Name ______________________ Class ______________ Date ________

4.4 Perpendicular Lines

Essential Question: What are the key ideas about perpendicular bisectors of a segment?

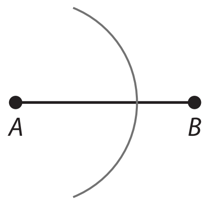
Resource Locker

Explore Constructing Perpendicular Bisectors and Perpendicular Lines

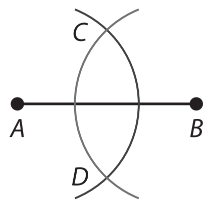
You can construct geometric figures without using measurement tools like a ruler or a protractor. By using geometric relationships and a compass and a straightedge, you can construct geometric figures with greater precision than figures drawn with standard measurement tools.

In Steps A–C, construct the perpendicular bisector of $\overline{AB}$.

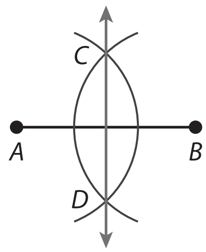
(A) Place the point of the compass at point *A*. Using a compass setting that is greater than half the length of $\overline{AB}$, draw an arc.



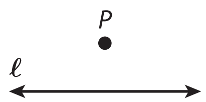
(B) Without adjusting the compass, place the point of the compass at point *B* and draw an arc intersecting the first arc in two places. Label the points of intersection *C* and *D*.



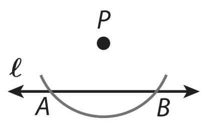
(C) Use a straightedge to draw $\overline{CD}$, which is the perpendicular bisector of $\overline{AB}$.



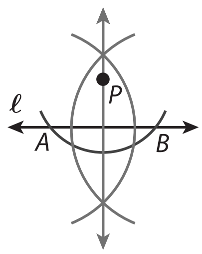
In Steps D–E, construct a line perpendicular to a line ℓ that passes through some point *P* that is not on ℓ.



(D) Place the point of the compass at *P*. Draw an arc that intersects line ℓ at two points, *A* and *B*.



(E) Use the methods in Steps A–C to construct the perpendicular bisector of $\overline{AB}$.



Because it is the perpendicular bisector of $\overline{AB}$, then the constructed line through *P* is perpendicular to line ℓ.

Reflect

1. In Step A of the first construction, why do you open the compass to a setting that is greater than half the length of $\overline{AB}$?

2. **What If?** Suppose Q is a point *on* line ℓ. Is the construction of a line perpendicular to ℓ through Q any different than constructing a perpendicular line through a point P *not* on the line, as in Steps D and E?

Explain 1 Proving the Perpendicular Bisector Theorem Using Reflections

You can use reflections and their properties to prove a theorem about perpendicular bisectors. These theorems will be useful in proofs later on.

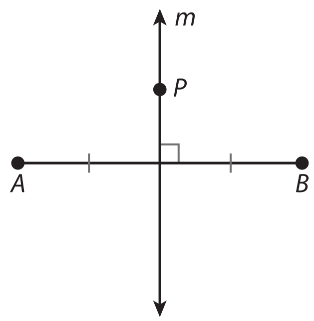
Perpendicular Bisector Theorem

If a point is on the perpendicular bisector of a segment, then it is equidistant from the endpoints of the segment.

Example 1 **Prove the Perpendicular Bisector Theorem.**

Given: P is on the perpendicular bisector m of $\overline{AB}$.

Prove: $PA = PB$



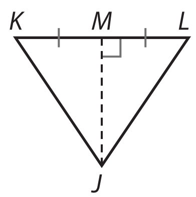
Consider the reflection across ________. Then the reflection of point P across line m is also ________ because point P lies on ________, which is the line of reflection.

Also, the reflection of ________ across line m is B by the definition of ________.

Therefore, $PA = PB$ because ________ preserves distance.

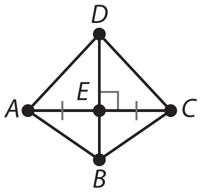
Reflect

3. **Discussion** What conclusion can you make about $\triangle KLJ$ in the diagram using the Perpendicular Bisector Theorem?



Your Turn

Use the diagram shown. $\overline{BD}$ is the perpendicular bisector of $\overline{AC}$.



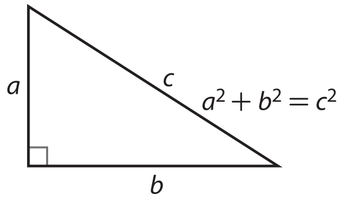
4. Suppose $ED = 16$ cm and $DA = 20$ cm. Find DC.

