Name ______ Class ______ Date ______

7.3 Triangle Inequalities

Essential Question: How can you use inequalities to describe the relationships among side lengths and angle measures in a triangle?

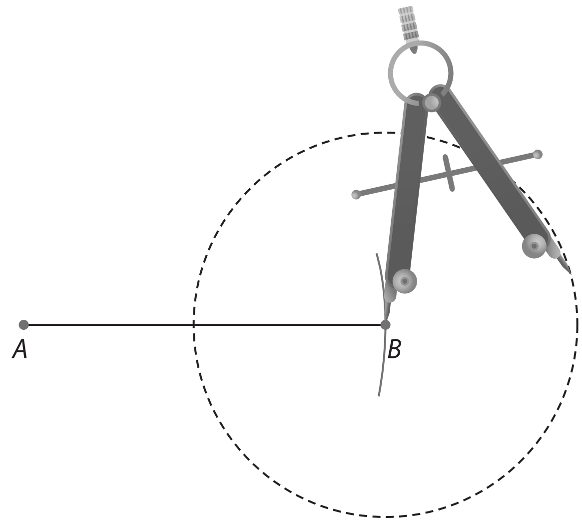
Resource Locker

Explore Exploring Triangle Inequalities

A triangle can have sides of different lengths, but are there limits to the lengths of any of the sides?

(A) Consider a $\triangle ABC$ where you know two side lengths, $AB = 4$ inches and $BC = 2$ inches. On a separate piece of paper, draw $\overline{AB}$ so that it is 4 inches long.

(B) To determine all possible locations for C with $\overline{BC} = 2$ inches, set your compass to 2 inches. Draw a circle with center at B.



(C) Choose and label a final vertex point C so it is located on the circle. Using a straightedge, draw the segments to form a triangle.

Are there any places on the circle where point C cannot lie? Explain.

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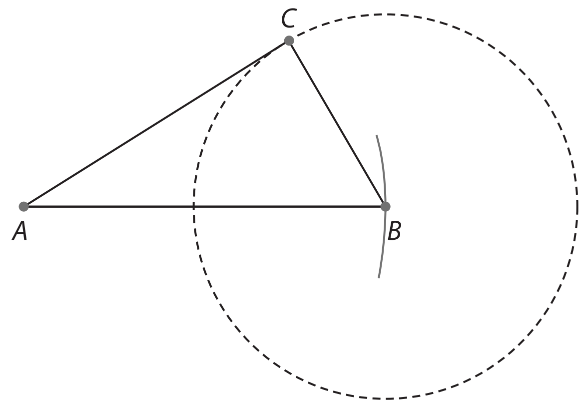
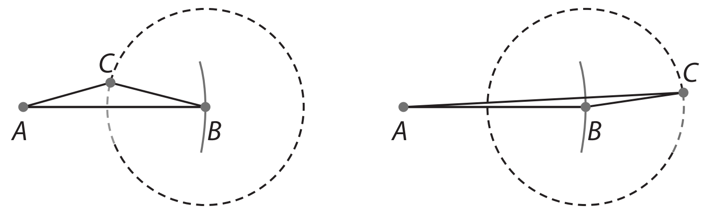


(D) Measure and record the lengths of the three sides of your triangle.

(E) The figures below show two other examples of $\triangle ABC$ that could have been formed. What are the values that $\overline{AC}$ approaches when point C approaches $\overline{AB}$?



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Reflect

1. Use the side lengths from your table to make the following comparisons. What do you notice?

$AB + BC \; ? \; AC$ $\quad$ $BC + AC \; ? \; AB$ $\quad$ $AC + AB \; ? \; BC$

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2. Measure the angles of some triangles with a protractor. Where is the smallest angle in relation to the shortest side? Where is the largest angle in relation to the longest side?

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3. **Discussion** How does your answer to the previous question relate to isosceles triangles or equilateral triangles?

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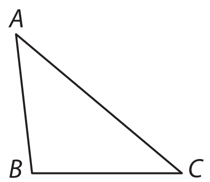
Explain 1 Using the Triangle Inequality Theorem

The Explore shows that the sum of the lengths of any two sides of a triangle is greater than the length of the third side. This can be summarized in the following theorem.

Triangle Inequality Theorem

The sum of any two side lengths of a triangle is greater than the third side length.



$AB + BC > AC$

$BC + AC > AB$

$AC + AB > BC$

To be able to form a triangle, each of the three inequalities must be true. So, given three side lengths, you can test to determine if they can be used as segments to form a triangle. To show that three lengths cannot be the side lengths of a triangle, you only need to show that one of the three triangle inequalities is false.

