

9.3 Properties of Rectangles, Rhombuses, and Squares

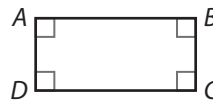


Resource Locker

Essential Question: What are the properties of rectangles, rhombuses, and squares?

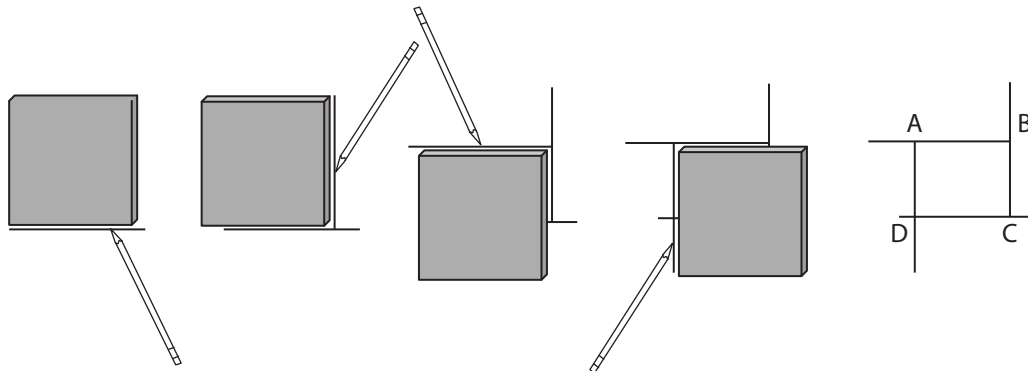
Explore Exploring Sides, Angles, and Diagonals of a Rectangle

A **rectangle** is a quadrilateral with four right angles. The figure shows rectangle $ABCD$.



Investigate properties of rectangles.

- A** Use a tile or pattern block and the following method to draw three different rectangles on a separate sheet of paper.



- B** Use a ruler to measure the sides and diagonals of each rectangle. Keep track of the measurements and compare your results to other students.

Reflect

1. Why does this method produce a rectangle? What must you assume about the tile?

2. **Discussion** Is every rectangle also a parallelogram? Make a conjecture based upon your measurements and explain your thinking.

3. Use your measurements to make two conjectures about the diagonals of a rectangle.

Conjecture: _____

Conjecture: _____



Explain 1 Proving Diagonals of a Rectangle are Congruent

You can use the definition of a rectangle to prove the following theorems.

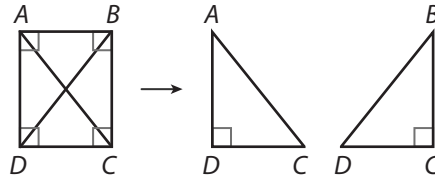
Properties of Rectangles

If a quadrilateral is a rectangle, then it is a parallelogram.
If a parallelogram is a rectangle, then its diagonals are congruent.

Example 1 Use a rectangle to prove the Properties of Rectangles Theorems.

Given: $ABCD$ is a rectangle.

Prove: $ABCD$ is a parallelogram; $\overline{AC} \cong \overline{BD}$.



(A)

Statements	Reasons
1. $ABCD$ is a rectangle.	1. Given
2. $\angle A$ and $\angle C$ are right angles.	2. Definition of
3. $\angle A \cong \angle C$	3. All right angles are congruent.
4. $\angle B$ and $\angle D$ are right angles.	4.
5. $\angle B \cong \angle D$	5.
6. $ABCD$ is a parallelogram.	6.
7. $\overline{AD} \cong \overline{CB}$	7. If a quadrilateral is a parallelogram, then its opposite sides are congruent.
8. $\overline{DC} \cong \overline{DC}$	8.
9. $\angle D$ and $\angle C$ are right angles.	9. Definition of rectangle
10. $\angle D \cong \angle C$	10. All right angles are congruent.
11.	11.
12.	12.

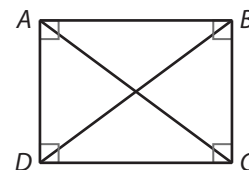
Reflect

4. **Discussion** A student says you can also prove the diagonals are congruent in Example 1 by using the SSS Triangle Congruence Theorem to show that $\triangle ADC \cong \triangle BCD$. Do you agree? Explain.

Your Turn

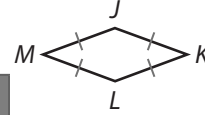
Find each measure.

5. $AD = 7.5$ cm and $DC = 10$ cm. Find DB .
6. $AB = 17$ cm and $BC = 12.75$ cm. Find DB .



Explain 2 Proving Diagonals of a Rhombus are Perpendicular

A **rhombus** is a quadrilateral with four congruent sides.
The figure shows rhombus $JKLM$.