5. Suppose $EC = 15$ cm and $BA = 25$ cm. Find BC.

Explain 2 Proving the Converse of the Perpendicular Bisector Theorem

The converse of the Perpendicular Bisector Theorem is also true. In order to prove the converse, you will use an *indirect proof* and the *Pythagorean Theorem*.

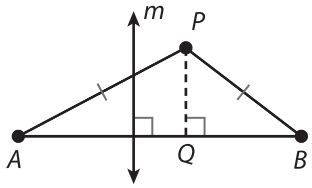
In an **indirect proof**, you assume that the statement you are trying to prove is false. Then you use logic to lead to a contradiction of given information, a definition, a postulate, or a previously proven theorem. You can then conclude that the assumption was false and the original statement is true.



Recall that the Pythagorean Theorem states that for a right triangle with legs of length a and b and a hypotenuse of length c, $a^2 + b^2 = c^2$.

Converse of the Perpendicular Bisector Theorem
If a point is equidistant from the endpoints of a segment, then it lies on the perpendicular bisector of the segment.

Example 2 **Prove the Converse of the Perpendicular Bisector Theorem**



Given: $PA = PB$

Prove: P is on the perpendicular bisector m of $\overline{AB}$.

Step A: Assume what you are trying to prove is false.

Assume that P is *not* on the perpendicular bisector m of ____________.
Then, when you draw a perpendicular line from P to the line containing A and B, it intersects $\overline{AB}$ at point Q, which is not the ____________ of $\overline{AB}$.

Step B: Complete the following to show that this assumption leads to a contradiction.

$\overline{PQ}$ forms two right triangles, $\triangle AQP$ and ________________.

So, $AQ^2 + QP^2 = PA^2$ and $BQ^2 + QP^2 =$ ☐ by the ________________ Theorem.

Subtract these equations:

$$AQ^2 + QP^2 = PA^2$$
$$\underline{BQ^2 + QP^2 = PB^2}$$
$$AQ^2 - BQ^2 = PA^2 - PB^2$$

However, $PA^2 - PB^2 = 0$ because ________________.

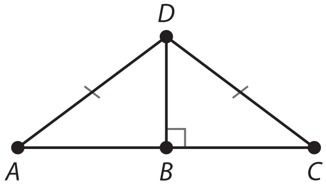
Therefore, $AQ^2 - BQ^2 = 0$. This means that $AQ^2 = BQ^2$ and $AQ = BQ$. This contradicts the fact that Q is not the midpoint of $\overline{AB}$. Thus, the initial assumption must be incorrect, and P must lie on the __________________________ of $\overline{AB}$.

Reflect

6. In the proof, once you know $AQ^2 = BQ^2$, why can you conclude that $AQ = BQ$?

Your Turn

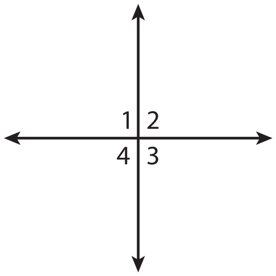
7. $\overline{AD}$ is 10 inches long. $\overline{BD}$ is 6 inches long. Find the length of $\overline{AC}$.



Explain 3 Proving Theorems about Right Angles

The symbol ⊥ means that two figures are perpendicular. For example, $\ell \perp m$ or $\overleftrightarrow{XY} \perp \overline{AB}$.

Example 3 **Prove each theorem about right angles.**



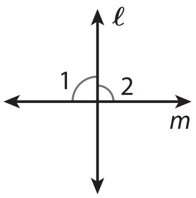
Ⓐ If two lines intersect to form one right angle, then they are perpendicular and they intersect to form four right angles.

Given: $m\angle 1 = 90°$ **Prove:** $m\angle 2 = 90°$, $m\angle 3 = 90°$, $m\angle 4 = 90°$

Statement	Reason
1. $m\angle 1 = 90°$	**1.** Given
2. ∠1 and ∠2 are a linear pair.	**2.** Given
3. ∠1 and ∠2 are supplementary.	**3.** Linear Pair Theorem
4. $m\angle 1 + m\angle 2 = 180°$	**4.** Definition of supplementary angles
5. $90° + m\angle 2 = 180°$	**5.** Substitution Property of Equality
6. $m\angle 2 = 90°$	**6.** Subtraction Property of Equality
7. $m\angle 2 = m\angle 4$	**7.** Vertical Angles Theorem
8. $m\angle 4 = 90°$	**8.** Substitution Property of Equality
9. $m\angle 1 = m\angle 3$	**9.** Vertical Angles Theorem
10. $m\angle 3 = 90°$	**10.** Substitution Property of Equality

Ⓑ If two intersecting lines form a linear pair of angles with equal measures, then the lines are perpendicular.

