Name ______________________ Class ______________ Date ________

5.4 SSS Triangle Congruence

Essential Question: What does the SSS Triangle Congruence Theorem tell you about triangles?

Resource Locker

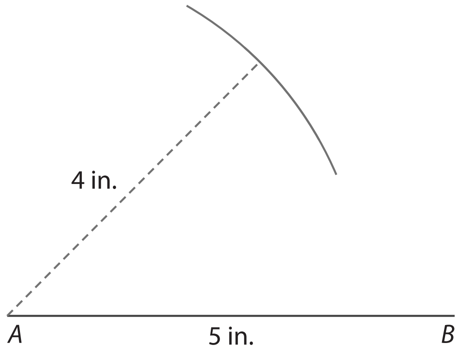
Explore Constructing Triangles Given Three Side Lengths

Two triangles are congruent if and only if a rigid motion transformation maps one triangle onto the other triangle. Many theorems can also be used to identify congruent triangles.

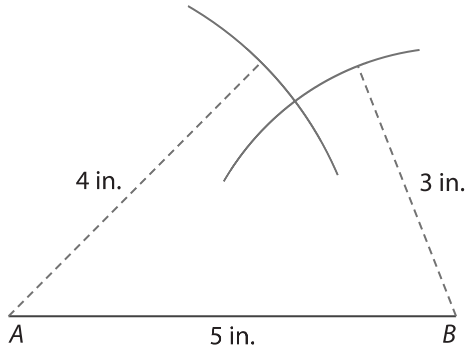
Follow these steps to construct a triangle with sides of length 5 in., 4 in., and 3 in. Use a ruler, compass, and either tracing paper or a transparency.

(A) Use a ruler to draw a line segment of length 5 inches. Label the endpoints *A* and *B*.

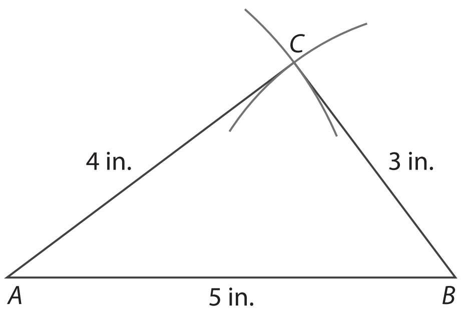
(B) Open a compass to 4 inches. Place the point of the compass on *A*, and draw an arc as shown.



(C) Now open the compass to 3 inches. Place the point of the compass on *B*, and draw a second arc.



(D) Next, find the intersection of the two arcs. Label the intersection C. Draw $\overline{AC}$ and $\overline{BC}$. Label the side lengths on the figure.



(E) Repeat steps A through D to draw $\triangle DEF$ on a separate piece of tracing paper. The triangle should have sides with the same lengths as $\triangle CAB$. Start with a segment that is 4 in. long. Label the endpoints *D* and *E* as shown.

(F) Compare $\triangle CAB$ and $\triangle DEF$. Are they congruent? How do you know?

Reflect

1. **Discussion** When you construct $\triangle CAB$, how do you know that the intersection of the two arcs is a distance of 4 inches from A and 3 inches from B?

2. Compare your triangles to those made by other students. Are they all congruent? Explain.

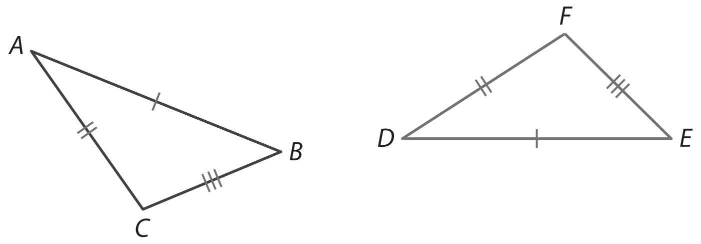
Explain 1 Justifying SSS Triangle Congruence

You can use rigid motions and the converse of the Perpendicular Bisector Theorem to justify this theorem.

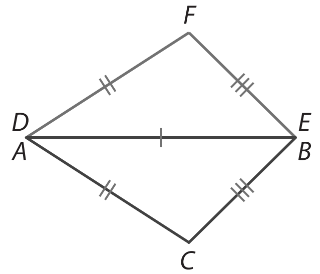
SSS Triangle Congruence Theorem

If three sides of one triangle are congruent to three sides of another triangle, then the triangles are congruent.

