Name ________________________ Class ______________ Date __________

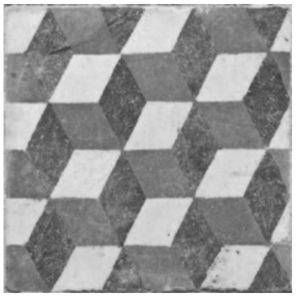
9.4 Conditions for Rectangles, Rhombuses, and Squares

Essential Question: How can you use given conditions to show that a quadrilateral is a rectangle, a rhombus, or a square?

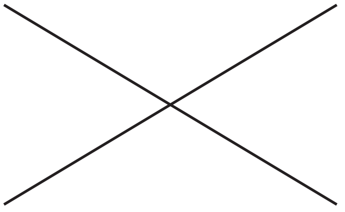
Resource Locker

Explore Properties of Rectangles, Rhombuses, and Squares

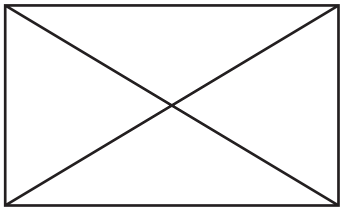
In this lesson we will start with given properties and use them to prove which special parallelogram it could be.



(A) Start by drawing two line segments of the same length that bisect each other but are not perpendicular. They will form an X shape, as shown.



(B) Connect the ends of the line segments to form a quadrilateral.



(C) Measure each of the four angles of the quadrilateral, and use those measurements to name the shape.

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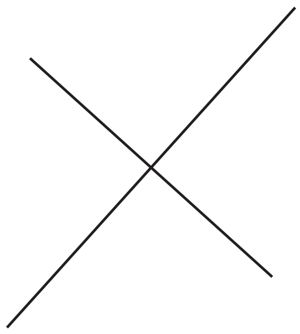
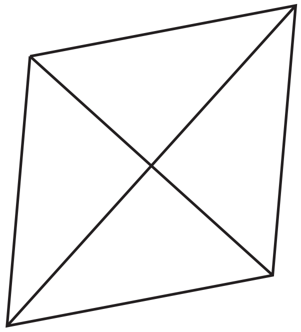
(D) Now, draw two line segments that are perpendicular and bisect each other but that are not the same length.



(E) Connect the ends of the line segments to form a quadrilateral.



(F) Measure each side length of the quadrilateral. Then use those measurements to name the shape.

____

Reflect

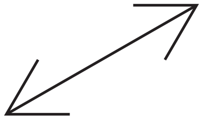
1. **Discussion** How are the diagonals of your rectangle in Step B different from the diagonals of your rhombus in Step E?

____

____

____

2. Draw a line segment. At each endpoint draw line segments so that four congruent angles are formed as shown. Then extend the segments so that they intersect to form a quadrilateral. Measure the sides. What do you notice? What kind of quadrilateral is it? How does the line segment relate to the angles drawn on either end of it?



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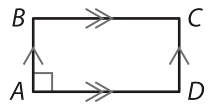
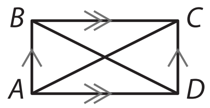
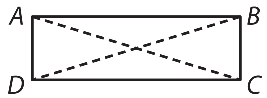
Explain 1 Proving that Congruent Diagonals Is a Condition for Rectangles

When you are given a parallelogram with certain properties, you can use the properties to determine whether the parallelogram is a rectangle.

Theorems: Conditions for Rectangles	
If one angle of a parallelogram is a right angle, then the parallelogram is a rectangle.	
If the diagonals of a parallelogram are congruent, then the parallelogram is a rectangle.	 $\overline{AC} \cong \overline{BD}$

Example 1 **Prove that if the diagonals of a parallelogram are congruent, then the parallelogram is a rectangle.**



Given: $ABCD$ is a parallelogram; $\overline{AC} \cong \overline{BD}$.

Prove: $ABCD$ is a rectangle.

Because ____, $\overline{AB} \cong \overline{CD}$.

It is given that $\overline{AC} \cong \overline{BD}$, and __________ by the Reflexive Property of Congruence.

