Name ______________________ Class ____________ Date ________

5.3 SAS Triangle Congruence

Essential Question: **What does the SAS Triangle Congruence Theorem tell you about triangles?**

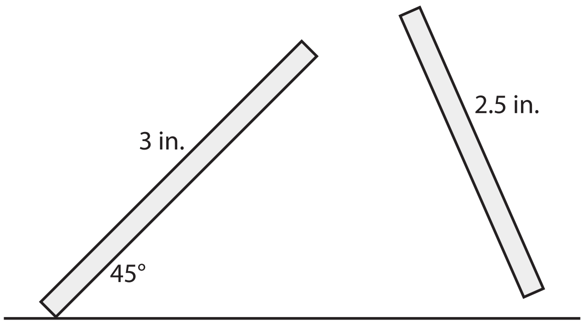
Resource Locker

Explore 1 Drawing Triangles Given Two Sides and an Angle

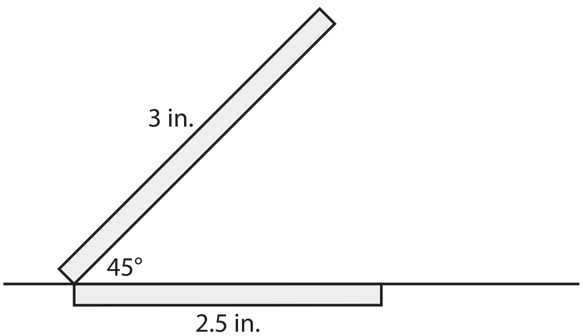
You know that when all corresponding parts of two triangles are congruent, then the triangles are congruent. Sometimes you can determine that triangles are congruent based on less information.

For this activity, cut two thin strips of paper, one 3 in. long and the other 2.5 in. long.

(A) On a sheet of paper use a straightedge to draw a horizontal line. Arrange the 3 in. strip to form a 45° angle, as shown. Next, arrange the 2.5 in. strip to complete the triangle. How many different triangles can you form? Support your answer with a diagram.



(B) Now arrange the two strips of paper to form a 45° angle so that the angle is *included* between the two consecutive sides, as shown. With this arrangement, can you construct more than one triangle? Why or why not?



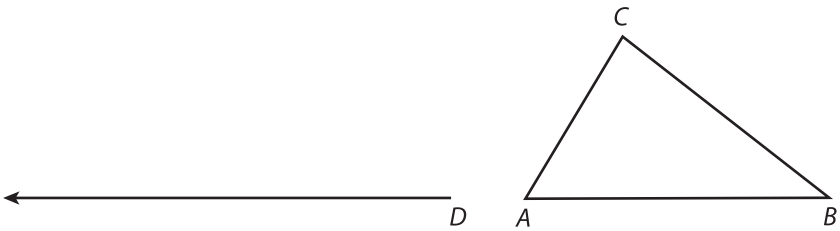
Reflect

1. **Discussion** If two triangles have two pairs of congruent corresponding sides and one pair of congruent corresponding angles, under what conditions can you conclude that the triangles must be congruent? Explain.

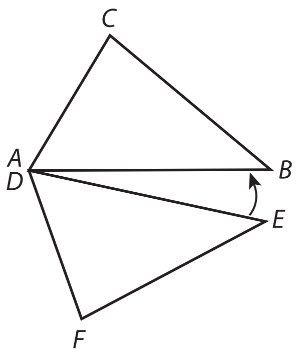
Explore 2 Justifying SAS Triangle Congruence

You can explain the results of Explore 1 using transformations.

(A) Construct $\triangle DEF$ by copying $\angle A$, side $\overline{AB}$, and side $\overline{AC}$. Let point D correspond to point A, point E correspond to point B, and point F correspond to point C, and place point E on the segment shown.



(B) The diagram illustrates one step in a sequence of rigid motions that will map $\triangle DEF$ onto $\triangle ABC$. Describe a complete sequence of rigid motions that will map $\triangle DEF$ onto $\triangle ABC$.



(C) What can you conclude about the relationship between $\triangle ABC$ and $\triangle DEF$? Explain your reasoning.

Reflect

2. Is it possible to map $\triangle DEF$ onto $\triangle ABC$ using a single rigid motion? If so, describe the rigid motion.

Explain 1 Deciding Whether Triangles are Congruent Using SAS Triangle Congruence

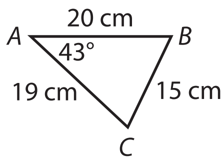
What you explored in the previous two activities can be summarized in a theorem. You can use this theorem and the definition of congruence in terms of rigid motions to determine whether two triangles are congruent.