Given: $m\angle 1 = m\angle 2$ **Prove:** $\ell \perp m$



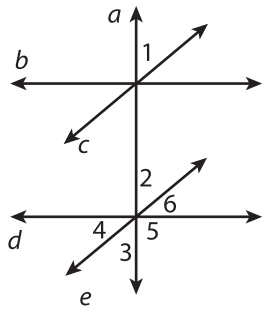
By the diagram, ∠1 and ∠2 form a linear pair so ∠1 and ∠2 are supplementary by the ____________________. By the definition of supplementary angles, $m\angle 1 + m\angle 2 =$ ________. It is also given that ____________________, so $m\angle 1 + m\angle 1 = 180°$ by the ______________________________. Adding gives $2 \cdot m\angle 1 = 180°$, and $m\angle 1 = 90°$ by the Division Property of Equality. Therefore, ∠1 is a right angle and $\ell \perp$ m by the ______________________________.

Reflect

8. State the converse of the theorem in Part B. Is the converse true?

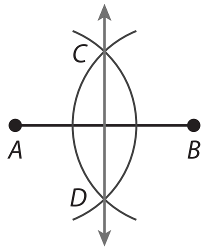
Your Turn

9. Given: $b \parallel d$, $c \parallel e$, $m\angle 1 = 50°$, and $m\angle 5 = 90°$. Use the diagram to find $m\angle 4$.



Elaborate

10. Discussion Explain how the converse of the Perpendicular Bisector Theorem justifies the compass-and-straightedge construction of the perpendicular bisector of a segment.

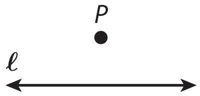


11. Essential Question Check-In How can you construct perpendicular lines and prove theorems about perpendicular bisectors?

Evaluate: Homework and Practice

- Online Homework
- Hints and Help
- Extra Practice

1. How can you construct a line perpendicular to line ℓ that passes through point P using paper folding?

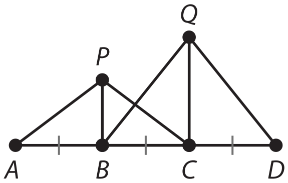


2. **Check for Reasonableness** How can you use a ruler and a protractor to check the construction in Elaborate Exercise 10?

3. Describe the point on the perpendicular bisector of a segment that is closest to the endpoints of the segment.

4. **Represent Real-World Problems** A field of soybeans is watered by a rotating irrigation system. The watering arm, $\overline{CD}$, rotates around its center point. To show the area of the crop of soybeans that will be watered, construct a circle with diameter CD.

Use the diagram to find the lengths. $\overline{BP}$ is the perpendicular bisector of $\overline{AC}$. $\overline{CQ}$ is the perpendicular bisector of $\overline{BD}$. $AB = BC = CD$.



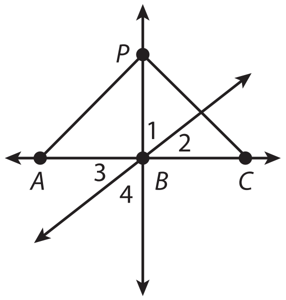
5. Suppose $AP = 5$ cm. What is the length of $\overline{PC}$?

6. Suppose $AP = 5$ cm and $BQ = 8$ cm. What is the length of $\overline{QD}$?

7. Suppose $AC = 12$ cm and $QD = 10$ cm. What is the length of $\overline{QC}$?

8. Suppose $PB = 3$ cm and $AD = 12$ cm . What is the length of $\overline{PC}$?

Given: $PA = PC$ and $BA = BC$. Use the diagram to find the lengths or angle measures described.

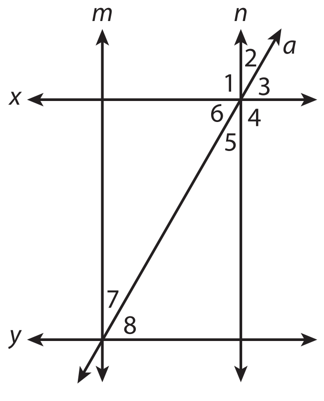


9. Suppose $m\angle 2 = 38°$. Find $m\angle 1$.

10. Suppose $PA = 10$ cm and $PB = 6$ cm. What is the length of $\overline{AC}$?

11. Find $m\angle 3 + m\angle 4$.

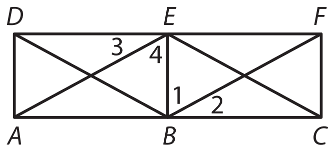
Given: $m \parallel n$, $x \parallel y$, and $y \perp m$. Use the diagram to find the angle measures.



