Name ______________________ Class ______________ Date ________

9.2 Conditions for Parallelograms

Essential Question: What criteria can you use to prove that a quadrilateral is a parallelogram?

Resource Locker

Explore Proving the Opposite Sides Criterion for a Parallelogram

You can prove that a quadrilateral is a parallelogram by using the definition of a parallelogram. That is, you can show that both pairs of opposite sides are parallel. However, there are other conditions that also guarantee that a quadrilateral is a parallelogram.

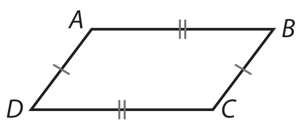
Theorem

If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

Complete the proof of the theorem.

Given: $\overline{AB} \cong \overline{CD}$ and $\overline{AD} \cong \overline{CB}$

Prove: $ABCD$ is a parallelogram.



(A) Draw diagonal $\overline{DB}$.

Why is it helpful to draw this diagonal?

____

____

(B) Use triangle congruence theorems and corresponding parts to complete the proof that the opposite sides are parallel so the quadrilateral is a parallelogram.

Statements	Reasons
1. Draw $\overline{DB}$.	**1.** Through any two points, there is exactly one line.
2. $\overline{DB} \cong \overline{DB}$	**2.**
3. $\overline{AB} \cong \overline{CD}$; $\overline{AD} \cong \overline{CB}$	**3.**
4. $\triangle ABD \cong \triangle CDB$	**4.**
5. $\angle ABD \cong \angle CDB$; $\angle ADB \cong \angle CBD$	**5.**
6. $\overline{AB} \parallel \overline{DC}$; $\overline{AD} \parallel \overline{BC}$	**6.**
7. $ABCD$ is a parallelogram.	**7.**

It is possible to combine the theorem from the Explore and the definition of a parallelogram to state the following condition for proving a quadrilateral is a parallelogram. You will prove this in the exercises.

Theorem

If one pair of opposite sides of a quadrilateral are parallel and congruent, then the quadrilateral is a parallelogram.

Reflect

1. **Discussion** A quadrilateral has two sides that are 3 cm long and two sides that are 5 cm long. A student states that the quadrilateral must be a parallelogram. Do you agree? Explain.

____

____

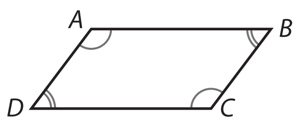
Explain 1 Proving the Opposite Angles Criterion for a Parallelogram

You can use relationships between angles to prove that a quadrilateral is a parallelogram.

Theorem

If both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

Example 1 **Prove that a quadrilateral is a parallelogram if its opposite angles are congruent.**



Given: $\angle A \cong \angle C$ and $\angle B \cong \angle D$ Prove: $ABCD$ is a parallelogram.

$m\angle A + m\angle B + m\angle C + m\angle D = 360°$ by ______________.

From the given information, $m\angle A = m\angle$ ☐ and $m\angle B = m\angle$ ☐. By substitution,

$m\angle A + m\angle D + m\angle A + m\angle D = 360°$ or $2m\angle$ ☐ $+ 2m\angle$ ☐ $= 360°$. Dividing

both sides by 2 gives ______________. Therefore, $\angle A$ and $\angle D$ are

supplementary and so $\overline{AB} \parallel \overline{DC}$ by the ______________

A similar argument shows that $\overline{AD} \parallel \overline{BC}$, so $ABCD$ is a parallelogram

by ______________.

Reflect

2. What property or theorem justifies dividing both sides of the equation by 2 in the above proof?

____

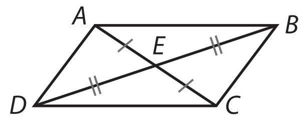
Explain 2 Proving the Bisecting Diagonals Criterion for a Parallelogram

You can use information about the diagonals in a given figure to show that the figure is a parallelogram.

Theorem

If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.

Example 2 **Prove that a quadrilateral whose diagonals bisect each other is a parallelogram.**



Given: $\overline{AE} \cong \overline{CE}$ and $\overline{DE} \cong \overline{BE}$

Prove: $ABCD$ is a parallelogram.