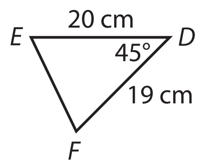
SAS Triangle Congruence Theorem

If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, then the triangles are congruent.

Example 1 **Determine whether the triangles are congruent. Explain your reasoning.**

Ⓐ Look for congruent corresponding parts.

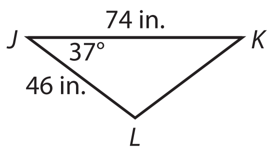


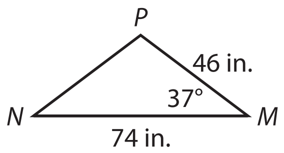


- Sides $\overline{DE}$ and $\overline{DF}$ do not correspond to side $\overline{BC}$, because they are not 15 cm long.
- $\overline{DE}$ corresponds to $\overline{AB}$, because $DE = AB = 20$ cm.
- $\overline{DF}$ corresponds to $\overline{AC}$, because $DF = AC = 19$ cm.
- $\angle A$ and $\angle D$ are corresponding angles because they are included between pairs of corresponding sides, but they don't have the same measure.

The triangles are not congruent, because there is no sequence of rigid motions that maps $\triangle ABC$ onto $\triangle DEF$.

Ⓑ





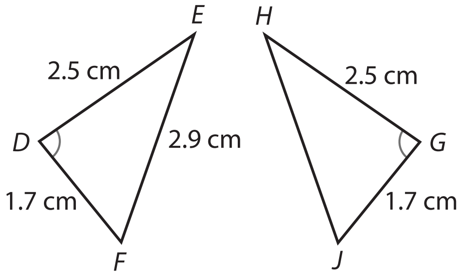
Look for congruent corresponding parts.

- $\overline{JL}$ corresponds to ______, because $JL =$ ______ $=$ ______ in.
- ______ corresponds to MN, because ______ $= MN = 74$ in.
- ______ corresponds to ______, because ______ $=$ ______ $=$ ______.

Two sides and the included angle of $\triangle JKL$ are congruent to two sides and the included angle of ________. $\triangle JKL \cong$ ______ by the ______________________________.

Your Turn

3. Determine whether the triangles are congruent. Explain your reasoning.



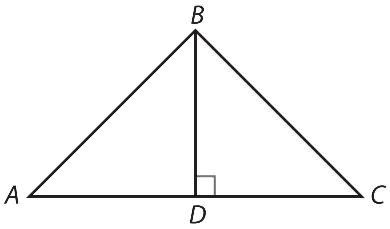
Explain 2 Proving Triangles Are Congruent Using SAS Triangle Congruence

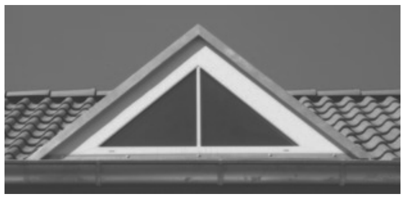
Theorems about congruent triangles can be used to show that triangles in real-world objects are congruent.

Example 2 **Write each proof.**

(A) Write a proof to show that the two halves of a triangular window are congruent if the vertical post is the perpendicular bisector of the base.

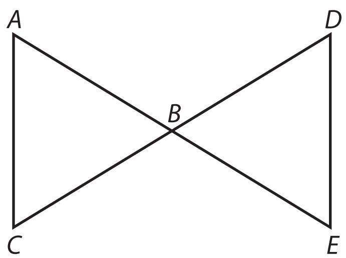
Given: $\overline{BD}$ is the perpendicular bisector of $\overline{AC}$.
Prove: $\triangle BDA \cong \triangle BDC$





It is given that $\overline{BD}$ is the perpendicular bisector of $\overline{AC}$. By the definition of a perpendicular bisector, $AD = CD$, which means $\overline{AD} \cong \overline{CD}$, and $\overline{BD} \perp \overline{AC}$, which means $\angle BDA$ and $\angle BDC$ are congruent right angles. In addition, $\overline{BD} \cong \overline{BD}$ by the reflexive property of congruence. So two sides and the included angle of $\triangle BDA$ are congruent to two sides and the included angle of $\triangle BDC$. The triangles are congruent by the SAS Triangle Congruence Theorem.

(B) Given: $\overline{CD}$ bisects $\overline{AE}$ and $\overline{AE}$ bisects $\overline{CD}$
Prove: $\triangle ABC \cong \triangle EBD$

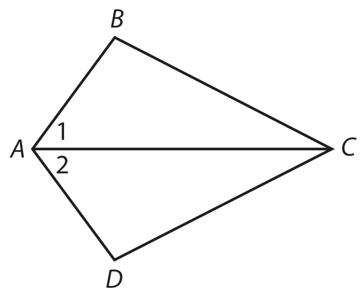


It is given that $\overline{CD}$ bisects $\overline{AE}$ and $\overline{AE}$ bisects $\overline{CD}$. So by the definition of a bisector, $AB = EB$ and ____________, which makes $\overline{AB} \cong \overline{EB}$ and ____________. $\angle ABC \cong$ ____________ because they are ________________. So two sides and the ____________ angle of $\triangle ABC$ are congruent to two sides and the ____________ angle of $\triangle EBD$. The triangles are congruent by the ________________________________.