So, ________________ by the SSS Triangle Congruence Theorem,

and __________________ by CPCTC. But these angles are ________________

since $\overline{AB} \parallel$ $\square$. Therefore, $m\angle BAD + m\angle CDA =$ $\square$. So

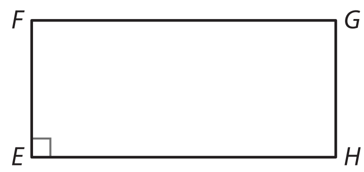
$m\angle BAD +$ $\square$ $=$ $\square$ by substitution, $2 \cdot m\angle BAD = 180°$,

and $m\angle BAD = 90°$. A similar argument shows that the other angles

of $ABCD$ are also ______ angles, so $ABCD$ is a __________.

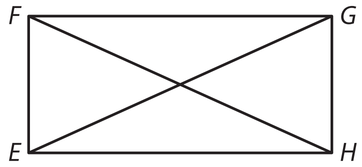
Reflect

3. **Discussion** Explain why this is a true condition for rectangles:
If one angle of a parallelogram is a right angle, then the parallelogram is a rectangle.



Your Turn

Use the given information to determine whether the quadrilateral is necessarily a rectangle. Explain your reasoning.

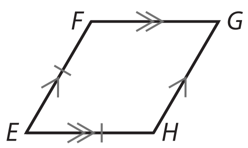
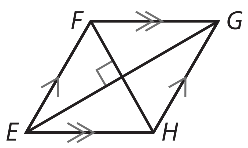
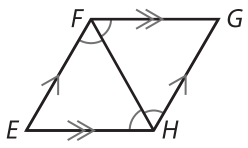


4. Given: $\overline{EF} \cong \overline{GF}$, $\overline{FG} \cong \overline{HE}$, $\overline{FH} \cong \overline{GE}$

5. Given: $m\angle FEG = 45°$, $m\angle GEH = 50°$

Explain 2 Proving Conditions for Rhombuses

You can also use given properties of a parallelogram to determine whether the parallelogram is a rhombus.

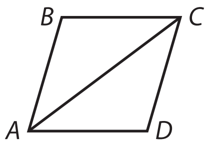
Theorems: Conditions for Rhombuses	
If one pair of consecutive sides of a parallelogram are congruent, then the parallelogram is a rhombus.	
If the diagonals of a parallelogram are perpendicular, then the parallelogram is a rhombus.	
If one diagonal of a parallelogram bisects a pair of opposite angles, then the parallelogram is a rhombus.	

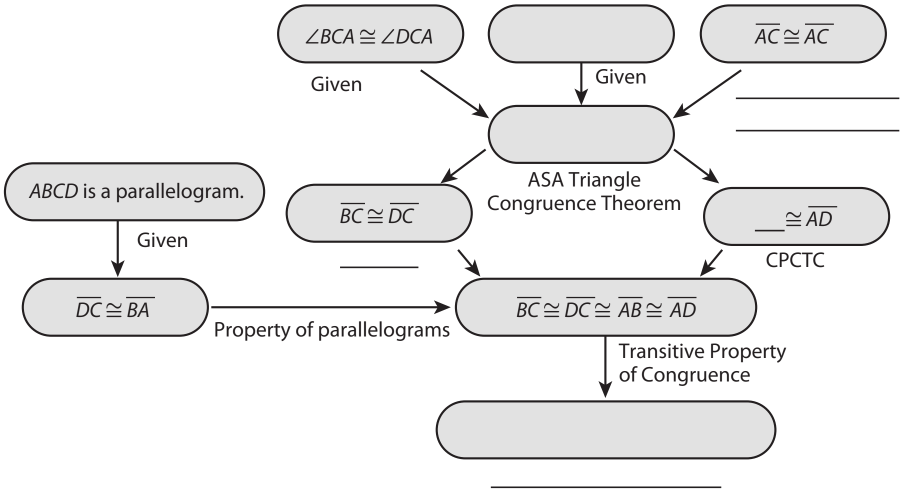
You will prove one of the theorems about rhombuses in Example 2 and the other theorems in Your Turn Exercise 6 and Evaluate Exercise 22.

Example 2 **Complete the flow proof that if one diagonal of a parallelogram bisects a pair of opposite angles, then the parallelogram is a rhombus.**

Given: *ABCD* is a parallelogram; ∠*BCA* ≅ ∠*DCA*; ∠*BAC* ≅ ∠*DAC*

Prove: *ABCD* is a rhombus.