Statements	Reasons
1. $\overline{AE} \cong \overline{CE}$, $\overline{DE} \cong \overline{BE}$	1.
2. $\angle AEB \cong \angle CED$, $\angle AED \cong \angle CEB$	2.
3.	3.
4. $\overline{AB} \cong \overline{CD}$, $\overline{AD} \cong \overline{CB}$	4.
5. $ABCD$ is a parallelogram.	5.

Reflect

3. **Critique Reasoning** A student claimed that you can also write the proof using the SSS Triangle Congruence Theorem since $\overline{AB} \cong \overline{CD}$ and $\overline{AD} \cong \overline{CB}$. Do you agree? Justify your response.

Explain 3 Using a Parallelogram to Prove the Concurrency of the Medians of a Triangle

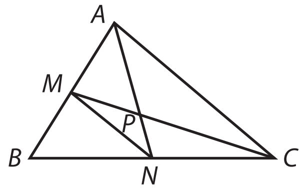
Sometimes properties of one type of geometric figure can be used to recognize properties of another geometric figure. Recall that you explored triangles and found that the medians of a triangle are concurrent at a point that is $\frac{2}{3}$ of the distance from each vertex to the midpoint of the opposite side. You can prove this theorem using one of the conditions for a parallelogram from this lesson.

Example 3 **Complete the proof of the Concurrency of Medians of a Triangle Theorem.**

Given: $\triangle ABC$

Prove: The medians of $\triangle ABC$ are concurrent at a point that is $\frac{2}{3}$ of the distance from each vertex to the midpoint of the opposite side.

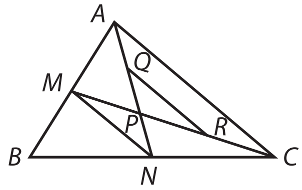
Let $\triangle ABC$ be a triangle such that M is the midpoint of $\overline{AB}$ and N is the midpoint of $\overline{BC}$. Label the point where the two medians intersect as P. Draw $\overline{MN}$.



$\overline{MN}$ is a midsegment of $\triangle ABC$ because it connects the midpoints of two sides of the triangle.

$\overline{MN}$ is parallel to ______ and $MN =$ ______ by the Triangle Midsegment Theorem.

Let Q be the midpoint of $\overline{PA}$ and let R be the midpoint of $\overline{PC}$.



Draw $\overline{QR}$.

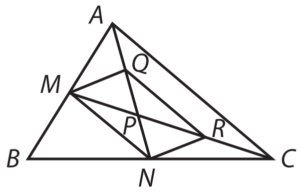
$\overline{QR}$ is a midsegment of $\triangle APC$ because it connects the midpoints of two sides of the triangle.

$\overline{QR}$ is parallel to ______ and $QR =$ ______ by the Triangle Midsegment Theorem.

So, you can conclude that $MN =$ ______ by substitution and that

$\overline{MN} \parallel \overline{QR}$ because ____.

Now draw $\overline{MQ}$ and $\overline{NR}$ and consider quadrilateral $MQRN$.



Quadrilateral $MQRN$ is a parallelogram because

____.

Since the diagonals of a parallelogram bisect each other, then $QP =$ ______.

Also, $AQ = QP$ since ________________________.

Therefore, $AQ = QP =$ ______. This shows that point P is located on $\overline{AN}$ at a point that is $\frac{2}{3}$ of the distance from A to N.

By similar reasoning, the diagonals of a parallelogram bisect each other, so $RP =$ ______.

Also, $CR = RP$ since ________________________.

Therefore, $CR = RP =$ ______. This shows that point P is located on $\overline{CM}$ at a point that is $\frac{2}{3}$ of the distance from C to M.

You can repeat the proof using any two medians of $\triangle ABC$. The same reasoning shows that the medians from vertices B and C intersect at a point that is also $\frac{2}{3}$ of the distance from C to M, so this point must also be point P. This shows that the three medians intersect at a unique point P and that the point is $\frac{2}{3}$ of the distance from each vertex to the midpoint of the opposite side.