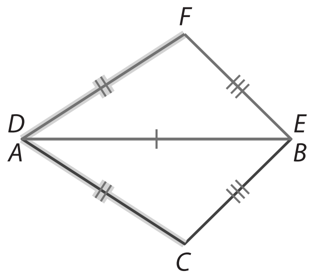
Example 1 **In the triangles shown, let $\overline{AB} \cong \overline{DE}$, $\overline{AC} \cong \overline{DF}$, and $\overline{BC} \cong \overline{EF}$. Use rigid motions to show that $\triangle ABC \cong \triangle DEF$.**



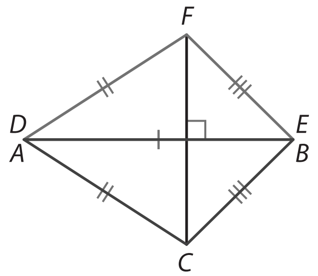
(A) Transform $\triangle ABC$ by a translation along $\overrightarrow{AD}$ followed by a rotation about point D, so that $\overline{AB}$ and $\overline{DE}$ coincide. The segments coincide because they are the same length.



Does a reflection across $\overline{AB}$ map point C to point F? To show this, notice that $DC = DF$, which means that point D is equidistant from point C and point F.

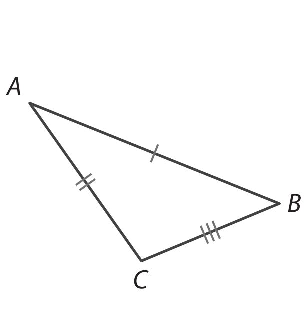


Therefore, point D lies on the perpendicular bisector of $\overline{CF}$ by the converse of the perpendicular bisector theorem. Because $EC = EF$, point E also lies on the perpendicular bisector of $\overline{CF}$.

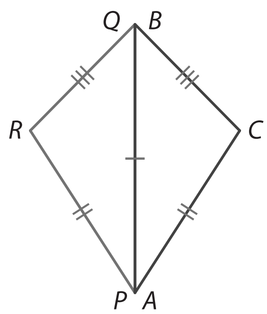


Since point D and point E both lie on the perpendicular bisector of $\overline{CF}$ and there is a unique line through any two points, $\overleftrightarrow{DE}$ is the perpendicular bisector of $\overline{CF}$. By the definition of reflection, the image of point C must be point F. Therefore, $\triangle ABC$ is mapped onto $\triangle DEF$ by a translation, followed by a rotation, followed by a reflection, and the two triangles are congruent.

(B) Show that $\triangle ABC \cong \triangle PQR$.



Triangle ABC is transformed by a sequence of rigid motions to form the figure shown below. Identify the sequence of rigid motions. (You will complete the proof on the following page.)

	**1.** **2.** **3.**

Complete the explanation by filling in the blanks with the name of a point, line segment, or geometric theorem.

Because $\overline{QR} \cong$ __________, point Q is equidistant from __________ and __________. Therefore, by the converse of the ______________________________ Theorem, point Q lies on the ______________________________ of $\overline{RC}$. Similarly, $\overline{PR} \cong$ __________. So point __________ lies on the perpendicular bisector of __________. Because two points determine a line, the line $\overleftrightarrow{PQ}$ is the ______________________________.

By the definition of reflection, the image of point C must be point __________. Therefore, $\triangle ABC \cong \triangle PQR$ because $\triangle ABC$ is mapped to __________ by a translation, a rotation, and a __________.

Reflect

3. Can you conclude that two triangles are congruent if two pairs of corresponding sides are congruent? Explain your reasoning and include an example.

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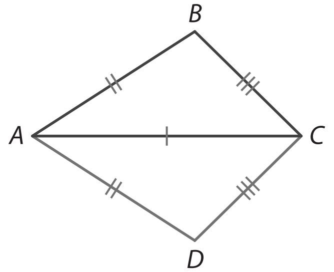
Your Turn

4. Use rigid motions and the converse of the perpendicular bisector theorem to explain why $\triangle ABC \cong \triangle ADC$.



Explain 2 Proving Triangles Are Congruent Using SSS Triangle Congruence

You can apply the SSS Triangle Congruence Theorem to confirm that triangles are congruent. Remember, if any one pair of corresponding parts of two triangles is not congruent, then the triangles are not congruent.

