Name ______________________ Class ______________ Date ________

4.3 Proving Lines are Parallel

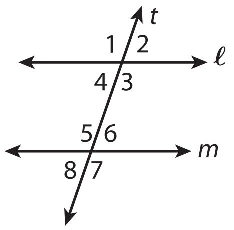
Essential Question: How can you prove that two lines are parallel?

Resource Locker

Explore Writing Converses of Parallel Line Theorems

You form the **converse** of and if-then statement "if p, then q" by swapping p and q. The converses of the postulate and theorems you have learned about lines cut by a transversal are true statements. In the Explore, you will write specific cases of each of these converses.

The diagram shows two lines cut by a transversal t. Use the diagram and the given statements in Steps A–D. You will complete the statements based on your work in Steps A–D.



Statements	
lines ℓ and m are parallel	$\angle 4 \cong \angle \square$
$\angle 6$ and $\angle \square$ are supplementary	$\angle \square \cong \angle 7$

(A) Use two of the given statements together to complete a statement about the diagram using the Same-Side Interior Angles Postulate.

By the postulate: If ______________________________, then $\angle 6$ and $\angle \square$ are supplementary.

(B) Now write the converse of the Same-Side Interior Angles Postulate using the diagram and your statement in Step A.

By its converse: If ______________________________,

then ______________________________.

(C) Repeat to illustrate the Alternate Interior Angles Theorem and its converse using the diagram and the given statements.

By the theorem: If ______________________________, then $\angle 4 \cong \angle \square$.

By its converse: If ______________________________,

then ______________________________.

(D) Use the diagram and the given statements to illustrate the Corresponding Angles Theorem and its converse.

By the theorem: If ______________________________, then $\angle \square \cong \angle 7$.

By its converse: ______________________________.

Reflect

1. How do you form the converse of a statement?

2. What kind of angles are $\angle 4$ and $\angle 6$ in Step C? What does the converse you wrote in Step C mean?

Explain 1 Proving that Two Lines are Parallel

The converses from the Explore can be stated formally as a postulate and two theorems. (You will prove the converses of the theorems in the exercises.)

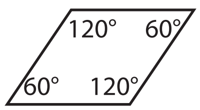
Converse of the Same-Side Interior Angles Postulate
If two lines are cut by a transversal so that a pair of same-side interior angles are supplementary, then the lines are parallel.

Converse of the Alternate Interior Angles Theorem
If two lines are cut by a transversal so that any pair of alternate interior angles are congruent, then the lines are parallel.

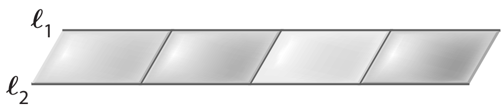
Converse of the Corresponding Angles Theorem
If two lines are cut by a transversal so that any pair of corresponding angles are congruent, then the lines are parallel.

You can use these converses to decide whether two lines are parallel.

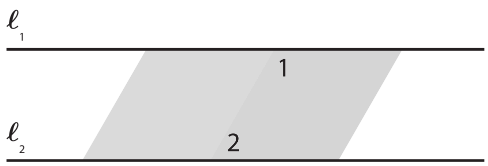
Example 1 **A mosaic designer is using quadrilateral-shaped colored tiles to make an ornamental design. Each tile is congruent to the one shown here.**



The designer uses the colored tiles to create the pattern shown here.



Ⓐ Use the values of the marked angles to show that the two lines ℓ_1 and ℓ_2 are parallel.

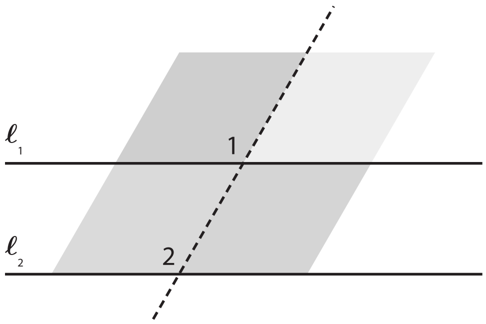


Measure of $\angle 1$: 120° Measure of $\angle 2$: 60°

Relationship between the two angles: They are supplementary.

Conclusion: $\ell_1 \parallel \ell_2$ by the Converse of the Same-Side Interior Angles Postulate.

(B) Now look at this situation. Use the values of the marked angles to show that the two lines are parallel.