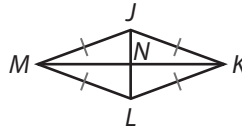
Properties of Rhombuses

If a quadrilateral is a rhombus, then it is a parallelogram.
If a parallelogram is a rhombus, then its diagonals are perpendicular.
If a parallelogram is a rhombus, then each diagonal bisects a pair of opposite angles.

Example 2 Prove that the diagonals of a rhombus are perpendicular.

Given: $JKLM$ is a rhombus.

Prove: $\overline{JL} \perp \overline{MK}$



Since $JKLM$ is a rhombus, $\overline{JM} \cong \square$. Because $JKLM$ is also a parallelogram, $\overline{MN} \cong \overline{KN}$ because _____ . By the Reflexive Property of Congruence, $\overline{JN} \cong \overline{JN}$, so $\triangle JNM \cong \triangle JNK$ by the _____. So, _____ by CPCTC. By the Linear Pair Theorem, $\angle JNM$ and $\angle JNK$ are supplementary. This means that $m\angle JNM + m\angle JNK = \square$. Since the angles are congruent, $m\angle JNM = m\angle JNK$ so by _____, $m\angle JNM + m\angle JNK = 180^\circ$ or $2m\angle JNK = 180^\circ$. Therefore, $m\angle JNK = \square$ and $\square \perp \overline{MK}$.

Reflect

7. What can you say about the image of J in the proof after a reflection across \overline{MK} ? Why?

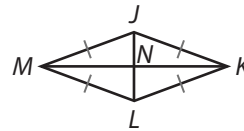
8. What property about the diagonals of a rhombus is the same as a property of all parallelograms? What special property do the diagonals of a rhombus have?

Your Turn

9. Prove that if a parallelogram is a rhombus, then each diagonal bisects a pair of opposite angles.

Given: $JKLM$ is a rhombus.

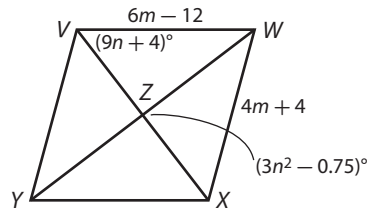
Prove: \overline{MK} bisects $\angle JML$ and $\angle JKL$;
 \overline{JL} bisects $\angle MJK$ and $\angle MLK$.





Explain 3 Using Properties of Rhombuses to Find Measures

Example 3 Use rhombus $VWXY$ to find each measure.



A Find XY .

All sides of a rhombus are congruent, so $\overline{VW} \cong \overline{WX}$ and $VW = WX$.

Substitute values for VW and WX . $6m - 12 = 4m + 4$

Solve for m . $m = 8$

Substitute the value of m to find VW . $VW = 6(8) - 12 = 36$

Because all sides of the rhombus are congruent, then $\overline{VW} \cong \overline{XY}$, and $XY = 36$.

B Find $\angle YVW$.

The diagonals of a rhombus are _____, so $\angle WZX$ is a right angle and

$m\angle WZX = \square$.

Since $m\angle WZX = (3n^2 - 0.75)^\circ$, then _____.

Solve for n . $3n^2 - 0.75 = 90$

$$n = \square$$

Substitute the value of n to find $m\angle WVZ$.

$$m\angle WVZ = \square$$

Since \overline{VX} bisects $\angle YVW$, then _____

Substitute 53.5° for $m\angle WVZ$. $m\angle YVW = 2(53.5^\circ) = 107^\circ$

Your Turn

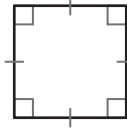
Use the rhombus $VWXY$ from Example 3 to find each measure.

10. Find $m\angle VYX$.

11. Find $m\angle XYZ$.

Explain 4 Investigating the Properties of a Square

A **square** is a quadrilateral with four sides congruent and four right angles.



Example 4 Explain why each conditional statement is true.

(A) If a quadrilateral is a square, then it is a parallelogram.

By definition, a square is a quadrilateral with four congruent sides.
Any quadrilateral with both pairs of opposite sides congruent is a parallelogram,
so a square is a parallelogram.

(B) If a quadrilateral is a square, then it is a rectangle.

By definition, a square is a quadrilateral with four _____.

By definition, a rectangle is also a quadrilateral with four _____.
Therefore, a square is a rectangle.

