Name _____ Class _____ Date _____

5.2 ASA Triangle Congruence

Essential Question: **What does the ASA Triangle Congruence Theorem tell you about triangles?**

Resource Locker

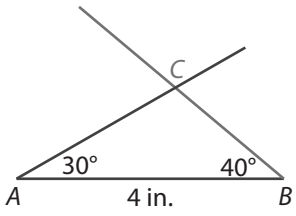
Explore 1 Drawing Triangles Given Two Angles and a Side

You have seen that two triangles are congruent if they have six pairs of congruent corresponding parts. However, it is not always possible to check all three pairs of corresponding sides and all three pairs of corresponding angles. Fortunately, there are shortcuts for determining whether two triangles are congruent.

A Draw a segment that is 4 inches long. Label the endpoints A and B.

B Use a protractor to draw a 30° angle so that one side is $\overline{AB}$ and its vertex is point A.

C Use a protractor to draw a 40° angle so that one side is $\overline{AB}$ and its vertex is point B. Label the point where the sides of the angles intersect as point C.



D Put your triangle and a classmate's triangle beside each other. Is there a sequence of rigid motions that maps one to the other? What does this tell you about the triangles?

_____

Reflect

1. In a polygon, the side that connects two consecutive angles is the *included side* of those two angles. Describe the triangle you drew using the term *included side*. Be as precise as possible.

_____

_____

2. **Discussion** Based on your results, how can you decide whether two triangles are congruent without checking that all six pairs of corresponding sides and corresponding angles are congruent?

_____

_____

_____

Explore 2 Justifying ASA Triangle Congruence

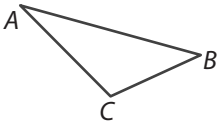
Explain the results of Explore 1 using transformations.

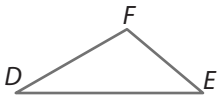
(A) Use tracing paper to make two copies of the triangle from Explore 1 as shown. Identify the corresponding parts you know to be congruent and mark these congruent parts on the figure.

$\angle A \cong$ _____

$\angle B \cong$ _____

$\overline{AB} \cong$ _____





(B) What can you do to show that these triangles are congruent?

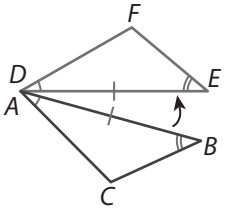
_____

(C) Translate $\triangle ABC$ so that point A maps to point D. What translation vector did you use?

_____

(D) Use a rotation to map point B to point E. What is the center of the rotation? What is the angle of the rotation?

_____

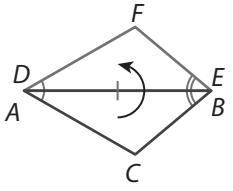


(E) How do you know the image of point B is point E?

_____

(F) What rigid motion do you think will map point C to point F?

_____



(G) To show that the image of point C is point F, notice that $\angle A$ is reflected across $\overleftrightarrow{DE}$, so the measure of the angle is preserved. Since $\angle A \cong \angle D$ you can conclude that the image of $\overline{AC}$ lies on _____. In particular, the image of point C must lie on _____. By similar reasoning, the image of $\overline{BC}$ lies on _____ and the image of point C must lie on _____. The only point that lies on both $\overline{DF}$ and $\overline{EF}$ is _____.

(H) Describe the sequence of rigid motions used to map $\triangle ABC$ to $\triangle DEF$.

_____

Reflect

3. **Discussion** Arturo said the argument in the activity works for any triangles with two pairs of congruent corresponding angles, and it is not necessary for the included sides to be congruent. Do you agree? Explain.

_____

_____

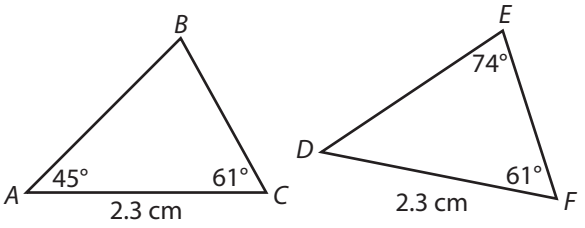
Explain 1 Deciding Whether Triangles Are Congruent Using ASA Triangle Congruence

You can state your findings about triangle congruence as a theorem. This theorem can help you decide whether two triangles are congruent.

ASA Triangle Congruence Theorem

If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, then the triangles are congruent.