Measure of ∠1: ________ Measure of ∠2: ________

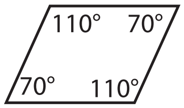
Relationship between the two angles: ____________________

Conclusion:

____

Reflect

3. **What If?** Suppose the designer had been working with this basic shape instead. Do you think the conclusions in Parts A and B would have been different? Why or why not?

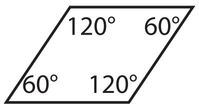


____

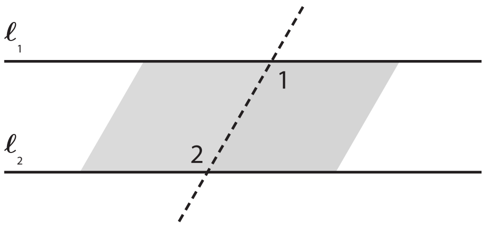
____

Your Turn

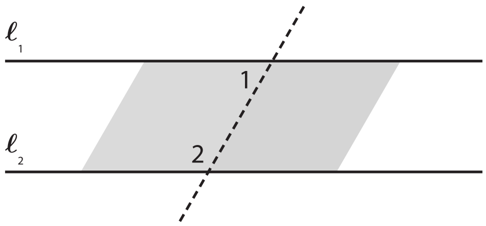
Explain why the lines are parallel given the angles shown. Assume that all tile patterns use this basic shape.



4.



5.



Explain 2 Constructing Parallel Lines

The Parallel Postulate guarantees that for any line ℓ, you can always construct a parallel line through a point that is not on ℓ.

The Parallel Postulate

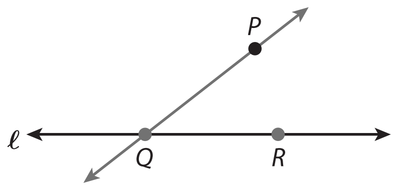
Through a point P not on line ℓ, there is exactly one line parallel to ℓ.

Example 2 **Use a compass and straightedge to construct parallel lines.**

(A) Construct a line m through a point P not on a line ℓ so that m is parallel to ℓ.

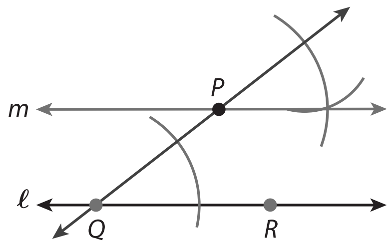
Step 1 Draw a line ℓ and a point P not on ℓ.

Step 2 Choose two points on ℓ and label them Q and R. Use a straightedge to draw $\overleftrightarrow{PQ}$.



Step 3 Use a compass to copy $\angle PQR$ at point P, as shown, to construct line m.

line $m \parallel$ line ℓ



(B) In the space provided, follow the steps to construct a line r through a point G not on a line s so that r is parallel to s.

Step 1 Draw a line s and a point G not on s.

Step 2 Choose two points on s and label them E and F. Use a straightedge to draw $\overleftrightarrow{GE}$.

Step 3 Use a compass to copy $\angle GEF$ at point G. Label the side of the angle as line r.
line $r \parallel$ line s

Reflect

6. **Discussion** Explain how you know that the construction in Part A or Part B produces a line passing through the given point that is parallel to the given line.

____

____

____

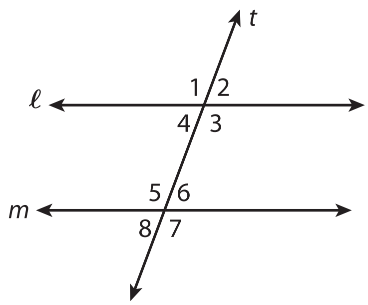
Your Turn

7. Construct a line m through P parallel to line ℓ.

Explain 3 Using Angle Pair Relationships to Verify Lines are Parallel

When two lines are cut by a transversal, you can use relationships of pairs of angles to decide if the lines are parallel.

Example 3 **Use the given angle relationships to decide whether the lines are parallel. Explain your reasoning.**



(A) $\angle 3 \cong \angle 5$

Step 1 Identify the relationship between the two angles.
$\angle 3$ and $\angle 5$ are congruent alternate interior angles.

Step 2 Are the lines parallel? Explain.
Yes, the lines are parallel by the Converse of the Alternate Interior Angles Theorem.

(B) $m\angle 4 = (x + 20)°$, $m\angle 8 = (2x + 5)°$, and $x = 15$.