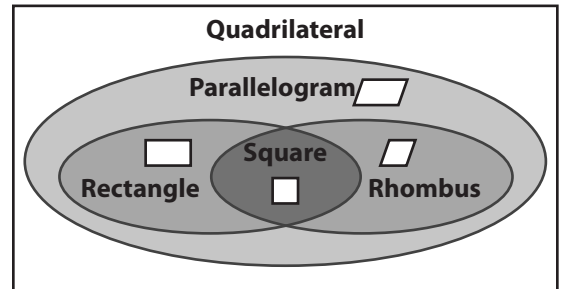
Your Turn

12. Explain why this conditional statement is true: If a quadrilateral is a square, then it is a rhombus.

13. Look at Part A. Use a different way to explain why this conditional statement is true: If a quadrilateral is a square, then it is a parallelogram.

Elaborate

14. Discussion The Venn diagram shows how quadrilaterals, parallelograms, rectangles, rhombuses, and squares are related to each other. From this lesson, what do you notice about the definitions and theorems regarding these figures?



15. Essential Question Check-In What are the properties of rectangles and rhombuses? How does a square relate to rectangles and rhombuses?



Evaluate: Homework and Practice

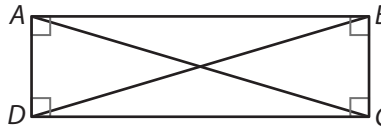


- Online Homework
- Hints and Help
- Extra Practice

1. Complete the paragraph proof of the Properties of Rectangles Theorems.

Given: $ABCD$ is a rectangle.

Prove: $ABCD$ is a parallelogram; $\overline{AC} \cong \overline{BD}$.



Proof that $ABCD$ is a _____: Since $ABCD$ is a rectangle, $\angle A$ and $\angle C$ are right angles. So $\angle A \cong \angle C$ because _____.

By similar reasoning, $\angle B \cong \angle D$. Therefore, $ABCD$ is a parallelogram because _____

Proof that the diagonals are congruent: Since $ABCD$ is a parallelogram,

$\overline{AD} \cong \overline{BC}$ because _____.

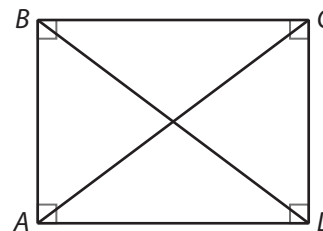
Also, _____ by the Reflexive Property of Congruence. By the definition of a rectangle, $\angle D$ and $\angle C$ are right angles, and so _____

because all right angles are _____. Therefore, $\triangle ADC \cong \triangle BCD$ by the _____

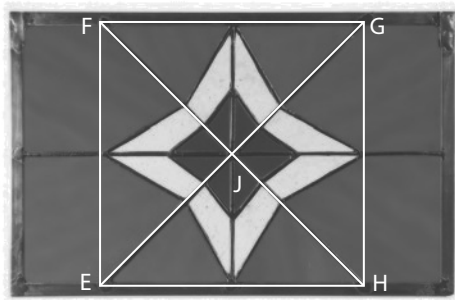
_____ and $\square \cong \square$ by CPCTC.

Find the lengths using rectangle $ABCD$.

- $AB = 21$; $AD = 28$. What is the value of $AC + BD$?
- $BC = 40$; $CD = 30$. What is the value of $BD - AC$?

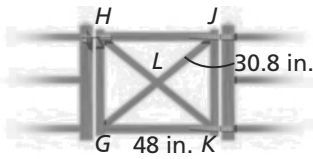


4. An artist connects stained glass pieces with lead strips. In this rectangular window, the strips are cut so that $FH = 34$ in. Find JG . Explain.



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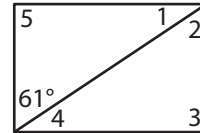
The rectangular gate has diagonal braces. Find each length.



5. Find HJ .

6. Find HK .

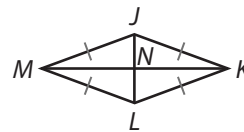
7. Find the measure of each numbered angle in the rectangle.



8. Complete the two-column proof that the diagonals of a rhombus are perpendicular.

Given: $JKLM$ is a rhombus.

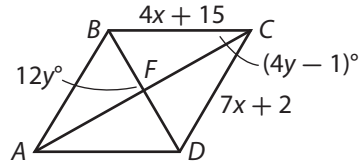
Prove: $\overline{JL} \perp \overline{MK}$



Statements	Reasons
1. $\overline{JM} \cong \overline{JK}$	1. Definition of rhombus
2. $\overline{MN} \cong \overline{KN}$	2.
3. $\overline{JN} \cong \overline{JN}$	3. Reflexive Property of Congruence
4.	4. SSS Triangle Congruence Theorem
5. $\angle JNM \cong \angle JNK$	5.
6. $\angle JNM$ and $\angle JNK$ are supplementary.	6.
7.	7. Definition of supplementary
8. $\angle JNM = \angle JNK$	8. Definition of congruence
9. <input type="text"/> + $\angle JNK = 180^\circ$	9. Substitution Property of Equality
10. $2m\angle JNK = 180^\circ$	10. Addition
11. $m\angle JNK = 90^\circ$	11. Division Property of Equality
12.	12. Definition of perpendicular lines