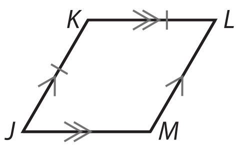


Your Turn

6. Prove that If one pair of consecutive sides of a parallelogram are congruent, then it is a rhombus.

Given: *JKLM* is a parallelogram. $\overline{JK} \cong \overline{KL}$

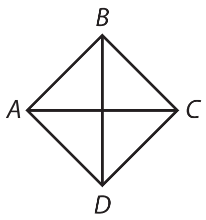
Prove: *JKLM* is a rhombus.



Explain 3 Applying Conditions for Special Parallelograms

In Example 3, you will decide whether you are given enough information to conclude that a figure is a particular type of special parallelogram.

Example 3 **Determine if the conclusion is valid. If not, tell what additional information is needed to make it valid.**



(A) **Given:** $\overline{AB} \cong \overline{CD}$; $\overline{BC} \cong \overline{DA}$; $\overline{AD} \perp \overline{DC}$; $\overline{AC} \perp \overline{BD}$

Conclusion: *ABCD* is a square.

To prove that a given quadrilateral is a square, it is sufficient to show that the figure is both a rectangle and a rhombus.

Step 1: Determine if *ABCD* is a parallelogram.

$\overline{AB} \cong \overline{CD}$ and $\overline{BC} \cong \overline{DA}$ are given. Since a quadrilateral with opposite sides congruent is a parallelogram, we know that *ABCD* is a parallelogram.

Step 2: Determine if *ABCD* is a rectangle.

Since $\overline{AD} \perp \overline{DC}$, by definition of perpendicular lines, $\angle ADC$ is a right angle. A parallelogram with one right angle is a rectangle, so *ABCD* is a rectangle.

Step 3: Determine if *ABCD* is a rhombus.

$\overline{AC} \perp \overline{BD}$. A parallelogram with perpendicular diagonals is a rhombus. So *ABCD* is a rhombus.

Step 4: Determine if *ABCD* is a square.

Since *ABCD* is a rectangle and a rhombus, it has four right angles and four congruent sides. So *ABCD* is a square by definition.

So, the conclusion is valid.

(B) **Given:** $\overline{AB} \cong \overline{BC}$

Conclusion: *ABCD* is a rhombus.

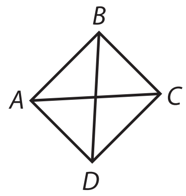
The conclusion is not valid. It is true that if two consecutive sides of a ________________ are congruent, then the __________________ is a ___________. To apply this theorem, however, you need to know that *ABCD* is a ________________. The given information is not sufficient to conclude that the figure is a parallelogram.

Reflect

7. Draw a figure that shows why this statement is not necessarily true: If one angle of a quadrilateral is a right angle, then the quadrilateral is a rectangle.

Your Turn

Determine if the conclusion is valid. If not, tell what additional information is needed to make it valid.



8. **Given:** $\angle ABC$ is a right angle.

Conclusion: *ABCD* is a rectangle.

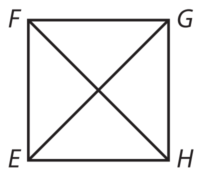
Elaborate

9. Look at the theorem boxes in Example 1 and Example 2. How do the diagrams help you remember the conditions for proving a quadrilateral is a special parallelogram?

10. *EFGH* is a parallelogram. In *EFGH*, $\overline{EG} \cong \overline{FH}$. Which conclusion is incorrect?

A. *EFGH* is a rectangle.

B. *EFGH* is a square.



11. **Essential Question Check-In** How are theorems about conditions for parallelograms different from the theorems regarding parallelograms used in the previous lesson?

Evaluate: Homework and Practice

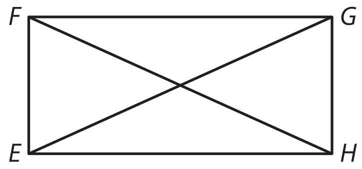
- Online Homework
- Hints and Help
- Extra Practice

1. Suppose Anna draws two line segments, $\overline{AB}$ and $\overline{CD}$ that intersect at point E. She draws them in such a way that $\overline{AB} \cong \overline{CD}$, $\overline{AB} \perp \overline{CD}$, and $\angle CAD$ is a right angle. What is the best name to describe $ACBD$? Explain.

