Name _____ Class _____ Date _____

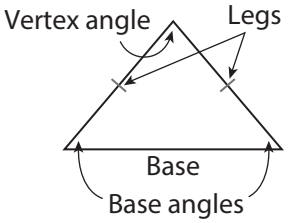
7.2 Isosceles and Equilateral Triangles

Essential Question: **What are the special relationships among angles and sides in isosceles and equilateral triangles?**

Resource Locker

Explore Investigating Isosceles Triangles

An **isosceles triangle** is a triangle with at least two congruent sides.



The congruent sides are called the **legs** of the triangle.

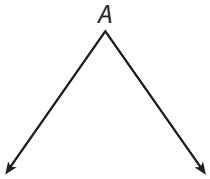
The angle formed by the legs is the **vertex angle**.

The side opposite the vertex angle is the **base**.

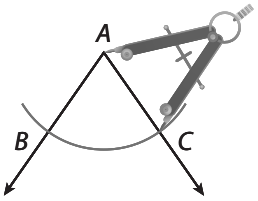
The angles that have the base as a side are the **base angles**.

In this activity, you will construct isosceles triangles and investigate other potential characteristics/properties of these special triangles.

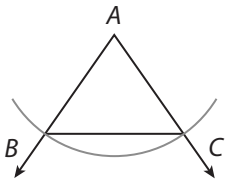
A Do your work in the space provided. Use a straightedge to draw an angle. Label your angle $\angle A$, as shown in the figure.



B Using a compass, place the point on the vertex and draw an arc that intersects the sides of the angle. Label the points B and C.



C Use the straightedge to draw line segment $\overline{BC}$.



D Use a protractor to measure each angle. Record the measures in the table under the column for Triangle 1.

	Triangle 1	Triangle 2	Triangle 3	Triangle 4
m∠*A*				
m∠*B*				
m∠*C*				

E Repeat steps A–D at least two more times and record the results in the table. Make sure ∠*A* is a different size each time.

Reflect

1. How do you know the triangles you constructed are isosceles triangles?

2. **Make a Conjecture** Looking at your results, what conjecture can be made about the base angles, ∠*B* and ∠*C*?

Explain 1 Proving the Isosceles Triangle Theorem and Its Converse

In the Explore, you made a conjecture that the base angles of an isosceles triangle are congruent. This conjecture can be proven so it can be stated as a theorem.

Isosceles Triangle Theorem

If two sides of a triangle are congruent, then the two angles opposite the sides are congruent.

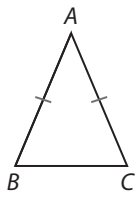
This theorem is sometimes called the Base Angles Theorem and can also be stated as "Base angles of an isosceles triangle are congruent."

Example 1 **Prove the Isosceles Triangle Theorem and its converse.**

Step 1 Complete the proof of the Isosceles Triangle Theorem.

Given: $\overline{AB} \cong \overline{AC}$

Prove: $\angle B \cong \angle C$



Statements	Reasons
1. $\overline{BA} \cong \overline{CA}$	**1.** Given
2. $\angle A \cong \angle A$	**2.**
3. $\overline{CA} \cong \overline{BA}$	**3.** Symmetric Property of Equality
4. $\triangle BAC \cong \triangle CAB$	**4.**
5.	**5.** CPCTC

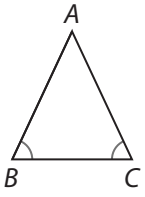
Step 2 Complete the statement of the Converse of the Isosceles Triangle Theorem.

If two _____ of a _____ are congruent, then the two _____ opposite those _____ are _____.

Step 3 Complete the proof of the Converse of the Isosceles Triangle Theorem.

Given: $\angle B \cong \angle C$

Prove: $\overline{AB} \cong \overline{AC}$



Statements	Reasons
1. $\angle ABC \cong \angle ACB$	**1.** Given
2.	**2.** Reflexive Property of Congruence
3. $\angle$ $\cong \angle ABC$	**3.** Symmetric Property of Equality
4. $\triangle ABC \cong \triangle$	**4.**
5. $\overline{AB} \cong \overline{AC}$	**5.**

Reflect

3. **Discussion** In the proofs of the Isosceles Triangle Theorem and its converse, how might it help to sketch a reflection of the given triangle next to the original triangle, so that vertex B is on the right?

Explain 2 Proving the Equilateral Triangle Theorem and Its Converse

An **equilateral triangle** is a triangle with three congruent sides.

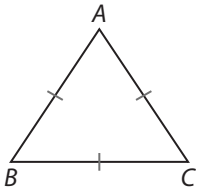
An **equiangular triangle** is a triangle with three congruent angles.