Example 2 **Prove that the triangles are congruent or explain why they are not congruent.**

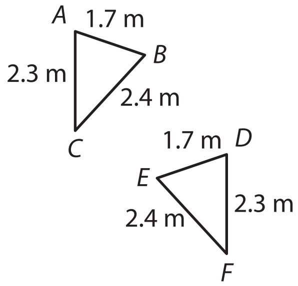
Ⓐ $AB = DE = 1.7$ m, so $\overline{AB} \cong \overline{DE}$.

$BC = EF = 2.4$ m, so $\overline{BC} \cong \overline{EF}$.

$AC = DF = 2.3$ m, so $\overline{AC} \cong \overline{DF}$.

The three sides of $\triangle ABC$ are congruent to the three sides of $\triangle DEF$.

$\triangle ABC \cong \triangle DEF$ by the SSS Triangle Congruence Theorem.



Ⓑ $DE =$ ______ $= 20$ cm, so ______________.

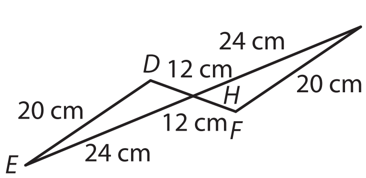
$DH =$ ______ $= 12$ cm, so ______________.

$EH =$ ______ $= 24$ cm, so ______________.

The three sides of $\triangle DEH$ are congruent to

the three sides of ______________, so the two triangles are

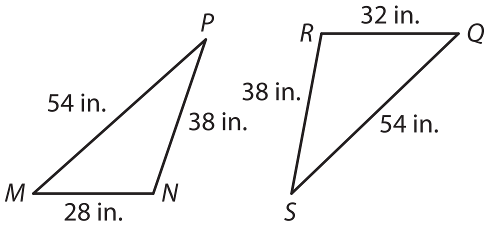
congruent by ________________________________.



Your Turn

Prove that the triangles are congruent or explain why they are not congruent.

5.



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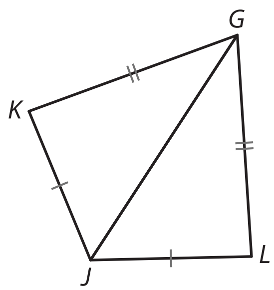

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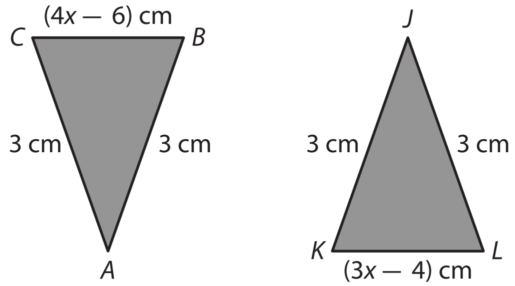
Explain 3 Applying Triangle Congruence

You can use the SSS Triangle Congruence Theorem and other triangle congruence theorems to solve many real-world problems that involve congruent triangles.

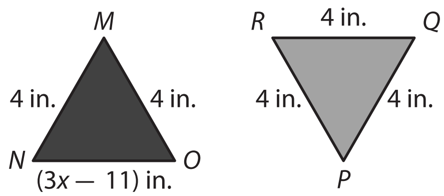
Example 3 **Find the value of x for which you can show the triangles are congruent.**

(A) Lexi bought matching triangular pendants for herself and her mom in the shapes shown. For what value of x can you use a triangle congruence theorem to show that the pendants are congruent? Which triangle congruence theorem can you use? Explain.



$\overline{AB} \cong \overline{JK}$ and $\overline{AC} \cong \overline{JL}$, because they have the same measure. So, if $\overline{BC} \cong \overline{KL}$, then $\triangle ABC \cong \triangle JKL$ by the SSS Triangle Congruence Theorem. Write an equation setting the lengths equal and solve for x. $4x - 6 = 3x - 4$; $x = 2$

(B) Adeline made a design using triangular tiles as shown. For what value of x can you use a triangle congruence theorem to show that the tiles are congruent? Which triangle congruence theorem can you use? Explain.



Notice that $\overline{PQ} \cong \overline{MN}$ and ____________ $\cong \overline{MO}$, because they have the same measure.