Reflect

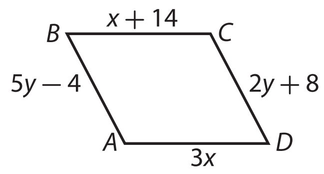
4. In the proof, how do you know that point P is located on $\overline{AN}$ at a point that is $\frac{2}{3}$ of the distance from A to N?

Explain 4 Verifying Figures Are Parallelograms

You can use information about sides, angles, and diagonals in a given figure to show that the figure is a parallelogram.

Example 4 **Show that each quadrilateral is a parallelogram for the given values of the variables.**

(A) $x = 7$ and $y = 4$



Step 1 Find BC and DA.

$BC = x + 14 = 7 + 14 = 21$

$DA = 3x = 3(7) = 21$

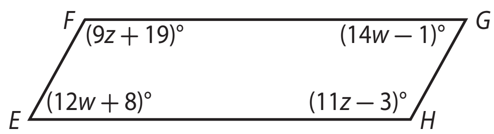
Step 2 Find AB and CD.

$AB = 5y - 4 = 5(4) - 4 = 16$

$CD = 2y + 8 = 2(4) + 8 = 16$

So, $BC = DA$ and $AB = CD$. $ABCD$ is a parallelogram since both pairs of opposite sides are congruent.

(B) $z = 11$ and $w = 4.5$



Step 1 Find $m\angle F$ and $m\angle H$.

$m\angle F =$ ____________ = ____________

$m\angle H =$ ____________ = ____________

Step 2 Find $m\angle E$ and $m\angle G$.

$m\angle E =$ ____________ = ____________

$m\angle G =$ ____________ = ____________

So, $m\angle F = m\angle$ ☐ and $m\angle E = m\angle$ ☐. $EFGH$ is a parallelogram since

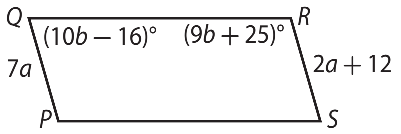
Reflect

5. What conclusions can you make about $\overline{FG}$ and $\overline{EH}$ in Part B? Explain.

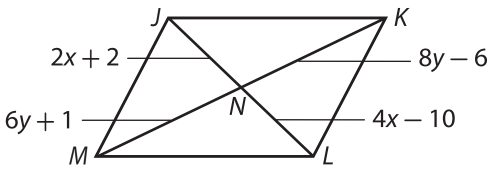
Your Turn

Show that each quadrilateral is a parallelogram for the given values of the variables.

6. $a = 2.4$ and $b = 9$



7. $x = 6$ and $y = 3.5$

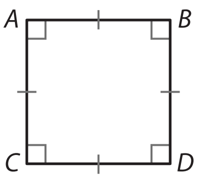


Elaborate

8. How are the theorems in this lesson different from the theorems in the previous lesson, Properties of Parallelograms?

9. Why is the proof of the Concurrency of the Medians of a Triangle Theorem in this lesson and not in the earlier module when the theorem was first introduced?

10. **Essential Question Check-In** Describe three different ways to show that quadrilateral *ABCD* is a parallelogram.