Equilateral Triangle Theorem

If a triangle is equilateral, then it is equiangular.

Example 2 **Prove the Equilateral Triangle Theorem and its converse.**

Step 1 Complete the proof of the Equilateral Triangle Theorem.



Given: $\overline{AB} \cong \overline{AC} \cong \overline{BC}$
Prove: $\angle A \cong \angle B \cong \angle C$

Given that $\overline{AB} \cong \overline{AC}$ we know that $\angle B \cong \angle$ ____ by the

_____.

It is also known that $\angle A \cong \angle B$ by the Isosceles Triangle Theorem, since _____.

Therefore, $\angle A \cong \angle C$ by _____.

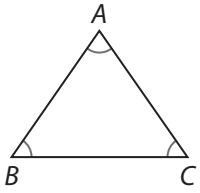
Finally, $\angle A \cong \angle B \cong \angle C$ by the _____ Property of Congruence.

The converse of the Equilateral Triangle Theorem is also true.

Converse of the Equilateral Triangle Theorem

If a triangle is equiangular, then it is equilateral.

Step 2 Complete the proof of the Converse of the Equilateral Triangle Theorem.



Given: $\angle A \cong \angle B \cong \angle C$

Prove: $\overline{AB} \cong \overline{AC} \cong \overline{BC}$

Because $\angle B \cong \angle C$, $\overline{AB} \cong$ ☐ by the

_____.

$\overline{AC} \cong \overline{BC}$ by the Converse of the Isosceles Triangle Theorem because

☐ $\cong \angle B$.

Thus, by the Transitive Property of Congruence, _____, and therefore, $\overline{AB} \cong \overline{AC} \cong \overline{BC}$.

Reflect

4. To prove the Equilateral Triangle Theorem, you applied the theorems of isosceles triangles. What can be concluded about the relationship between equilateral triangles and isosceles triangles?

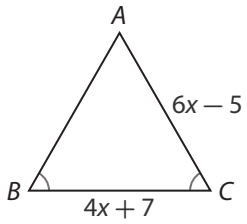
_____

Explain 3 Using Properties of Isosceles and Equilateral Triangles

You can use the properties of isosceles and equilateral triangles to solve problems involving these theorems.

Example 3 **Find the indicated measure.**

(A) Katie is stitching the center inlay onto a banner that she created to represent her new tutorial service. It is an equilateral triangle with the following dimensions in centimeters. What is the length of each side of the triangle?



To find the length of each side of the triangle, first find the value of x.

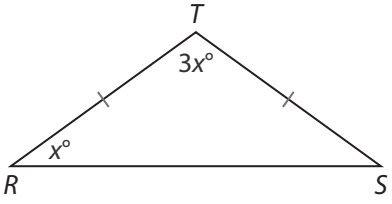
$\overline{AC} \cong \overline{BC}$	Converse of the Equilateral Triangle Theorem
$AC = BC$	Definition of congruence
$6x - 5 = 4x + 7$	Substitution Property of Equality
$x = 6$	Solve for x.

Substitute 6 for x into either $6x - 5$ or $4x + 7$.

$6(6) - 5 = 36 - 5 = 31$ or $4(6) + 7 = 24 + 7 = 31$

So, the length of each side of the triangle is 31 cm.

(B) $m\angle T$



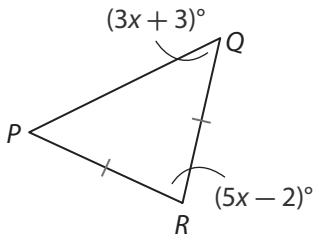
To find the measure of the vertex angle of the triangle, first find the value of ____.

$m\angle R = m\angle S = x°$	☐ Theorem
$m\angle R + m\angle S + \square = 180°$	Triangle Sum Theorem
$x + x + 3x = 180$	Substitution Property of Equality
$\square = 180$	Addition Property of Equality
$x = \square$	☐ Property of Equality

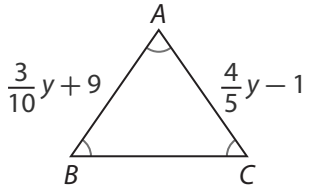
So, $m\angle T = 3x° = 3(\square)° = \square°$.

Your Turn

5. Find m∠*P*.



6. Katie's tutorial service is going so well that she is having shirts made with the equilateral triangle emblem. She has given the t-shirt company these dimensions. What is the length of each side of the triangle in centimeters?



Elaborate

7. **Discussion** Consider the vertex and base angles of an isosceles triangle. Can they be right angles? Can they be obtuse? Explain.

8. **Essential Question Check-In** Discuss how the sides of an isosceles triangle relate to its angles.