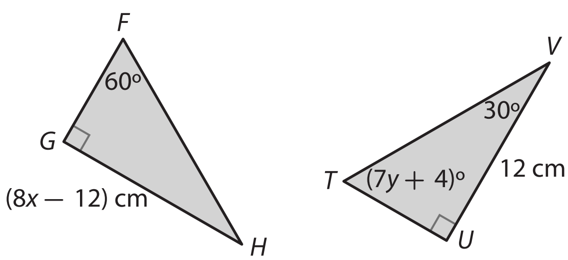
If $\overline{NO} \cong \overline{QR}$, then $\triangle MNO \cong$ ____________ by the ____________ Triangle Congruence Theorem.

Write an equation setting the lengths equal and solve for x.

____

Your Turn

7. Craig made a mobile using geometric shapes including triangles shaped as shown. For what value of x and y can you use a triangle congruence theorem to show that the triangles are congruent? Which triangle congruence theorem can you use? Explain.



Elaborate

8. An isosceles triangle has two sides of equal length. If we ask everyone in class to construct an isosceles triangle that has one side of length 8 cm and another side of length 12 cm, how many sets of congruent triangles might the class make?

9. **Essential Question Check-In** How do you explain the SSS Triangle Congruence Theorem?

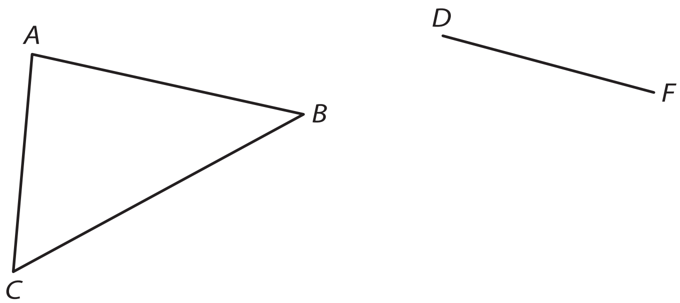
Evaluate: Homework and Practice

Personal Math Trainer

- Online Homework
- Hints and Help
- Extra Practice

Use a compass and a straightedge to complete the drawing of $\triangle DEF$ so that it is congruent to $\triangle ABC$.

1.



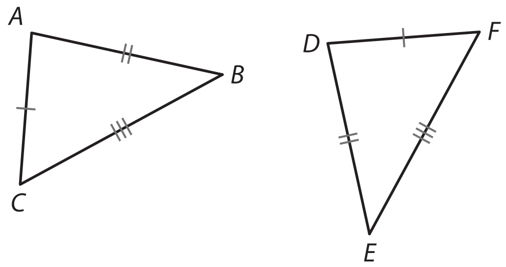
On a separate piece of paper, use a compass and a ruler to construct two congruent triangles with the given side lengths. Label the lengths of the sides.

2. 3 in., 3.5 in., 4 in.

3. 3 cm, 11 cm, 12 cm

Identify a sequence of rigid motions that maps one side of $\triangle ABC$ onto one side of $\triangle DEF$.

4.



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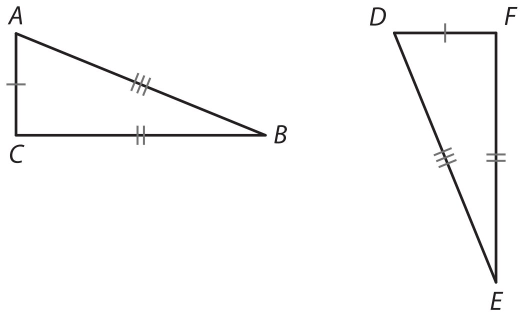

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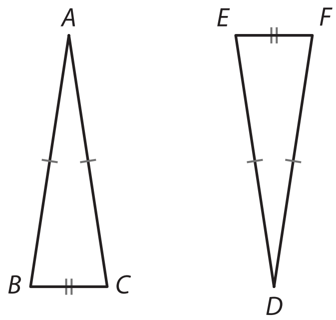

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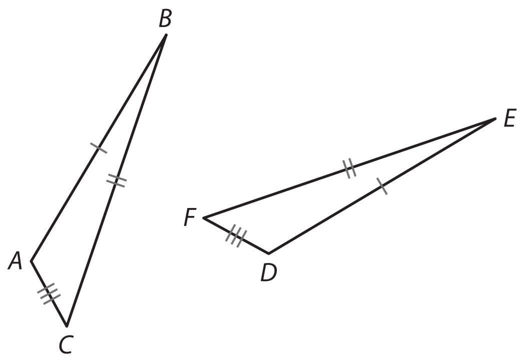
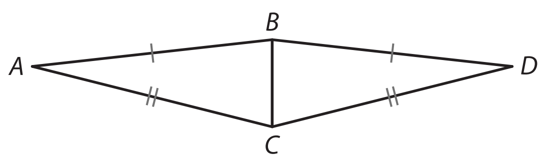


In each figure, identify the perpendicular bisector and the line segment it bisects, and explain how to use the information to show that the two triangles are congruent.