Example 1 **Determine whether the triangles are congruent. Explain your reasoning.**



(A) **Step 1** Find m∠*D*.

$$m\angle D + m\angle E + m\angle F = 180°$$
$$m\angle D + 74° + 61° = 180°$$
$$m\angle D + 135° = 180°$$
$$m\angle D = 45°$$

Step 2 Compare the angle measures and side lengths.

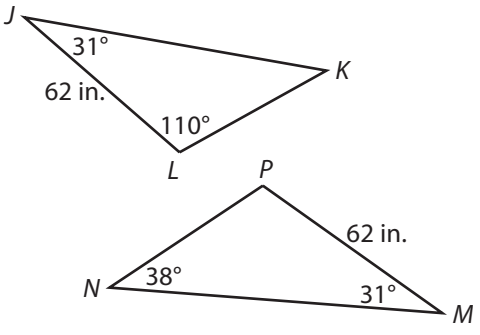
$m\angle A = m\angle D = 45°$, $AC = DF = 2.3$ cm, and $m\angle C = m\angle F = 61°$

So, $\angle A \cong \angle D$, $\overline{AC} \cong \overline{DF}$, and $\angle C \cong \angle F$.

$\angle A$ and $\angle C$ include side $\overline{AC}$, and $\angle D$ and $\angle F$ include side $\overline{DF}$.

So, $\triangle ABC \cong \triangle DEF$ by the ASA Triangle Congruence Theorem.

(B) **Step 1** Find m∠*P*.



$$m\angle M + m\angle N + m\angle P = 180°$$
$$\square° + \square° + m\angle P = 180°$$
$$\square° + m\angle P = 180°$$
$$m\angle P = \square°$$

Step 2 Compare the angle measures and side lengths.

None of the angles in $\triangle MNP$ has a measure of ☐.
Therefore, there [is/is not] a sequence of rigid motions that maps $\triangle MNP$ onto $\triangle JKL$, and $\triangle MNP$ [is/is not] congruent to $\triangle JKL$.

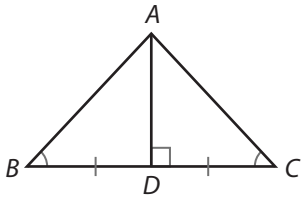
Reflect

4. In Part B, do you need to find m∠*K*? Why or why not?

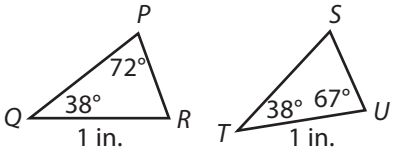
Your Turn

Determine whether the triangles are congruent. Explain your reasoning.

5.



6.



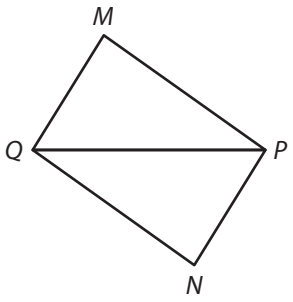
Explain 2 Proving Triangles Are Congruent Using ASA Triangle Congruence

The ASA Triangle Congruence Theorem may be used as a reason in a proof.

Example 2 **Write each proof.**

(A) Given: $\angle MQP \cong \angle NPQ$, $\angle MPQ \cong \angle NQP$

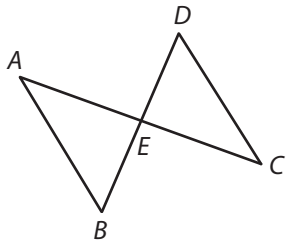
Prove: $\triangle MQP \cong \triangle NPQ$



Statements	Reasons
1. $\angle MQP \cong \angle NPQ$	**1.** Given
2. $\angle MPQ \cong \angle NQP$	**2.** Given
3. $\overline{QP} \cong \overline{QP}$	**3.** Reflexive Property of Congruence
4. $\triangle MQP \cong \triangle NPQ$	**4.** ASA Triangle Congruence Theorem

Ⓑ Given: $\angle A \cong \angle C$, E is the midpoint of $\overline{AC}$.

Prove: $\triangle AEB \cong \triangle CED$



Statements	Reasons
1. $\angle A \cong \angle C$	**1.**
2. E is the midpoint of $\overline{AC}$.	**2.**
3. $\overline{AE} \cong \overline{CE}$	**3.**
4. $\angle AEB \cong \angle CED$	**4.**
5. $\triangle AEB \cong \triangle CED$	**5.**

Reflect

7. In Part B, suppose the length of $\overline{AB}$ is 8.2 centimeters. Can you determine the length of any other segments in the figure? Explain.