Your Turn

4. Given: $\overline{AB} \cong \overline{AD}$ and $\angle 1 \cong \angle 2$

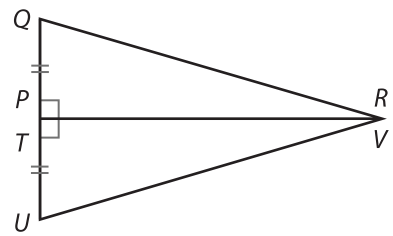
Prove: $\triangle BAC \cong \triangle DAC$



Elaborate

5. Explain why the corresponding angles must be *included* angles in order to use the SAS Triangle Congruence Theorem.

6. Jeffrey draws $\triangle PQR$ and $\triangle TUV$. He uses a translation to map point P to point T and point R to point V as shown. What should be his next step in showing the triangles are congruent? Why?



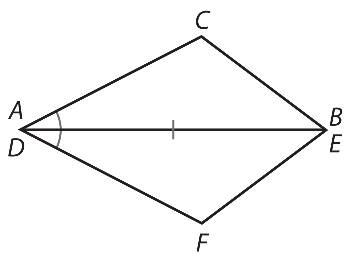
7. **Essential Question Check-In** If two triangles share a common side, what else must be true for the SAS Triangle Congruence Theorem to apply?

Evaluate: Homework and Practice

Personal Math Trainer

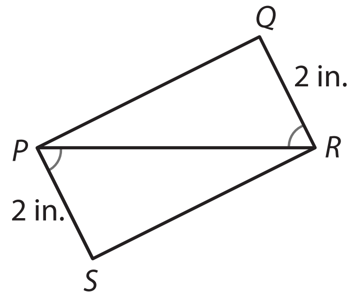
- Online Homework
- Hints and Help
- Extra Practice

1. Sarah performs rigid motions mapping point A to point D and point B to point E, as shown. Does she have enough information to confirm that the triangles are congruent? Explain your reasoning.



Determine whether the triangles are congruent. Explain your reasoning.

2.



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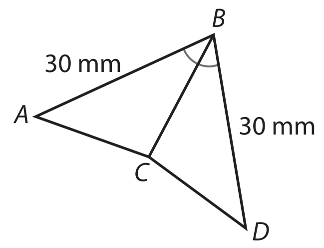

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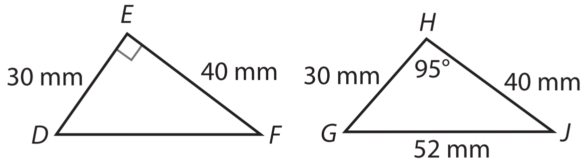

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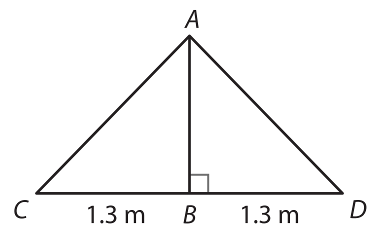
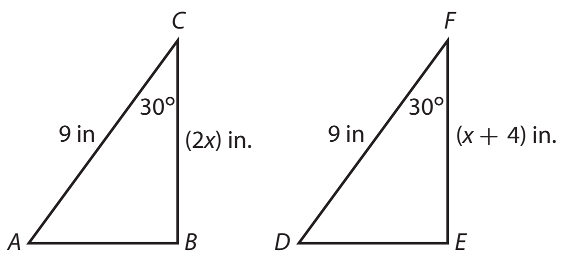


Find the value of the variable that results in congruent triangles. Explain.

6.