12. Suppose $m\angle 7 = 30°$. Find $m\angle 3$.

13. Suppose $m\angle 1 = 90°$. What is $m\angle 2 + m\angle 3 + m\angle 5 + m\angle 6$?

Use this diagram of trusses for a railroad bridge in Exercise 14.



14. Suppose $\overline{BE}$ is the perpendicular bisector of $\overline{DF}$. Which of the following statements do you know are true? Select all that apply. Explain your reasoning.

A. $BD = BF$

B. $m\angle 1 + m\angle 2 = 90°$

C. E is the midpoint of $\overline{DF}$.

D. $m\angle 3 + m\angle 4 = 90°$

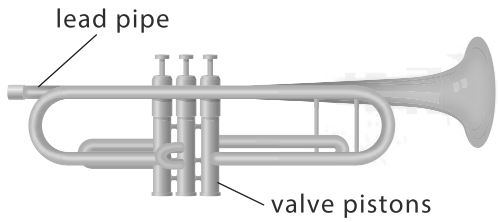
E. $\overline{DA} \perp \overline{AC}$

15. Algebra Two lines intersect to form a linear pair with equal measures. One angle has the measure $2x°$ and the other angle has the measure $(20y - 10)°$. Find the values of x and y. Explain your reasoning.

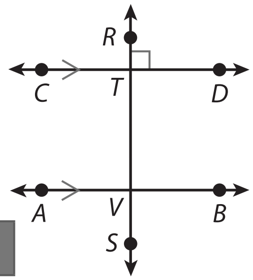
16. Algebra Two lines intersect to form a linear pair of congruent angles. The measure of one angle is $(8x + 10)°$ and the measure of the other angle is $\left(\frac{15y}{2}\right)°$. Find the values of x and y. Explain your reasoning.

H.O.T. Focus on Higher Order Thinking

17. Communicate Mathematical Ideas The valve pistons on a trumpet are all perpendicular to the lead pipe. Explain why the valve pistons must be parallel to each other.



18. **Justify Reasoning** Prove the theorem: In a plane, if a transversal is perpendicular to one of two parallel lines, then it is perpendicular to the other.



Given: $\overline{RS} \perp \overline{CD}$ and $\overline{AB} \parallel \overline{CD}$ Prove: $\overline{RS} \perp \overline{AB}$

Statements	Reasons

19. **Analyze Mathematical Relationships** Complete the indirect proof to show that two supplementary angles cannot both be obtuse angles.

Given: $\angle 1$ and $\angle 2$ are supplementary.

Prove: $\angle 1$ and $\angle 2$ cannot both be obtuse.

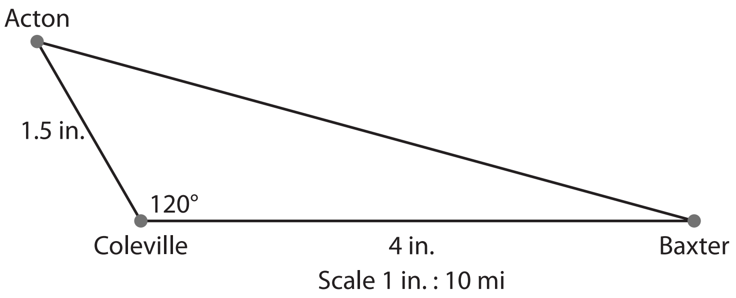
Assume that two supplementary angles *can* both be obtuse angles. So, assume that $\angle 1$ and $\angle 2$ ____________. Then $m\angle 1 > 90°$ and $m\angle 2 >$ ☐ by ____________________. Adding the two inequalities, $m\angle 1 + m\angle 2 >$ ☐. However, by the definition of supplementary angles, ______________. So $m\angle 1 + m\angle 2 > 180°$ contradicts the given information.

This means the assumption is ______, and therefore

____________________.

Lesson Performance Task

A utility company wants to build a wind farm to provide electricity to the towns of Acton, Baxter, and Coleville. Because of concerns about noise from the turbines, the residents of all three towns do not want the wind farm built close to where they live. The company comes to an agreement with the residents to build the wind farm at a location that is equally distant from all three towns.



a. Use the drawing to draw a diagram of the locations of the towns using a scale of 1 in. : 10 mi. Draw the 4-inch and 1.5-inch lines with a 120° angle between them. Write the actual distances between the towns on your diagram.

b. Estimate where you think the wind farm will be located.

c. Use what you have learned in this lesson to find the exact location of the wind farm. What is the approximate distance from the wind farm to each of the three towns?