Example 1 **Use the Triangle Inequality Theorem to tell whether a triangle can have sides with the given lengths. Explain.**

(A) 4, 8, 10

$4 + 8 \overset{?}{>} 10$ | $4 + 10 \overset{?}{>} 8$ | $8 + 10 \overset{?}{>} 4$

$12 > 10$ ✓ | $14 > 8$ ✓ | $18 > 4$ ✓

Conclusion: The sum of each pair of side lengths is greater than the third length. So, a triangle can have side lengths of 4, 8, and 10.

(B) 7, 9, 18

$\square + \square \overset{?}{>} 18$ | $\square + \square \overset{?}{>} 9$ | $9 + \square \overset{?}{>} \square$

$\square > 18$ $\square$ | $\square > 9$ $\square$ | $\square > \square$ $\square$

Conclusion: ______________________________

Reflect

4. Can an isosceles triangle have these side lengths? Explain. 5, 5, 10

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5. How do you know that the Triangle Inequality Theorem applies to all equilateral triangles?

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Your Turn

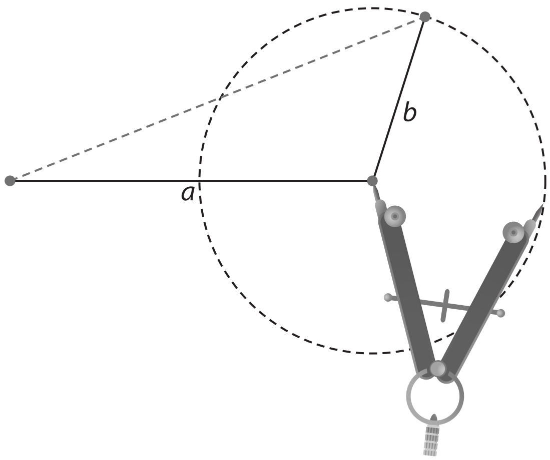
Determine if a triangle can be formed with the given side lengths. Explain your reasoning.

6. 12 units, 4 units, 17 units

7. 24 cm, 8 cm, 30 cm

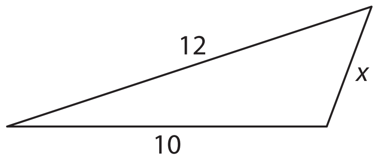
Explain 2 Finding Possible Side Lengths in a Triangle

From the Explore, you have seen that if given two side lengths for a triangle, there are an infinite number of side lengths available for the third side. But the third side is also restricted to values determined by the Triangle Inequality Theorem.



Example 2 **Find the range of values for x using the Triangle Inequality Theorem.**

(A) Find possible values for the length of the third side using the Triangle Inequality Theorem.



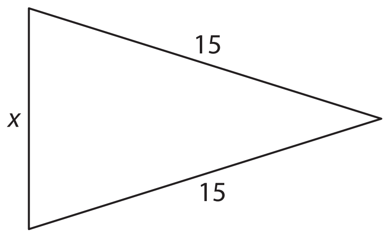
$x + 10 > 12$ $\quad$ $x + 12 > 10$ $\quad$ $10 + 12 > x$

$x > 2$ $\quad$ $x > -2$ $\quad$ $22 > x$

$2 < x < 22$

Ignore the inequality with a negative value, since a triangle cannot have a negative side length. Combine the other two inequalities to find the possible values for x.

(B)



☐ + ☐ > ☐ $\quad$ ☐ + ☐ > ☐ $\quad$ ☐ + ☐ > ☐

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☐ $< x <$ ☐

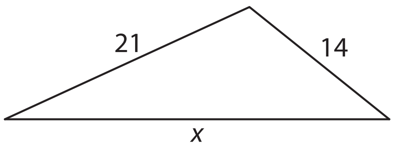
Reflect

8. **Discussion** Suppose you know that the length of the base of an isosceles triangle is 10, but you do not know the lengths of its legs. How could you use the Triangle Inequality Theorem to find the range of possible lengths for each leg? Explain.

Your Turn

Find the range of values for *x* using the Triangle Inequality Theorem.

9.



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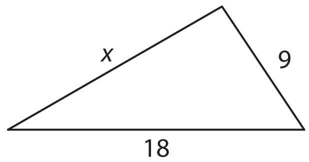
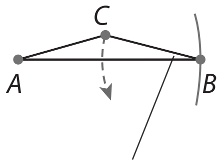
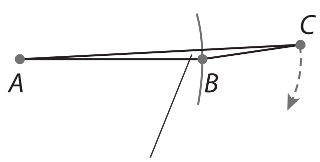


Explain 3 Ordering a Triangle's Angle Measures Given Its Side Lengths

From the Explore Step D, you can see that changing the length of $\overline{AC}$ also changes the measure of $\angle B$ in a predictable way.