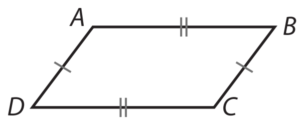
Evaluate: Homework and Practice

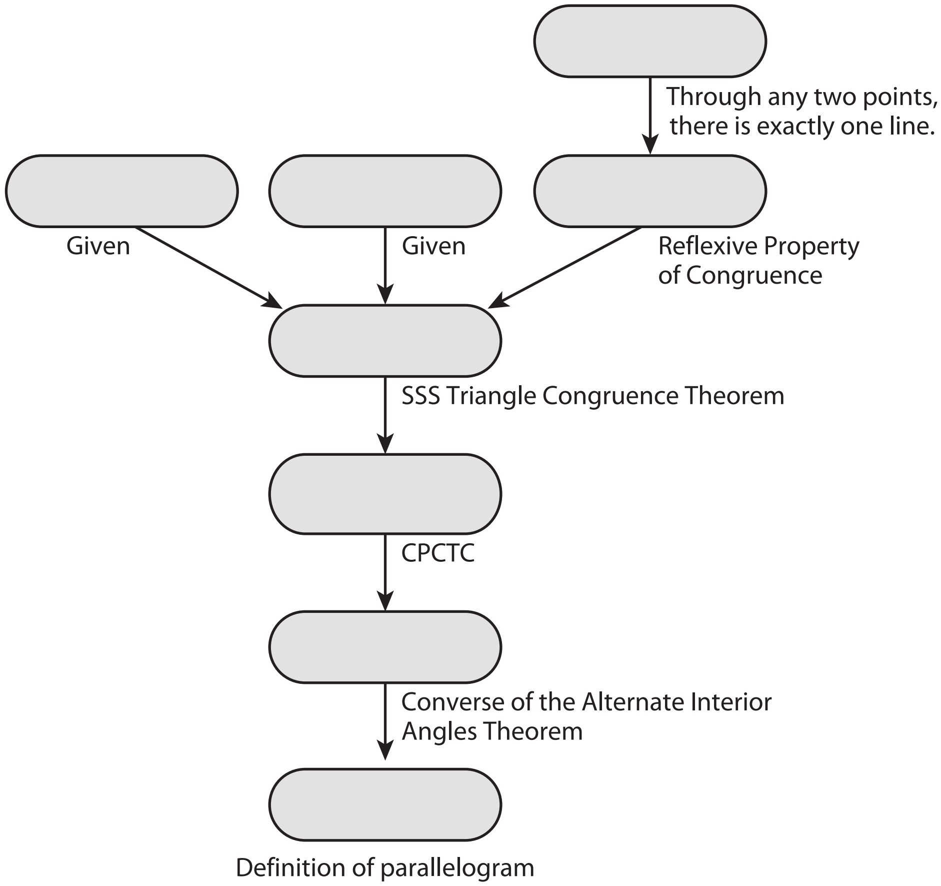
Personal Math Trainer
- Online Homework
- Hints and Help
- Extra Practice

1. You have seen a proof that if both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram. Write the proof as a flow proof.

Given: $\overline{AB} \cong \overline{CD}$ and $\overline{AD} \cong \overline{CB}$

Prove: $ABCD$ is a parallelogram.

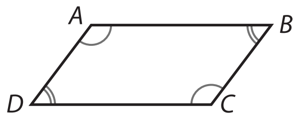




2. You have seen a proof that if both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram. Write the proof as a two-column proof.

Given: $\angle A \cong \angle C$ and $\angle B \cong \angle D$

Prove: $ABCD$ is a parallelogram.

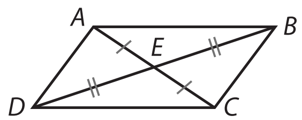


Statements	Reasons
1.	1.

3. You have seen a proof that if the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram. Write the proof as a paragraph proof.

Given: $\overline{AE} \cong \overline{CE}$ and $\overline{DE} \cong \overline{BE}$

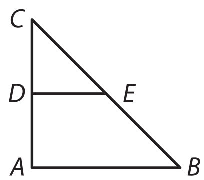
Prove: $ABCD$ is a parallelogram.



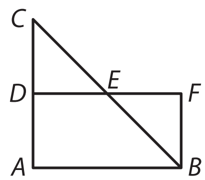
4. Complete the following proof of the Triangle Midsegment Theorem.

Given: D is the midpoint of $\overline{AC}$, and E is the midpoint of $\overline{BC}$.

Prove: $\overline{DE} \parallel \overline{AB}$, $DE = \frac{1}{2}AB$



Extend $\overline{DE}$ to form $\overline{DF}$ such that $\overline{DE} \cong \overline{FE}$. Then draw $\overline{BF}$, as shown.



It is given that E is the midpoint of $\overline{CB}$, so $\overline{CE} \cong$ ______.

By the Vertical Angles Theorem, $\angle CED \cong$ ______.

So, $\triangle CED \cong$ ______ by ______________________________.

Since corresponding parts of congruent triangles are congruent, $\overline{CD} \cong$ ______.