8.



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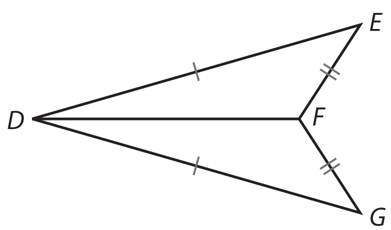
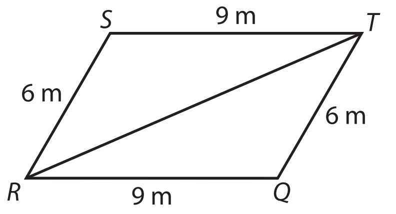


Prove that the triangles are congruent or explain why this is not possible.

10.



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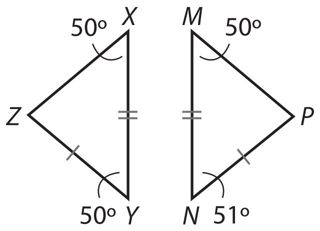

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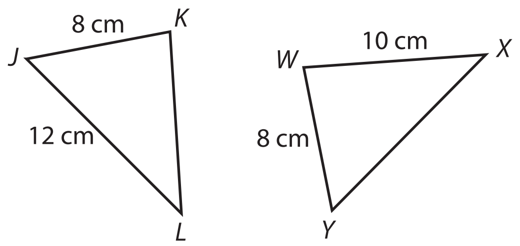

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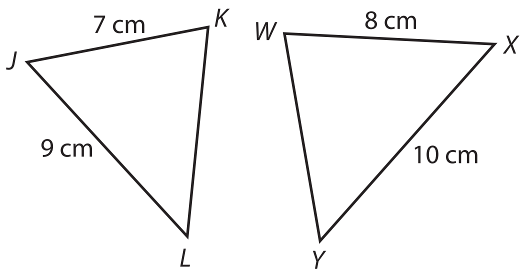
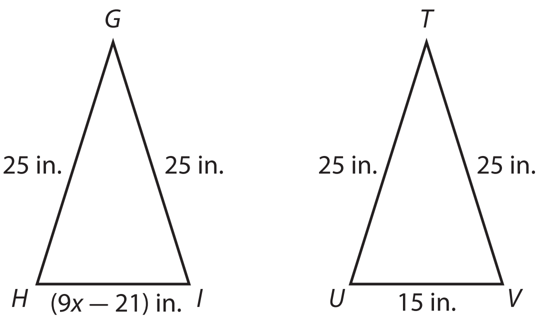
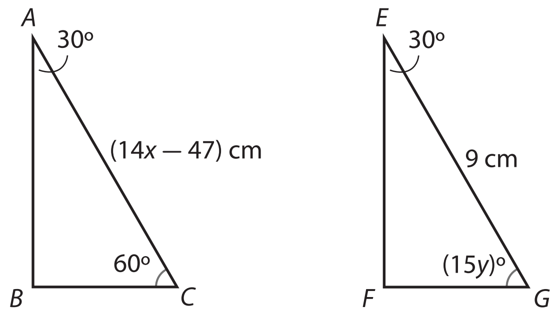


14. Carol bought two chairs with triangular backs. For what value of *x* can you use a triangle congruence theorem to show that the triangles are congruent? Which triangle congruence theorem can you use? Explain.



15. For what values of x and y can you use a triangle congruence theorem to show that the triangles are congruent? Which triangle congruence theorem can you use? Explain.



Find all possible solutions for x such that $\triangle ABC$ is congruent to $\triangle DEF$. One or more of the problems may have no solution.

16. $\triangle ABC$: sides of length 6, 8, and x.
$\triangle DEF$: sides of length 6, 9, and $x - 1$.