Evaluate: Homework and Practice

- Online Homework
- Hints and Help
- Extra Practice

1. Use a straightedge. Draw a line. Draw an acute angle with vertex A along the line. Then use a compass to copy the angle. Place the compass point at another point B along the line and draw the copied angle so that the angle faces the original angle. Label the intersection of the angle sides as point C. Look at the triangle you have formed. What is true about the two base angles of $\triangle ABC$? What do you know about $\overline{CA}$ and $\overline{CB}$? What kind of triangle did you form? Explain your reasoning.

2. Prove the Isosceles Triangle Theorem as a paragraph proof.

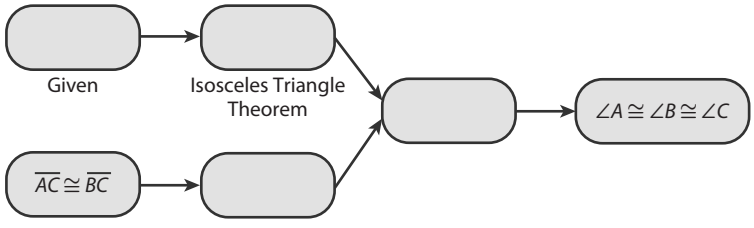
Given: $\overline{AB} \cong \overline{AC}$

Prove: $\angle B \cong \angle C$

3. Complete the flow proof of the Equilateral Triangle Theorem.

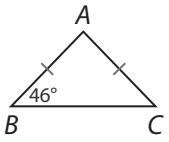
Given: $\overline{AB} \cong \overline{AC} \cong \overline{BC}$

Prove: $\angle A \cong \angle B \cong \angle C$

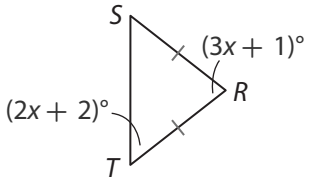


Find the measure of the indicated angle.

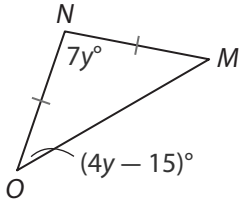
4. m∠*A*



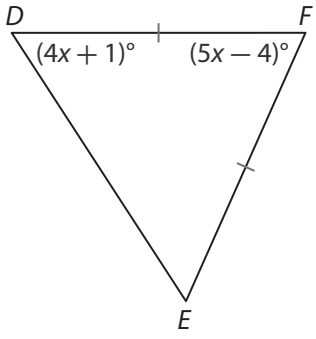
5. m∠*R*



6. m∠*O*

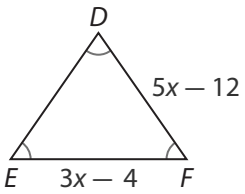


7. m∠*E*

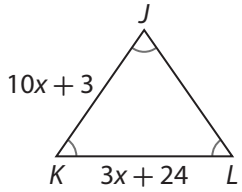


Find the length of the indicated side.

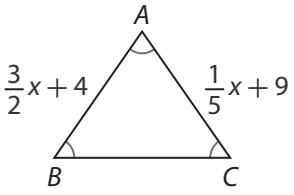
8. $\overline{DE}$



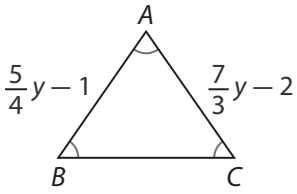
9. $\overline{KL}$



10. $\overline{AB}$

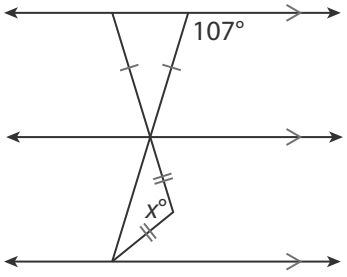


11. $\overline{BC}$

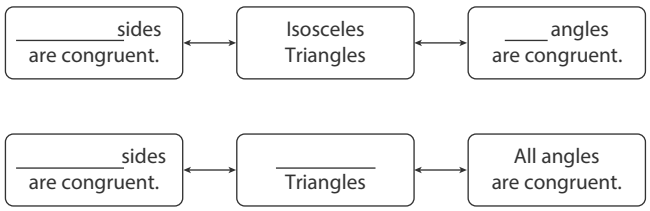


12. Given $\triangle JKL$ with $m\angle J = 63°$ and $m\angle L = 54°$, is the triangle an acute, isosceles, obtuse, or right triangle?