D is the midpoint of $\overline{AC}$, so $\overline{CD} \cong$ ______.

By the Transitive Property of Congruence, $\overline{AD} \cong$ ______.

Also, since corresponding parts of congruent triangles are congruent, $\angle CDE \cong$ ______.

So, $\overline{AC} \parallel \overline{FB}$ by ______________________________.

This shows that $DFBA$ is a parallelogram because ______________________

____.

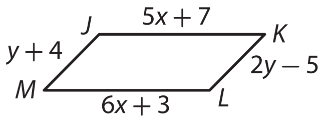
By the definition of parallelogram, $\overline{DE}$ is parallel to ______.

Since opposite sides of a parallelogram are congruent, $AB =$ ______.

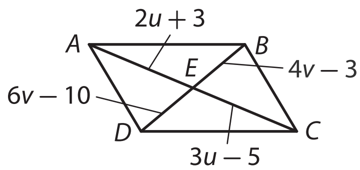
$\overline{DE} \cong \overline{FE}$, so $DE = \frac{1}{2}\boxed{}$ and by substitution, $DE = \frac{1}{2}\boxed{}$.

Show that each quadrilateral is a parallelogram for the given values of the variables.

5. $x = 4$ and $y = 9$

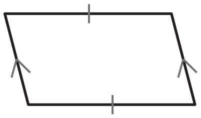


6. $u = 8$ and $v = 3.5$



Determine if each quadrilateral must be a parallelogram. Justify your answer.

7.



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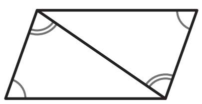

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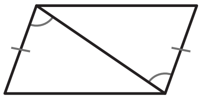

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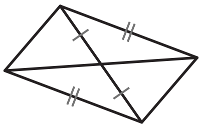

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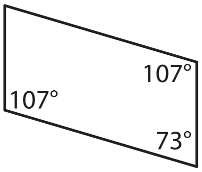

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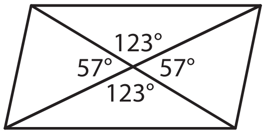
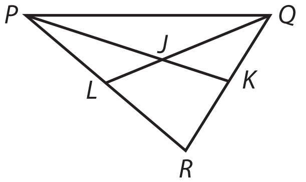


13. **Communicate Mathematical Ideas** Kalil wants to write the proof that the medians of a triangle are concurrent at a point that is $\frac{2}{3}$ of the distance from each vertex to the midpoint of the opposite side. He starts by drawing $\triangle PQR$ and two medians, $\overline{PK}$ and $\overline{QL}$. He labels the point of intersection as point J, as shown. What segment should Kalil draw next? What conclusions can he make about this segment? Explain.



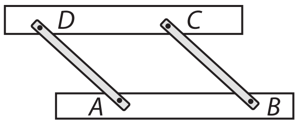
14. **Critical Thinking** Jasmina said that you can draw a parallelogram using the following steps.

1. Draw a point P.
2. Use a ruler to draw a segment that is 1 inch long with its midpoint at P.
3. Use the ruler to draw a segment that is 2 inches long with its midpoint at P.
4. Use the ruler to connect the endpoints of the segments to form a parallelogram.

Does Jasmina's method always work? Is there ever a time when it would not produce a parallelogram? Explain.

15. **Critique Reasoning** Matthew said that there is another condition for parallelograms. He said that if a quadrilateral has two congruent diagonals, then the quadrilateral is a parallelogram. Do you agree? If so, explain why. If not, give a counterexample to show why the condition does not work.

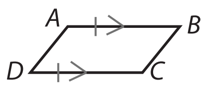
16. A parallel rule can be used to plot a course on a navigation chart. The tool is made of two rulers connected at hinges to two congruent crossbars, $\overline{AD}$ and $\overline{BC}$. You place the edge of one ruler on your desired course and then move the second ruler over the compass rose on the chart to read the bearing for your course. If $\overline{AD} \parallel \overline{BC}$, why is $\overline{AB}$ always parallel to $\overline{CD}$?



