment (the pre-image) is a line segment whose length is the product of the scale factor and the length of the pre-image.

Use the following steps to apply a dilation by a factor of 3, with center at the point O, to $\overleftrightarrow{AC}$.



(A) To locate the point A', draw a ray from O through A. Place A' on this ray so that the distance from O to A' is three times the distance from O to A.

(B) To locate point B', draw a ray from O through B. Place B' on this ray so that the distance from O to B' is three times the distance from O to B.

(C) To locate point C', draw a ray from O through C. Place C' on this ray so that the distance from O to C' is three times the distance from O to C.

(D) Draw a line through A', B', and C'.

(E) Measure $\overline{AB}$, $\overline{AC}$, and $\overline{BC}$. Measure $\overline{A'B'}$, $\overline{A'C'}$, and $\overline{B'C'}$. Make a conjecture about the lengths of segments that have been dilated.

Reflect

3. Make a conjecture about the length of the image of a 4 cm segment after a dilation with scale factor k. Can the image ever be shorter than the preimage?

4. What can you say about the image of a segment under a dilation? Does your answer depend upon the location of the segment? Explain

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Explain 1 Applying Properties of Dilations

The **center of dilation** is the fixed point about which all other points are transformed by a dilation. The ratio of the lengths of corresponding sides in the image and the preimage is called the **scale factor**.

Properties of Dilations

- Dilations preserve angle measure.
- Dilations preserve betweenness.
- Dilations preserve collinearity.
- Dilations preserve orientation.
- Dilations map a line segment (the pre-image) to another line segment whose length is the product of the scale factor and the length of the pre-image.
- Dilations map a line not passing through the center of dilation to a parallel line and leave a line passing through the center unchanged.

Example 1 **Determine if the transformation on the coordinate plane is a dilation. If it is, give the scale factor.**

(A) Preserves angle measure: yes

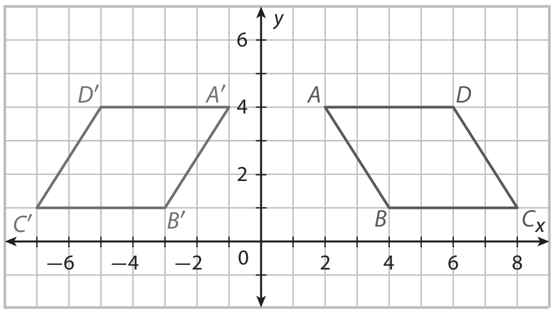
Preserves betweenness: yes

Preserves collinearity: yes

Preserves orientation: no

Ratio of corresponding sides: 1 : 1

Is this transformation a dilation? No, it does not preserve orientation.



(B) Preserves angle measure (Y/N) ______________

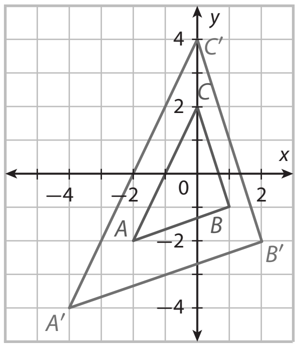
Preserves betweenness (Y/N) ______________

Preserves collinearity (Y/N) ______________

Preserves orientation (Y/N) ______________

Scale Factor ______________

Is this transformation a dilation? ______________



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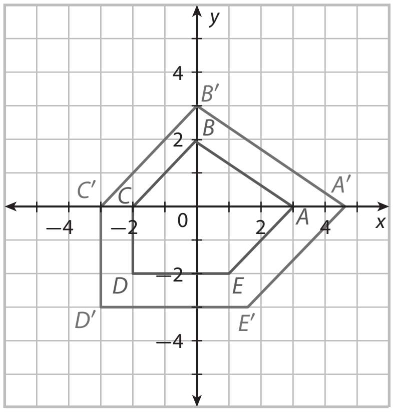
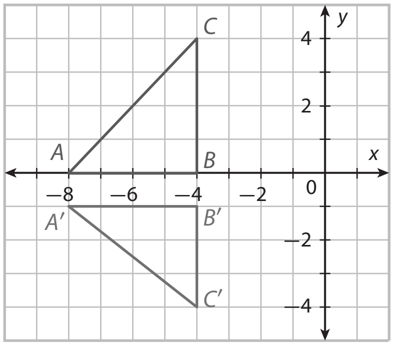
Your Turn

Determine if the transformations are dilations.

5.



6.



Explain 2 Determining the Center and Scale of a Dilation

When you have a figure and its image after dilation, you can find the center of dilation by drawing lines that connect corresponding vertices. These lines will intersect at the center of dilation.