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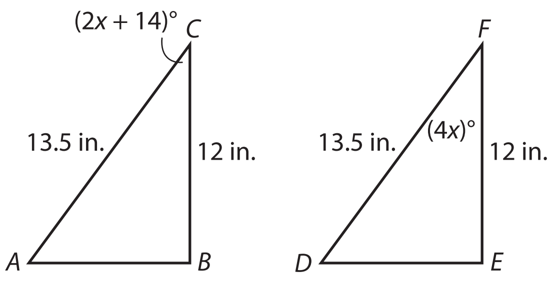
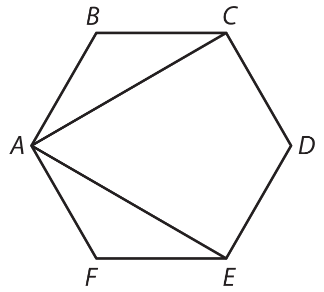
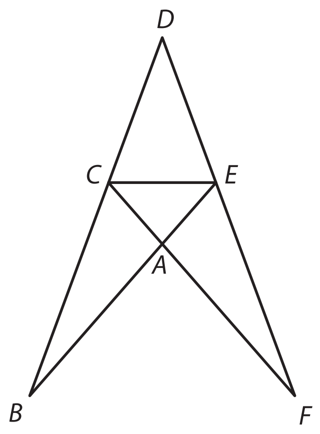


8. Given that polygon *ABCDEF* is a regular hexagon, prove that $\overline{AC} \cong \overline{AE}$.

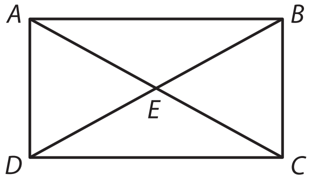


Statements	Reasons

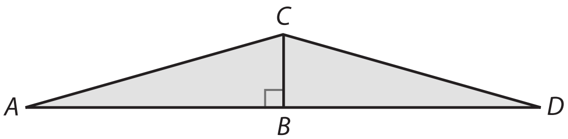
9. A product designer is designing an easel with extra braces as shown in the diagram. Prove that if $\overline{BD} \cong \overline{FD}$ and $\overline{CD} \cong \overline{ED}$, then the braces $\overline{BE}$ and $\overline{FC}$ are also congruent.



10. An artist is framing a large picture and wants to put metal poles across the back to strengthen the frame as shown in the diagram. If the metal poles are both the same length and they bisect each other, prove that $\overline{AB} \cong \overline{CD}$ and $\overline{AD} \cong \overline{CB}$.



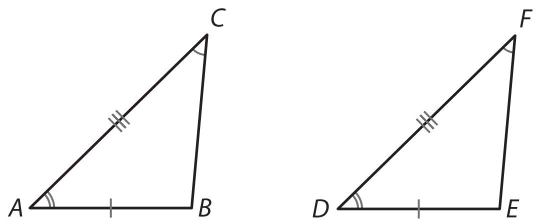
11. The figure shows a side panel of a skateboard ramp. Kalim wants to confirm that the right triangles in the panel are congruent.



a. What measurements should Kalim take if he wants to confirm that the triangles are congruent by SAS? Explain.

b. What measurements should Kalim take if he wants to confirm that the triangles are congruent by ASA? Explain.

12. Which of the following are reasons that justify why the triangles are congruent? Select all that apply.



A. SSA Triangle Congruence Theorem

B. SAS Triangle Congruence Theorem

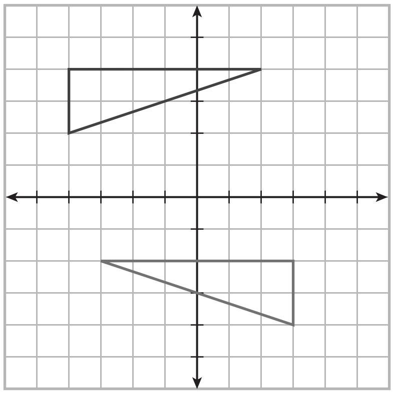
C. ASA Triangle Congruence Theorem

D. Converse of CPCTC

E. CPCTC

H.O.T. Focus on Higher Order Thinking

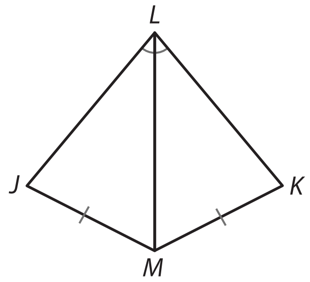
13. Multi-Step Refer to the following diagram to answer each question.



a. Use a triangle congruence theorem to explain why these triangles are congruent.

b. Describe a sequence of rigid motions to map the top triangle onto the bottom triangle to confirm that they are congruent.

14. **Explain the Error** Mark says that the diagram confirms that a given angle and two given side lengths determine a unique triangle even if the angle is not an included angle. Explain Mark's error.



15. **Justify Reasoning** The opposite sides of a rectangle are congruent. Can you conclude that a diagonal of a rectangle divides the rectangle into two congruent triangles? Justify your response.

Lesson Performance Task

The diagram of the Great Pyramid at Giza gives the approximate lengths of edge $\overline{AB}$ and slant height $\overline{AC}$. The slant height is the perpendicular bisector of $\overline{BD}$. Find the perimeter of $\triangle ABD$. Explain how you found the answer.