17. $\triangle ABC$: sides of length 3, $x + 1$, and 14.
$\triangle DEF$: sides of length 13, $x - 9$, and $2x - 6$

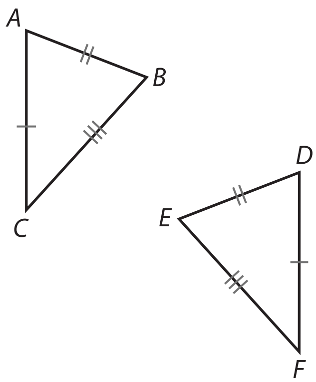
18. $\triangle ABC$: sides of length 17, 17, and $2x + 1$.
$\triangle DEF$: sides of length 17, 17, and $3x - 9$

19. $\triangle ABC$: sides of length 19, 25, and $5x - 2$.
$\triangle DEF$: sides of length 25, 28, and $4 - y$

20. $\triangle ABC$: sides of length 8, $x - y$, and $x + y$
$\triangle DEF$: sides of length 8, 15, and 17

21. $\triangle ABC$: sides of length 9, x, and $2x - y$
$\triangle DEF$: sides of length 8, 9, and $2y - x$

22. These statements are part of an explanation for the SSS Triangle Congruence Theorem. Write the numbers 1 to 6 to place these strategies in a logical order. The statements refer to triangles ABC and DEF shown here.



______________ Rotate the image of $\triangle ABC$ about E, so that the image of $\overline{BC}$ coincides with $\overline{EF}$.

______________ Apply the definition of reflection to show D is the reflection of A across $\overleftrightarrow{EF}$.

______________ Conclude that $\triangle ABC \cong \triangle DEF$ because a sequence of rigid motions maps one triangle onto the other.

______________ Translate $\triangle ABC$ along $\overrightarrow{BE}$.

______________ Define $\overleftrightarrow{EF}$ as the perpendicular bisector of the line connecting D and the image of A.

______________ Identify E, and then F, as equidistant from D and the image of A.

23. Determine whether the given information is sufficient to guarantee that two triangles are congruent. Select the correct answer for each lettered part.

A. The triangles have three pairs of congruent corresponding angles. ◯ sufficient ◯ not sufficient

B. The triangles have three pairs of congruent corresponding sides. ◯ sufficient ◯ not sufficient

C. The triangles have two pairs of congruent corresponding sides and one pair of congruent corresponding angles. ◯ sufficient ◯ not sufficient

D. The triangles have two pairs of congruent corresponding angles and one pair of congruent corresponding sides. ◯ sufficient ◯ not sufficient

E. Two angles and the included side of one triangle are congruent to two angles and the included side of the other triangle. ◯ sufficient ◯ not sufficient

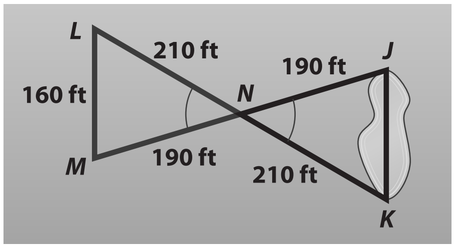
F. Two sides and the included angle of one triangle are congruent to two sides and the included angle of the other triangle. ◯ sufficient ◯ not sufficient

24. Make a Conjecture Does a version of SSS congruence apply to quadrilaterals? Provide an example to support your answer.

25. Are two triangles congruent if all pairs of corresponding angles are congruent? Support your answer with an example.

H.O.T. Focus on Higher Order Thinking

26. Explain the Error Ava wants to know the distance *JK* across a pond. She locates points as shown. She says that the distance across the pond must be 160 ft by the SSS Triangle Congruence Theorem. Explain her error.

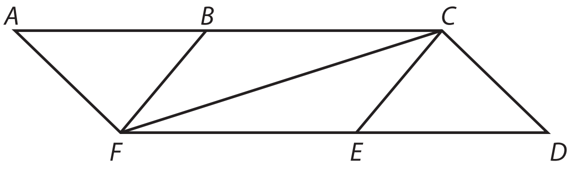


27. Analyze Relationships Write a proof.

Given: $\angle BFC \cong \angle ECF$, $\angle BCF \cong \angle EFC$

$\overline{AB} \cong \overline{DE}$, $\overline{AF} \cong \overline{DC}$

Prove: $\triangle ABF \cong \triangle DEC$



Statements	Reasons

Lesson Performance Task

Mike and Michelle each hope to get a contract with the city to build benches for commuters to sit on while waiting for buses. The benches must be stable so that they don't collapse, and they must be attractive. Their designs are shown. Judge the two benches on stability and attractiveness. Explain your reasoning.