2. Write a two-column proof that if the diagonals of a parallelogram are congruent, then the parallelogram is a rectangle.

Given: $EFGH$ is a parallelogram; $\overline{EG} \cong \overline{HF}$.

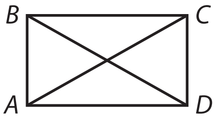
Prove: $EFGH$ is a rectangle.



Statements	Reasons
1.	1.

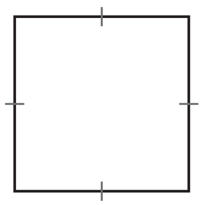
Determine whether each quadrilateral must be a rectangle. Explain.

3.



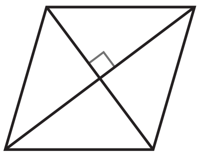
Given: $BD = AC$

4.

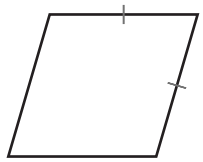


Each quadrilateral is a parallelogram. Determine whether each parallelogram is a rhombus or not.

5.

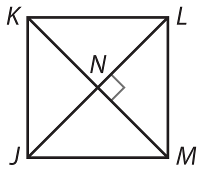


6.

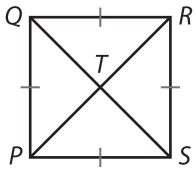


Give one characteristic about each figure that would make the conclusion valid.

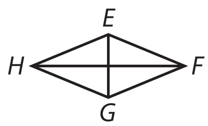
7. Conclusion: *JKLM* is a rhombus.



8. Conclusion: *PQRS* is a square.



Determine if the conclusion is valid. If not, tell what additional information is needed to make it valid.



9. Given: $\overline{EG}$ and $\overline{FH}$ bisect each other. $\overline{EG} \perp \overline{FH}$

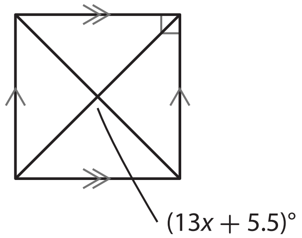
Conclusion: *EFGH* is a rhombus.

10. $\overline{FH}$ bisects $\angle EFG$ and $\angle EHG$.

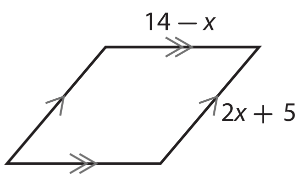
Conclusion: *EFGH* is a rhombus.

Find the value of *x* that makes each parallelogram the given type.

11. square

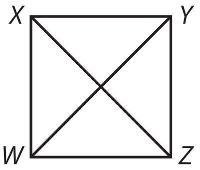


12. rhombus

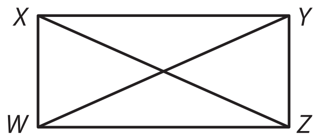


In Exercises 13–16, Determine which quadrilaterals match the figure: parallelogram, rhombus, rectangle, or square? List all that apply.

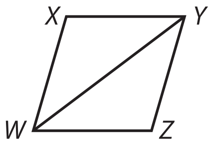
13. Given: $\overline{WY} \cong \overline{XZ}$, $\overline{WY} \perp \overline{XZ}$, $\overline{XY} \cong \overline{ZW}$



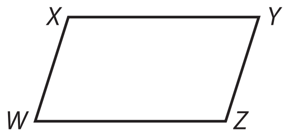
14. Given: $\overline{XY} \cong \overline{ZW}$, $\overline{WY} \cong \overline{ZX}$



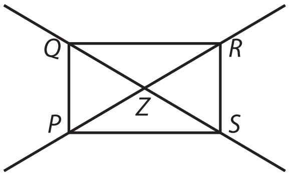
15. Given: $\overline{XY} \cong \overline{ZW}$, $\angle XWY \cong \angle YWZ$, $\angle XYW \cong \angle ZYW$



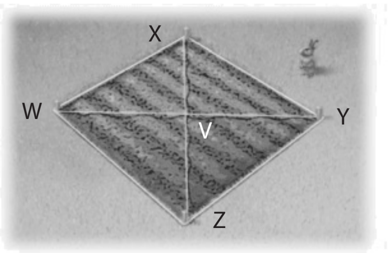
16. Given: $m\angle WXY = 130°$, $m\angle XWZ = 50°$, $m\angle WZY = 130°$



17. **Represent Real-World Problems** A framer uses a clamp to hold together pieces of a picture frame. The pieces are cut so that $\overline{PQ} \cong \overline{RS}$ and $\overline{QR} \cong \overline{SP}$. The clamp is adjusted so that PZ, QZ, RZ, and SZ are all equal lengths. Why must the frame be a rectangle?