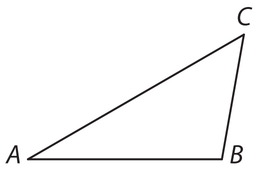
As side *AC* gets shorter, m∠*B* approaches 0°



As side *AC* gets longer, m∠*B* approaches 180°

Side-Angle Relationships in Triangles

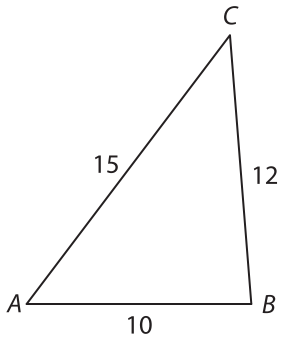
If two sides of a triangle are not congruent, then the larger angle is opposite the longer side.



$AC > BC$
$m\angle B > m\angle A$

Example 3 **For each triangle, order its angle measures from least to greatest.**

Ⓐ



Longest side length: AC

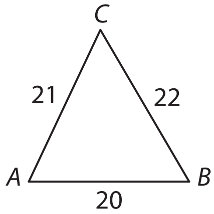
Greatest angle measure: $m\angle B$

Shortest side length: AB

Least angle measure: $m\angle C$

Order of angle measures from least to greatest: $m\angle C$, $m\angle A$, $m\angle B$

Ⓑ



Longest side length: ______________

Greatest angle measure: ______________

Shortest side length: ______________

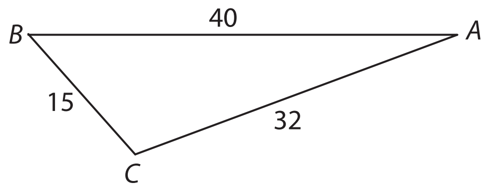
Least angle measure: ______________

Order of angle measures from least to greatest: ______________

Your Turn

For each triangle, order its angle measures from least to greatest.

11.



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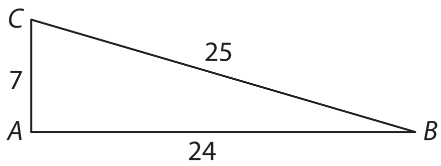
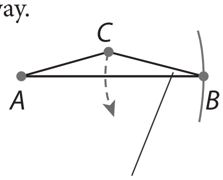
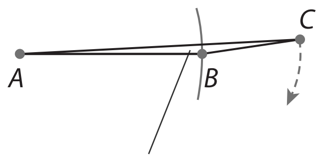


Explain 4 Ordering a Triangle's Side Lengths Given Its Angle Measures

From the Explore Step D, you can see that changing the the measure of $\angle B$ also changes length of $\overline{AC}$ in a predictable way.



As $m\angle B$ approaches 0°, side AC gets shorter



As $m\angle B$ approaches 180°, side AC gets longer

Angle-Side Relationships in Triangles

If two angles of a triangle are not congruent, then the longer side is opposite the larger angle.

Example 4 **For each triangle, order the side lengths from least to greatest.**

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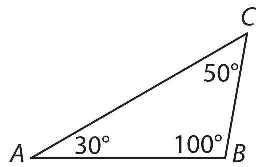


Greatest angle measure: m∠*B*

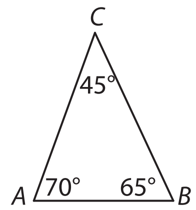
Longest side length: *AC*

Least angle measure: m∠*A*

Shortest side length: *BC*

Order of side lengths from least to greatest: *BC*, *AB*, *AC*

Ⓑ



Greatest angle measure: ______________

Longest side length: ______________

Least angle measure: ______________

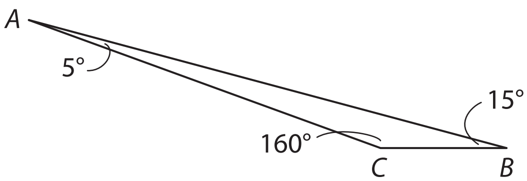
Shortest side length: ______________

Order of side lengths from least to great: ______________

Your Turn

For each triangle, order the side lengths from least to greatest.

13.