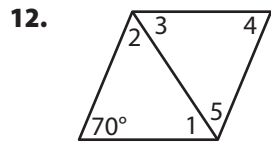
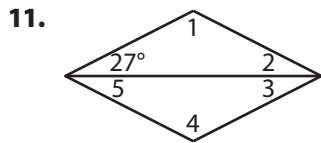
$ABCD$ is a rhombus. Find each measure.

9. Find AB .



10. Find $m\angle ABC$.

Find the measure of each numbered angle in the rhombus.



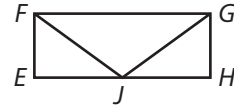
13. Select the word that best describes when each of the following statements are true. Select the correct answer for each lettered part.

- | | | | |
|------------------------------------|------------------------------|---------------------------------|-----------------------------|
| A. A rectangle is a parallelogram. | <input type="radio"/> always | <input type="radio"/> sometimes | <input type="radio"/> never |
| B. A parallelogram is a rhombus. | <input type="radio"/> always | <input type="radio"/> sometimes | <input type="radio"/> never |
| C. A square is a rhombus. | <input type="radio"/> always | <input type="radio"/> sometimes | <input type="radio"/> never |
| D. A rhombus is a square. | <input type="radio"/> always | <input type="radio"/> sometimes | <input type="radio"/> never |
| E. A rhombus is a rectangle. | <input type="radio"/> always | <input type="radio"/> sometimes | <input type="radio"/> never |

14. Use properties of special parallelograms to complete the proof.

Given: $EFGH$ is a rectangle. J is the midpoint of \overline{EH} .

Prove: $\triangle FJG$ is isosceles.



Statements	Reasons
1. $EFGH$ is a rectangle. J is the midpoint of \overline{EH} .	1. Given
2. $\angle E$ and $\angle H$ are right angles.	2. Definition of rectangle
3. $\angle E \cong \angle H$	3.
4. $EFGH$ is a parallelogram.	4.
5.	5.
6.	6.
7.	7.
8.	8.
9.	9.

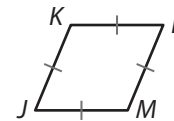
15. **Explain the Error** Find and explain the error in this paragraph proof. Then describe a way to correct the proof.

Given: $JKLM$ is a rhombus.

Prove: $JKLM$ is a parallelogram.

Proof: It is given that $JKLM$ is a rhombus. So, by the definition of a rhombus,

$\overline{JK} \cong \overline{LM}$, and $\overline{KL} \cong \overline{MJ}$. If a quadrilateral is a parallelogram, then its opposite sides are congruent. So $JKLM$ is a parallelogram.



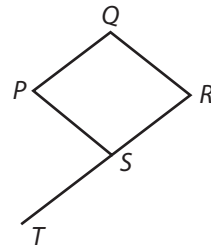
The opening of a soccer goal is shaped like a rectangle.

16. Draw a rectangle to represent a soccer goal. Label the rectangle $ABCD$ to show that the distance between the goalposts, \overline{BC} , is three times the distance from the top of the goalpost to the ground. If the perimeter of $ABCD$ is 64 feet, what is the length of \overline{BC} ?



17. In your rectangle from Evaluate 16, suppose the distance from B to D is $(y + 10)$ feet, and the distance from A to C is $(2y - 5.3)$ feet. What is the approximate length of \overline{AC} ?

18. $PQRS$ is a rhombus, with $PQ = (7b - 5)$ meters and $QR = (2b - 0.5)$ meters. If S is the midpoint of \overline{RT} , what is the length of \overline{RT} ?



19. Communicate Mathematical Ideas List the properties that a square “inherits” because it is each of the following quadrilaterals.

a. a parallelogram

b. a rectangle

c. a rhombus

H.O.T. Focus on Higher Order Thinking

Justify Reasoning For the given figure, describe any rotations or reflections that would carry the figure onto itself. Explain.

20. A rhombus that is not a square

21. A rectangle that is not a square

22. A square

23. Analyze Relationships Look at your answers for Exercises 20–22. How does your answer to Exercise 22 relate to your answers to Exercises 20 and 21? Explain.

Lesson Performance Task

The portion of the Arkansas state flag that is not red is a rhombus. On one flag, the diagonals of the rhombus measure 24 inches and 36 inches. Find the area of the rhombus. Justify your reasoning.