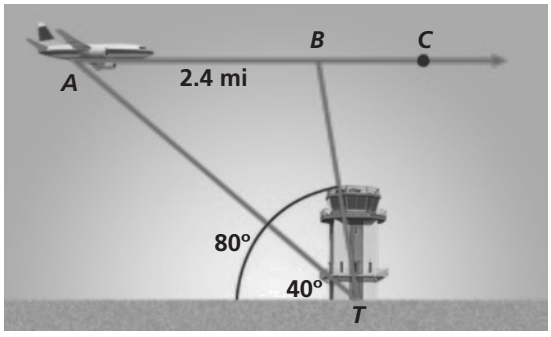
13. Find x. Explain your reasoning. The horizontal lines are parallel.



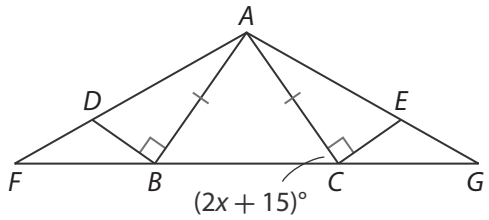
14. **Summarize** Complete the diagram to show the cause and effect of the theorems covered in the lesson. Explain why the arrows show the direction going both ways.



15. A plane is flying parallel to the ground along $\overrightarrow{AC}$. When the plane is at A, an air-traffic controller in tower T measures the angle to the plane as 40°. After the plane has traveled 2.4 miles to B, the angle to the plane is 80°. How can you find BT?



16. John is building a doghouse. He decides to use the roof truss design shown. If $m\angle DBF = 35°$, what is the measure of the vertex angle of the isosceles triangle?



17. The measure of the vertex angle of an isosceles triangle is 12 more than 5 times the measure of a base angle. Determine the sum of the measures of the base angles.

18. **Justify Reasoning** Determine whether each of the following statements is true or false. Select the correct answer for each lettered part. Explain your reasoning.

a. All isosceles triangles have at least two acute angles. ◯ True ◯ False

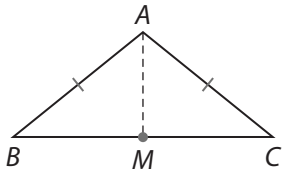
b. If the perimeter of an equilateral triangle is P, then the length of each of its sides is $\frac{P}{3}$. ◯ True ◯ False

c. All isosceles triangles are equilateral triangles. ◯ True ◯ False

d. If you know the length of one of the legs of an isosceles triangle, you can determine its perimeter. ◯ True ◯ False

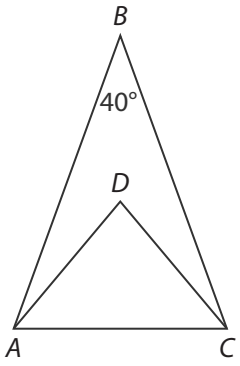
e. The exterior angle of an equilateral triangle is obtuse. ◯ True ◯ False

19. **Critical Thinking** Prove $\angle B \cong \angle C$, given point M is the midpoint of $\overline{BC}$.



Statements	Reasons

20. Given that $\triangle ABC$ is an isosceles triangle and $\overline{AD}$ and $\overline{CD}$ are angle bisectors, what is m$\angle ADC$?



H.O.T. Focus on Higher Order Thinking

21. **Analyze Relationships** Isosceles right triangle ABC has a right angle at B and $\overline{AB} \cong \overline{CB}$. $\overline{BD}$ bisects angle B, and point D is on $\overline{AC}$. If $\overline{BD} \perp \overline{AC}$, describe triangles ABD and CBD. Explain. HINT: Draw a diagram.

Communicate Mathematical Ideas **Follow the method to construct a triangle. Then use what you know about the radius of a circle to explain the congruence of the sides.**

22. Construct an isosceles triangle. Explain how you know that two sides are congruent.

- Use a compass to draw a circle. Mark two different points on the circle.
- Use a straightedge to draw a line segment from the center of the circle to each of the two points on the circle (radii).
- Draw a line segment (chord) between the two points on the circle.

I know two sides are congruent because _____

_____

_____

23. Construct an equilateral triangle. Explain how you know the three sides are congruent.

- Use a compass to draw a circle.
- Draw another circle of the same size that goes through the center of the first circle. (Both should have the same radius length.)
- Mark one point where the circles intersect.
- Use a straightedge to draw line segments connecting both centers to each other and to the intersection point.

I know the three sides are congruent because _____

_____

_____

_____

_____

Lesson Performance Task

The control tower at airport A is in contact with an airplane flying at point P, when it is 5 miles from the airport, and 30 seconds later when it is at point Q, 4 miles from the airport. The diagram shows the angles the plane makes with the ground at both times. If the plane flies parallel to the ground from P to Q at constant speed, how fast is it traveling?