18. **Represent Real-World Problems** A city garden club is planting a square garden. They drive pegs into the ground at each corner and tie strings between each pair. The pegs are spaced so that $\overline{WX} \cong \overline{XY} \cong \overline{YZ} \cong \overline{ZW}$. How can the garden club use the diagonal strings to verify that the garden is a square?



19. A quadrilateral is formed by connecting the midpoints of a rectangle. Which of the following could be the resulting figure? Select all that apply.

◯ parallelogram ◯ rectangle

◯ rhombus ◯ square

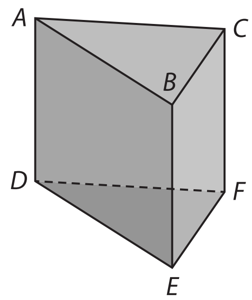
H.O.T. Focus on Higher Order Thinking

20. Critical Thinking The diagonals of a quadrilateral are perpendicular bisectors of each other. What is the best name for this quadrilateral? Explain your answer.

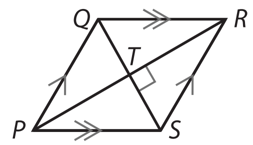
21. Draw Conclusions Think about the relationships between angles and sides in this triangular prism to decide if the given face is a rectangle.

Given: $\overline{AC} \cong \overline{DF}$, $\overline{AB} \cong \overline{DE}$, $\overline{AB} \perp \overline{BC}$, $\overline{DE} \perp \overline{EF}$, $\overline{BE} \perp \overline{EF}$, $\overline{BC} \parallel \overline{EF}$

Prove: *EBCF* is a rectangle.



22. Justify Reasoning Use one of the other rhombus theorems to prove that if the diagonals of a parallelogram are perpendicular, then the parallelogram is a rhombus.



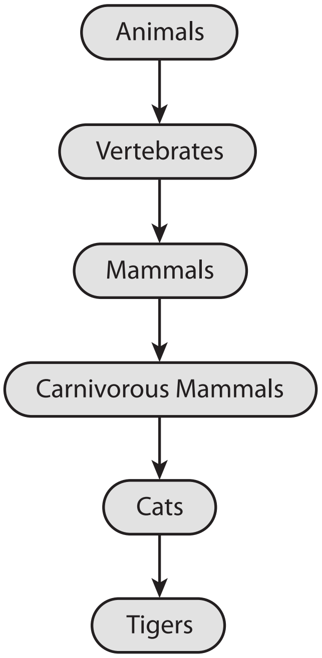
Given: *PQRS* is a parallelogram. $\overline{PR} \perp \overline{QS}$

Prove: *PQRS* is a rhombus.

Statements	Reasons
1. *PQRS* is a parallelogram.	**1.** Given
2. $\overline{PT} \cong$	**2.** Diagonals of a parallelogram bisect each other.
3. $\overline{QT} \cong$	**3.** Reflexive Property of Congruence
4. $\overline{PR} \perp \overline{QS}$	**4.** Given
5. $\angle QTP$ and $\angle QTR$ are right angles.	**5.**
6. $\angle QTP \cong \angle QTR$	**6.**
7. $\triangle QTP \cong \triangle QTR$	**7.**
8. $\overline{QP} \cong$	**8.** CPCTC
9. *PQRS* is a rhombus.	**9.**

Lesson Performance Task

The diagram shows the organizational ladder of groups to which tigers belong.



a. Use the terms below to create a similar ladder in which each term is a subset of the term above it.

Parallelogram	Geometric figures	Squares
Quadrilaterals	Figures	Rhombuses

b. Decide which of the following statements is true. Then write three more statements like it, using terms from the list in part (a).

If a figure is a rhombus, then it is a parallelogram.

If a figure is a parallelogram, then it is a rhombus.

c. Explain how you can use the ladder you created above to write if-then statements involving the terms on the list.