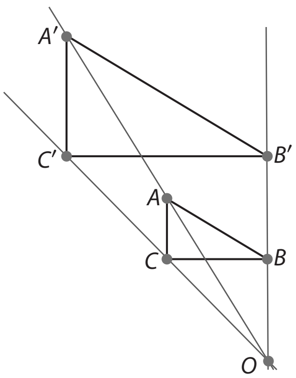
Example 2 **Determine the center of dilation and the scale factor of the dilation of the triangles.**

Ⓐ Draw $\overleftrightarrow{AA'}$, $\overleftrightarrow{BB'}$, and $\overleftrightarrow{CC'}$. The point where the lines cross is the center of dilation. Label the intersection *O*. Measure to find the scale factor.

$OA = 25$ mm $\quad OB = 13$ mm $\quad OC = 19$ mm

$OA' = 50$ mm $\quad OB' = 26$ mm $\quad OC' = 38$ mm

The scale factor is 2 to 1.



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Ⓑ Draw $\overleftrightarrow{AA'}$, $\overleftrightarrow{BB'}$, and $\overleftrightarrow{CC'}$. Measure from each point to the intersection O to the nearest millimeter.

$OA =$ ________

$OA' =$ ________

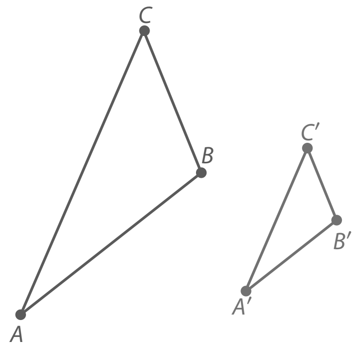
$OB =$ ________

$OB' =$ ________

$OC =$ ________

$OC' =$ ________

The scale factor is ________.

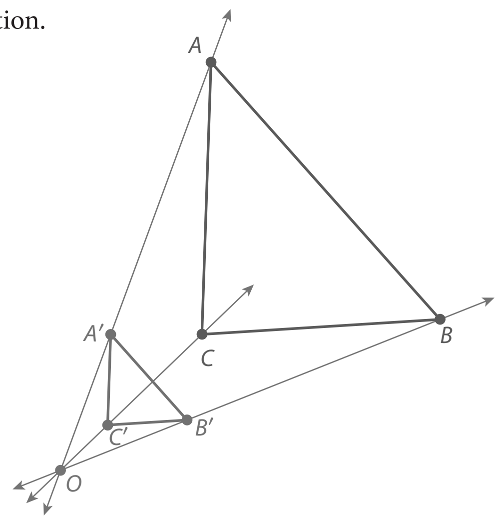


Reflect

7. For the dilation in Your Turn 5, what is the center of dilation? Explain how you can tell without drawing lines.

Your Turn

8. Determine the center of dilation and the scale factor of the dilation.



$OA' =$ ________ cm, $OA =$ ________

The scale factor of the dilation is ________.

Elaborate

9. How is the length of the image of a line segment under a dilation related to the length of its preimage?

10. **Discussion** What is the result of dilating a figure using a scale factor of 1? For this dilation, does the center of dilation affect the position of the image relative to the preimage? Explain.

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11. **Essential Question Check-In** In general how does a dilation transform a figure?

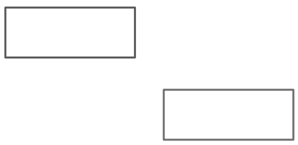
Evaluate: Homework and Practice

- Online Homework
- Hints and Help
- Extra Practice

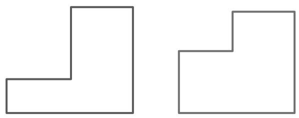
1. Consider the definition of a dilation. A dilation is a transformation that can change the size of a polygon but leaves the shape unchanged. In a dilation, how are the ratios of the measures of the corresponding sides related?

Tell whether one figure appears to be a dilation of the other figure Explain.

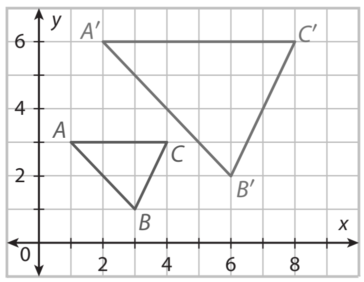
2.



3.