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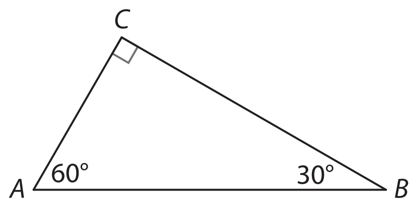

Elaborate

15. When two sides of a triangle are congruent, what can you conclude about the angles opposite those sides?

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16. What can you conclude about the side opposite the obtuse angle in an obtuse triangle?

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17. **Essential Question Check-In** Suppose you are given three values that could represent the side lengths of a triangle. How can you use one inequality to determine if the triangle exists?

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Evaluate: Homework and Practice

- Online Homework
- Hints and Help
- Extra Practice

Use a compass and straightedge to decide whether each set of lengths can form a triangle.

1. 7 cm, 9 cm, 18 cm

2. 2 in., 4 in., 5 in.

3. 1 in., 2 in., 10 in.

4. 9 cm, 10 cm, 11 cm

Determine whether a triangle can be formed with the given side lengths.

5. 10 ft, 3 ft, 15 ft

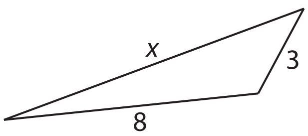
6. 12 in., 4 in., 15 in.

7. 9 in., 12 in., and 18 in.

8. 29 m, 59 m, and 89 m

Find the range of possible values for *x* using the Triangle Inequality Theorem.

9.



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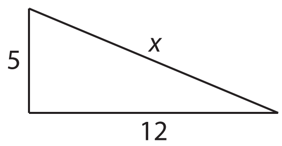
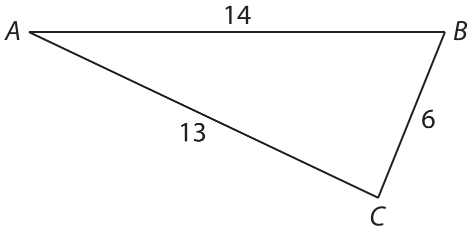


11. A triangle with side lengths 22.3, 27.6, and x

12. **Analyze Relationships** Suppose a triangle has side lengths AB, BC, and x, where $AB = 2 \cdot BC$. Find the possible range for x in terms of BC.

For each triangle, write the order of the angle measures from least to greatest.

13.



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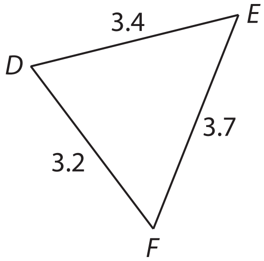
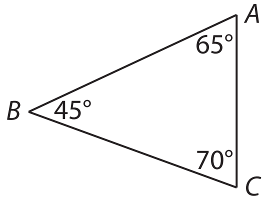


15. **Analyze Relationships** Suppose a triangle has side lengths PQ, QR, and PR, where $PR = 2PQ = 3QR$. Write the angle measures in order from least to greatest.

For each triangle, write the side lengths in order from least to greatest.

16.



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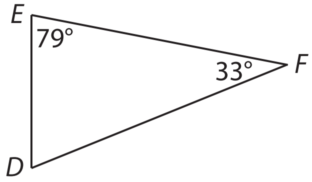
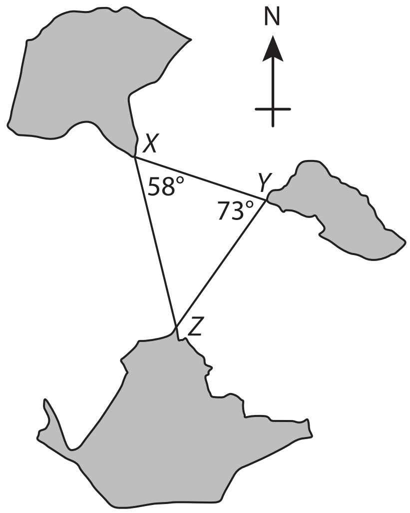


18. In $\triangle JKL$, $m\angle J = 53°$, $m\angle K = 68°$, and $m\angle L = 59°$.

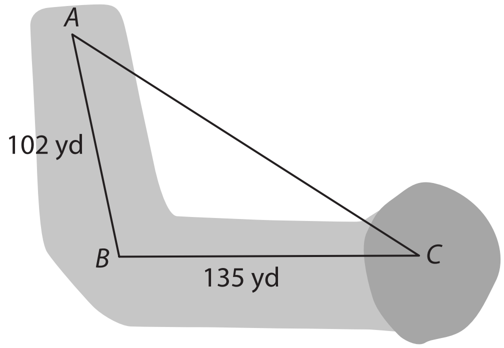
19. In $\triangle PQR$, $m\angle P = 102°$ and $m\angle Q = 25°$.