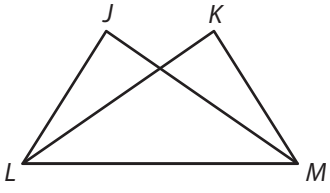
_____

Your Turn

Write each proof.

8. Given: $\angle JLM \cong \angle KML$, $\angle JML \cong \angle KLM$

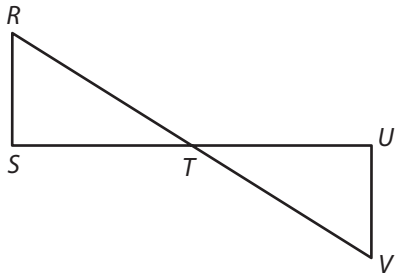
Prove: $\triangle JML \cong \triangle KLM$



Statements	Reasons

9. Given: $\angle S$ and $\angle U$ are right angles, $\overline{RV}$ bisects $\overline{SU}$.

Prove: $\triangle RST \cong \triangle VUT$



Statements	Reasons

Elaborate

10. **Discussion** Suppose you and a classmate both draw triangles with a 30° angle, a 70° angle, and a side that is 3 inches long. How will they compare? Explain your reasoning.

11. **Discussion** How can a diagram show you that corresponding parts of two triangles are congruent without providing specific angle measures or side lengths?

12. **Essential Question Check-In** What must be true in order for you to use the ASA Triangle Congruence Theorem to prove that triangles are congruent?

Evaluate: Homework and Practice

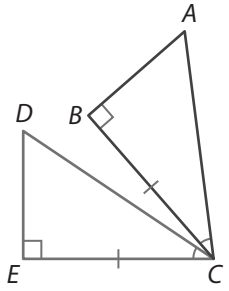
- Online Homework
- Hints and Help
- Extra Practice

1. Natasha draws a segment $\overline{PQ}$ that is 6 centimeters long. She uses a protractor to draw a 60° angle so that one side is $\overline{PQ}$ and its vertex is point *P*. Then she uses a protractor to draw an 35° angle so that one side is $\overline{PQ}$ and its vertex is point *Q*.

a. Draw a triangle following the instructions that Natasha used. Label the vertices and the known side and angle measures.

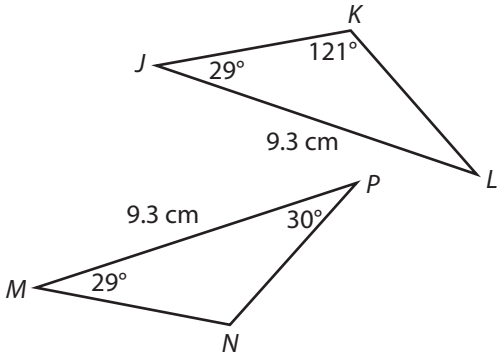
b. Will there be a sequence of rigid motions that will map your triangle onto Natasha's triangle? Explain.

2. Tomas drew two triangles, as shown, so that $\angle B \cong \angle E$, $\overline{BC} \cong \overline{EC}$, and $\angle ACB \cong \angle DCE$. Describe a sequence of one or more rigid motions Tomas can use to show that $\triangle ABC \cong \triangle DEC$.

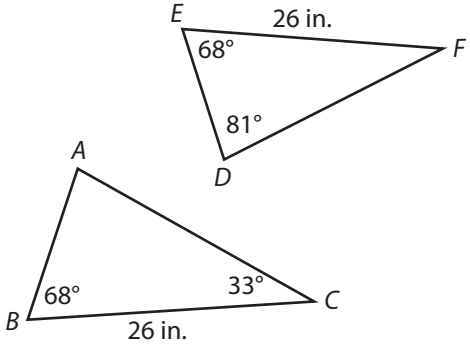


Determine whether the triangles are congruent. Explain your reasoning.

3.

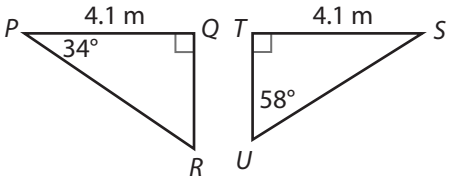


4.

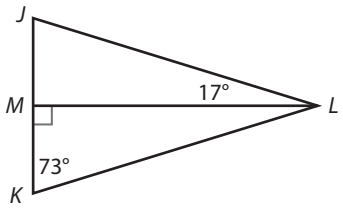


Determine whether the triangles are congruent. Explain your reasoning.