4. Is the scale factor of the dilation of $\triangle ABC$ equal to $\frac{1}{2}$? Explain.



5. Square A is a dilation of square B. What is the scale factor?

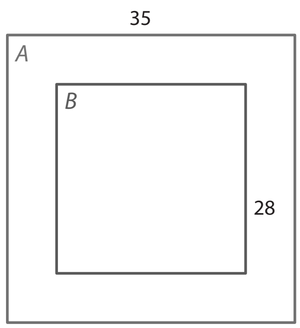
a. $\frac{1}{7}$

b. $\frac{4}{5}$

c. $\frac{5}{4}$

d. 7

e. $\frac{25}{16}$



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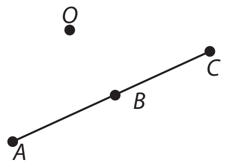
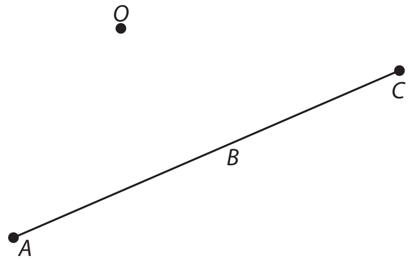
6. Apply a dilation to $\overline{AC}$ with a scale factor of 2 and center at the point *O*.

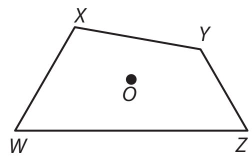


7. Apply a dilation to $\overline{AC}$ with a scale factor of $\frac{1}{3}$ and center at the point *O*.

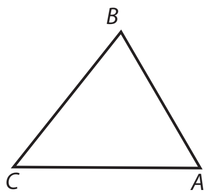


8. What happens when a triangle is dilated using one of the vertices as the center of dilation?

9. Draw an image of *WXYZ*. The center of the dilation is *O*, and the scale factor is 2.



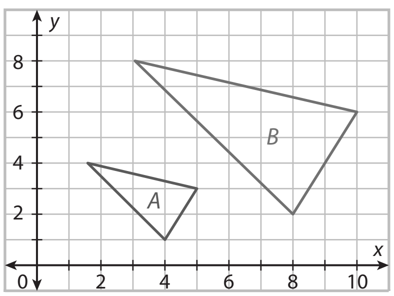
10. Draw an image of $\triangle ABC$. The center of dilation is *C*, and the scale factor is 1.5.



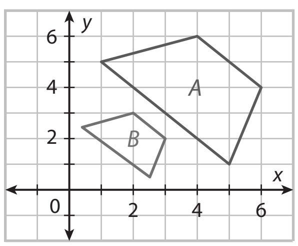
11. Compare dilations to rigid motions. How are they the same? How are they different?

Determine if the transformation of figure *A* to figure *B* on the coordinate plane is a dilation. Verify ratios of corresponding side lengths for a dilation.

12.



13.



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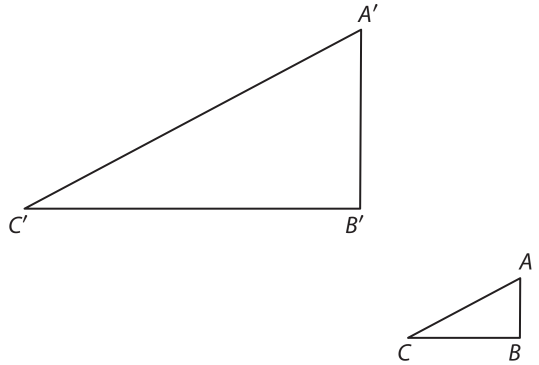
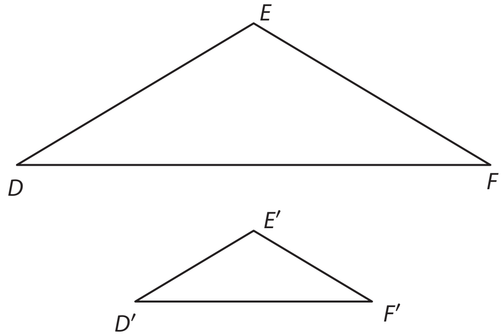
Determine the center of dilation and the scale factor of the dilation.

14.



The scale factor is ________.

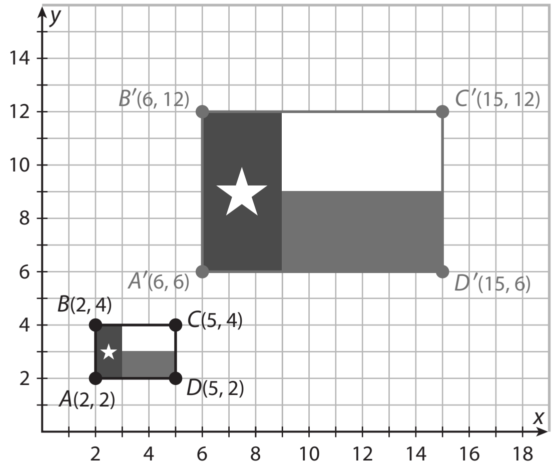
15.