20. **Represent Real-World Problems** Rhonda is traveling from New York City to Paris and is trying to decide whether to fly via Frankfurt or to get a more expensive direct flight. Given that it is 3,857 miles from New York City to Frankfurt and another 278 miles from Frankfurt to Paris, what is the range of possible values for the direct distance from New York City to Paris?

21. Represent Real-World Problems A large ship is sailing between three small islands. To do so, the ship must sail between two pairs of islands, avoiding sailing between a third pair. The safest route is to avoid the closest pair of islands. Which is the safest route for the ship?



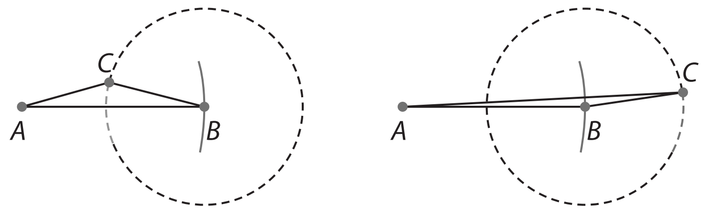
22. Represent Real-World Problems A hole on a golf course is a dogleg, meaning that it bends in the middle. A golfer will usually start by driving for the bend in the dogleg (from A to B), and then using a second shot to get the ball to the green (from B to C). Sandy believes she may be able to drive the ball far enough to reach the green in one shot, avoiding the bend (from A direct to C). Sandy knows she can accurately drive a distance of 250 yd. Should she attempt to drive for the green on her first shot? Explain.



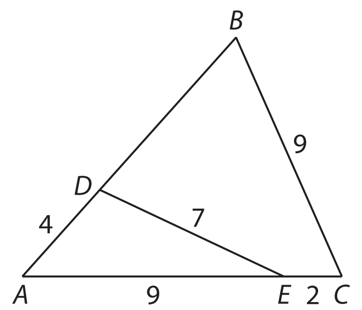
23. Represent Real-World Problems Three cell phone towers form a triangle, $\triangle PQR$. The measure of $\angle Q$ is 10° less than the measure of $\angle P$. The measure of $\angle R$ is 5° greater than the measure of $\angle Q$. Which two towers are closest together?

24. Algebra In $\triangle PQR$, $PQ = 3x + 1$, $QR = 2x - 2$, and $PR = x + 7$. Determine the range of possible values of x.

25. In any triangle ABC, suppose you know the lengths of $\overline{AB}$ and $\overline{BC}$, and suppose that $AB > BC$. If x is the length of the third side, $\overline{AC}$, use the Triangle Inequality Theorem to prove that $AB - BC < x < AB + BC$. That is, x must be between the difference and the sum of the other two side lengths. Explain why this result makes sense in terms of the constructions shown in the figure.



26. Given the information in the diagram, prove that $m\angle DEA < m\angle ABC$.



27. An isosceles triangle has legs with length 11 units. Which of the following could be the perimeter of the triangle? Choose all that apply. Explain your reasoning.

a. 22 units

b. 24 units

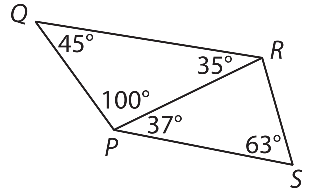
c. 34 units

d. 43 units

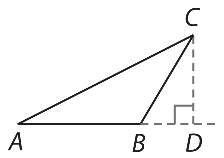
e. 44 units

H.O.T. Focus on Higher Order Thinking

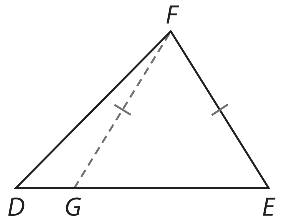
28. Communicate Mathematical Ideas Given the information in the diagram, prove that $PQ < PS$.



29. Justify Reasoning In obtuse $\triangle ABC$, $m\angle A < m\angle B$. The auxiliary line segment $\overline{CD}$ perpendicular to $\overrightarrow{AB}$ (extended beyond B) creates right triangles ADC and BDC. Describe how you could use the Pythagorean Theorem to prove that $BC < AC$.



30. Make a Conjecture In acute $\triangle DEF$, $m\angle D < m\angle E$. The auxiliary line segment $\overline{FG}$ creates $\triangle EFG$, where $EF = FG$. What would you need to prove about the points D, G, and E to prove that $\angle DGF$ is obtuse, and therefore that $EF < DF$? Explain.



Lesson Performance Task

As captain of your orienteering team, it's your job to map out the shortest distance from point A to point H on the map. Justify each of your decisions.