5.



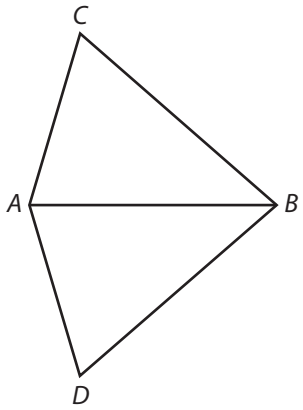
6.



Write each proof.

7. **Given:** $\overline{AB}$ bisects $\angle CAD$ and $\angle CBD$.

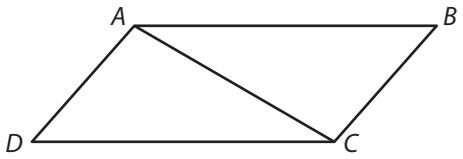
Prove: $\triangle CAB \cong \triangle DAB$



Statements	Reasons
1. $\overline{AB}$ bisects $\angle CAD$ and $\angle CBD$.	**1.**
2. $\angle CAB \cong \angle DAB$	**2.** Definition of bisector
3.	**3.** Definition of bisector
4.	**4.** Reflexive Property of Congruence
5. $\triangle CAB \cong \triangle DAB$	**5.**

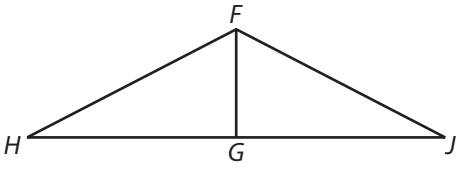
8. **Given:** $\overline{AB}$ is parallel to $\overline{CD}$, $\angle ACB \cong \angle CAD$.

Prove: $\triangle ABC \cong \triangle CDA$

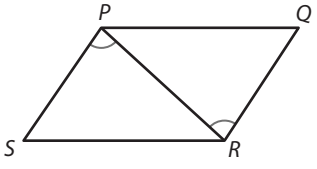


9. **Given:** $\angle H \cong \angle J$, G is the midpoint of $\overline{HJ}$, $\overline{FG}$ is perpendicular to $\overline{HJ}$.

Prove: $\triangle FGH \cong \triangle FGJ$



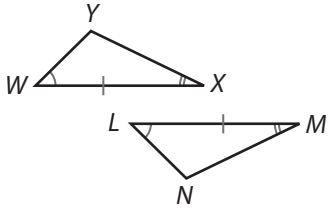
10. The figure shows quadrilateral $PQRS$. What additional information do you need in order to conclude that $\triangle SPR \cong \triangle QRP$ by the ASA Triangle Congruence Theorem? Explain.



11. **Communicate Mathematical Ideas** In the figure, $\overline{WX}$ is parallel to $\overline{LM}$.

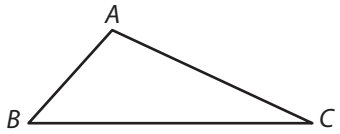
a. Describe a sequence of two rigid motions that maps $\triangle LMN$ to $\triangle WXY$.

b. How can you be sure that point N maps to point Y?

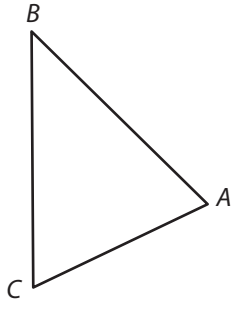


Use a compass and straightedge and the ASA Triangle Congruence Theorem to construct a triangle that is congruent to $\triangle ABC$.

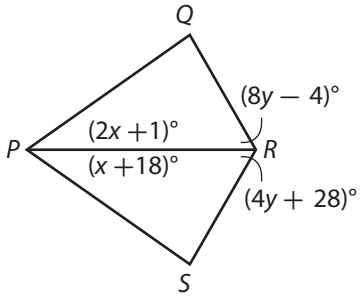
12.



13.



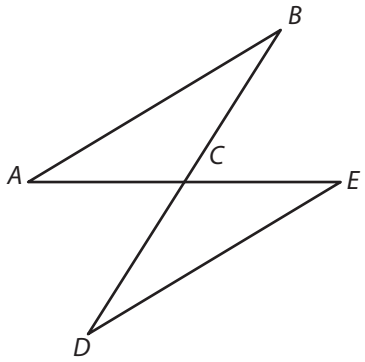
14. **Multi-Step** For what values of the variables is $\triangle QPR$ congruent to $\triangle SPR$? In this case, what is $m\angle Q$?