The scale factor is ________.

16. You work at a photography store. A customer has a picture that is 4.5 inches tall. The customer wants a reduced copy of the picture to fit a space of 1.8 inches tall on a postcard. What scale factor should you use to reduce the picture to the correct size?

17. **Computer Graphics** An artist uses a computer program to enlarge a design, as shown. What is the scale factor of the dilation?



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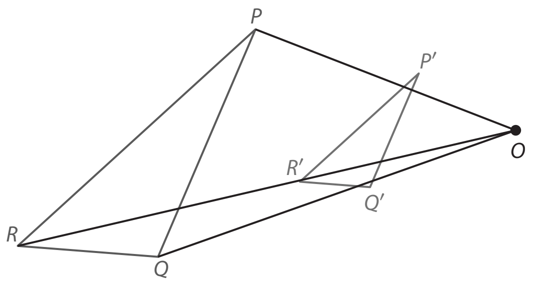
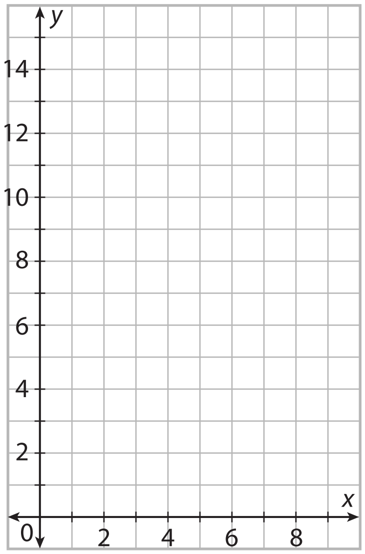
18. **Explain the Error** What mistakes did the student make when trying to determine the center of dilation? Determine the center of dilation.



H.O.T. Focus on Higher Order Thinking



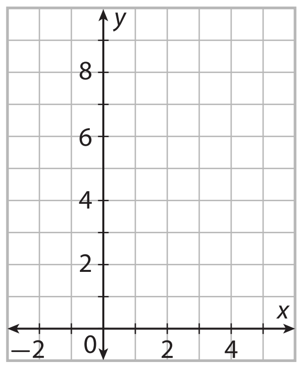
19. Draw $\triangle DEF$ with vertices D $(3, 1)$ E $(3, 5)$ F $(0, 5)$.

a. Determine the perimeter and the area of $\triangle DEF$.

b. Draw an image of $\triangle DEF$ after a dilation having a scale factor of 3, with the center of dilation at the origin $(0, 0)$. Determine the perimeter and area of the image.

c. How is the scale factor related to the ratios $\frac{\text{perimeter } \triangle D'E'F'}{\text{perimeter } \triangle DEF}$ and $\frac{\text{area } \triangle D'E'F'}{\text{area } \triangle DEF}$?

20. Draw $\triangle WXY$ with vertices $(4, 0)$, $(4, 8)$, and $(-2, 8)$.



a. Dilate $\triangle WXY$ using a factor of $\frac{1}{4}$ and the origin as the center. Then dilate its image using a scale factor of 2 and the origin as the center. Draw the final image.

b. Use the scale factors given in part (a) to determine the scale factor you could use to dilate $\triangle WXY$ with the origin as the center to the final image in one step.

c. Do you get the same final image if you switch the order of the dilations in part (a)? Explain your reasoning.

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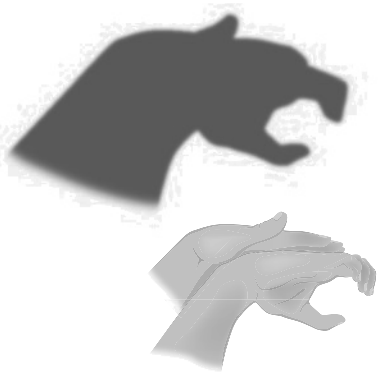
Lesson Performance Task

You've hung a sheet on a wall and lit a candle. Now you move your hands into position between the candle and the sheet and, to the great amusement of your audience, create an image of an animal on the sheet.

Compare and contrast what you're doing with what happens when you draw a dilation of a triangle on a coordinate plane. Point out ways that dilations and hand puppets are alike and ways they are different. Discuss measures that are preserved in hand-puppet projections and those that are not. Some terms you might like to discuss:

- pre-image
- image
- center of dilation
- scale factor
- transformation
- input
- output



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