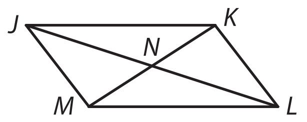
17. Write a two-column proof to prove that a quadrilateral with a pair of opposite sides that are parallel and congruent is a parallelogram.

Given: $\overline{AB} \cong \overline{CD}$ and $\overline{AB} \parallel \overline{CD}$

Prove: $ABCD$ is a parallelogram. (*Hint*: Draw $\overline{DB}$.)

Statements	Reasons
1.	**1.** 

18. Does each set of given information guarantee that quadrilateral $JKLM$ is a parallelogram? Select the correct answer for each lettered part.



A. $JN = 25$ cm, $JL = 50$ cm, $KN = 13$ cm, $KM = 26$ cm ◯ Yes ◯ No

B. $\angle MJL \cong \angle KLJ$, $\overline{JM} \cong \overline{LK}$ ◯ Yes ◯ No

C. $\overline{JM} \cong \overline{JK}$, $\overline{KL} \cong \overline{LM}$ ◯ Yes ◯ No

D. $\angle MJL \cong \angle MLJ$, $\angle KJL \cong \angle KLJ$ ◯ Yes ◯ No

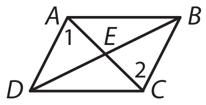
E. $\triangle JKN \cong \triangle LMN$ ◯ Yes ◯ No

H.O.T. Focus on Higher Order Thinking

19. Explain the Error A student wrote the two-column proof below. Explain the student's error and explain how to write the proof correctly.

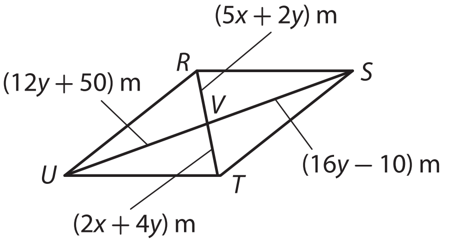
Given: $\angle 1 \cong \angle 2$, E is the midpoint of $\overline{AC}$.

Prove: $ABCD$ is a parallelogram.

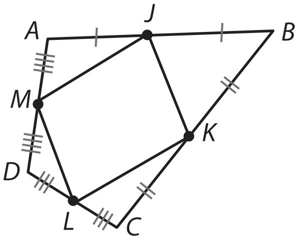


Statements	Reasons
1. $\angle 1 \cong \angle 2$	**1.** Given
2. E is the midpoint of $\overline{AC}$.	**2.** Given
3. $\overline{AE} \cong \overline{CE}$	**3.** Definition of midpoint
4. $\angle AED \cong \angle CEB$	**4.** Vertical angles are congruent.
5. $\triangle AED \cong \triangle CEB$	**5.** ASA Triangle Congruence Theorem
6. $\overline{AD} \cong \overline{CB}$	**6.** Corresponding parts of congruent triangles are congruent
7. $ABCD$ is a parallelogram.	**7.** If a pair of opposite sides of a quadrillateral are congruent, then the quadrillateral is a parallelogram.

20. Persevere in Problem Solving The plan for a city park shows that the park is a quadrilateral with straight paths along the diagonals. For what values of the variables is the park a parallelogram? In this case, what are the lengths of the paths?



21. Analyze Relationships When you connect the midpoints of the consecutive sides of any quadrilateral, the resulting quadrilateral is a parallelogram. Use the figure below to explain why this is true. (*Hint:* Draw a diagonal of *ABCD*.)



Lesson Performance Task

In this lesson you've learned three theorems for confirming that a figure is a parallelogram.

- If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.
- If both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram.
- If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.

For each of the following situations, choose one of the three theorems and use it in your explanation. You should choose a different theorem for each explanation.

a. You're an amateur astronomer, and one night you see what appears to be a parallelogram in the constellation of Lyra. Explain how you could verify that the figure is a parallelogram.

b. You have a frame shop and you want to make an interesting frame for an advertisement for your store. You decide that you'd like the frame to be a parallelogram but not a rectangle. Explain how you could construct the frame.

c. You're using a toolbox with cantilever shelves like the one shown here. Explain how you can confirm that the brackets that attach the shelves to the box form a parallelogram ABCD.