Write each proof.

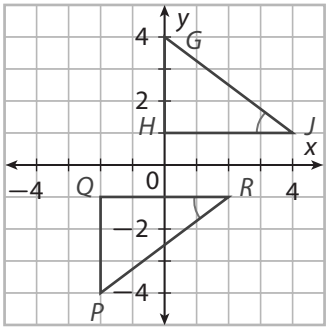
15. Given: $\angle A \cong \angle E$, C is the midpoint of $\overline{AE}$.

Prove: $\overline{AB} \cong \overline{ED}$



16. The figure shows $\triangle GHJ$ and $\triangle PQR$ on a coordinate plane.

a. Explain why the triangles are congruent using the ASA Triangle Congruence Theorem.



b. Explain why the triangles are congruent using rigid motions.

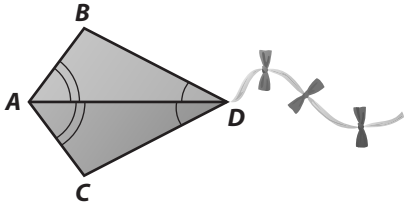
17. **Justify Reasoning** A factory makes triangular traffic signs. Each sign is an equilateral triangle with three 60° angles. Explain why two signs that each have a side 36 inches long must be congruent.

18. **Represent Real-World Problems** Rob is making the kite shown in the figure.

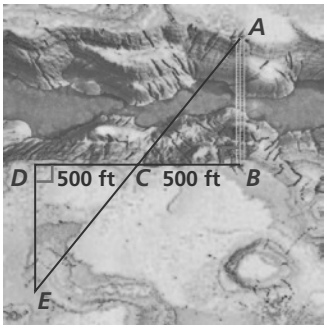
a. Can Rob conclude that $\triangle ABD \cong \triangle ACD$? Why or why not?

b. Rob says that $AB = AC$ and $BD = CD$. Do you agree? Explain.

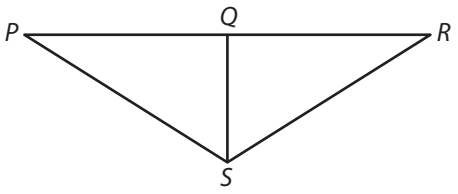
c. Given that $BD = x + 15$ cm and $AB = x$ cm, write an expression for the distance around the kite in centimeters.



19. In order to find the distance across a canyon, Mariela sites a tree across the canyon (point A) and locates points on her side of the canyon as shown. Explain how she can use this information to find the distance AB across the canyon.



20. Determine whether each of the following provides enough information to prove that $\triangle SQP \cong \triangle SQR$. Select the correct answer for each lettered part.



a. Q is the midpoint of $\overline{PR}$. ◯ Yes ◯ No

b. $\angle P \cong \angle R$ ◯ Yes ◯ No

c. $\angle SQP$ is a right angle, $\angle PSQ \cong \angle RSQ$ ◯ Yes ◯ No

d. $\angle SQP$ is a right angle, $m\angle P = 32°$, $m\angle RSQ = 58°$. ◯ Yes ◯ No

e. $\angle P \cong \angle R$, $\angle PSQ \cong \angle RSQ$ ◯ Yes ◯ No

H.O.T. Focus on Higher Order Thinking

21. **Counterexamples** Jasmine said that the ASA Triangle Congruence Theorem works for quadrilaterals. That is, if two angles and the included side of one quadrilateral are congruent to two angles and the included side of another quadrilateral, then the quadrilaterals are congruent. Sketch and mark a figure of two quadrilaterals as a counterexample to show that Jasmine is incorrect.

22. **Critique Reasoning** $\triangle ABC$ and $\triangle DEF$ are both right triangles and both triangles contain a 30° angle. Both triangles have a side that is 9.5 mm long. Yoshio claims that he can use the ASA Triangle Congruence Theorem to show that the triangles are congruent. Do you agree? Explain.

23. **Draw Conclusions** Do you think there is an ASAS Congruence Theorem for quadrilaterals? Suppose two quadrilaterals have a pair of congruent consecutive angles with a pair of congruent included sides and an additional pair of congruent corresponding sides. Must the quadrilaterals be congruent? Justify your response.

Lesson Performance Task

The flag of the Congo Republic consists of green and red right triangles separated by a yellow parallelogram. Construct an argument to prove that $\triangle BAF \cong \triangle EDC$.