Step 1 Identify the relationship between the two angles.

$m\angle 4 = (x + 20)°$ $\qquad$ $m\angle 8 = (2x + 5)°$

$= \left(\boxed{} + 20\right)° = \boxed{}$ $\qquad$ $= \left(2 \cdot \boxed{} + 5\right)° = \boxed{}$

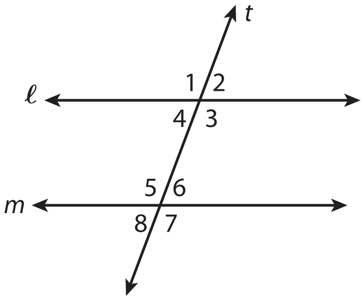
So, ________ and ________ are ______________________ angles.

Step 2 Are the lines parallel? Explain.

____

Your Turn

Identify the type of angle pair described in the given condition. How do you know that lines ℓ and m are parallel?



8. $m\angle 3 + m\angle 6 = 180°$

9. $\angle 2 \cong \angle 6$

Elaborate

10. How are the converses in this lesson different from the postulate/theorems in the previous lesson?

11. **What If?** Suppose two lines are cut by a transversal such that alternate interior angles are both congruent and supplementary. Describe the lines.

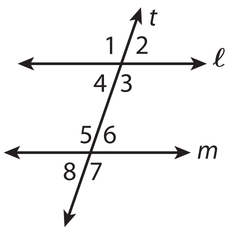
12. **Essential Question Check-In** Name two ways to test if a pair of lines is parallel, using the interior angles formed by a transversal crossing the two lines.

Evaluate: Homework and Practice

Personal Math Trainer
- Online Homework
- Hints and Help
- Extra Practice

The diagram shows two lines cut by a transversal *t*. Use the diagram and the given statements in Exercises 1–3 on the facing page.

Statements
lines ℓ and m are parallel
$m\angle\boxed{} + m\angle 3 = 180°$
$\angle 1 \cong \angle\boxed{}$
$\angle\boxed{} \cong \angle 6$



1. Use two of the given statements together to complete statements about the diagram to illustrate the Corresponding Angles Theorem. Then write its converse.

By the theorem: If ______________________________, then $\angle 1 \cong \angle$ $\boxed{}$.

By its converse: ______________________________

2. Use two of the given statements together to complete statements about the diagram to illustrate the Same-Side Interior Angles Postulate. Then write its converse.

By the postulate: If ______________________, then $m\angle$ $\boxed{}$ $+ m\angle 3 = 180°$.

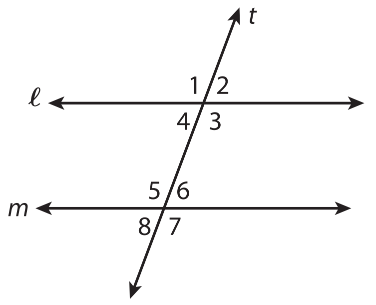
By its converse: ______________________________

3. Use two of the given statements together to complete statements about the diagram to illustrate the Alternate Interior Angles Theorem. Then write its converse.

By the theorem: If ______________________________, then $\angle$ $\boxed{}$ $\cong \angle 6$.

By its converse: ______________________________

4. **Matching** Match the angle pair relationship on the left with the name of a postulate or theorem that you could use to prove that lines ℓ and m in the diagram are parallel.



A. $\angle 2 \cong \angle 6$

B. $\angle 3 \cong \angle 5$

C. $\angle 4$ and $\angle 5$ are supplementary.

D. $\angle 4 \cong \angle 8$

E. $m\angle 3 + m\angle 6 = 180°$

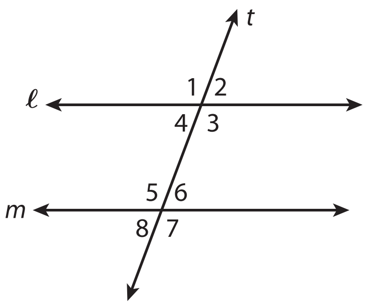
F. $\angle 4 \cong \angle 6$

________ Converse of the Corresponding Angles Theorem

________ Converse of the Same-Side Interior Angles Postulate

________ Converse of the Alternate Interior Angles Theorem

Use the diagram for Exercises 5–8.



5. What must be true about $\angle 7$ and $\angle 3$ for the lines to be parallel? Name the postulate or theorem.

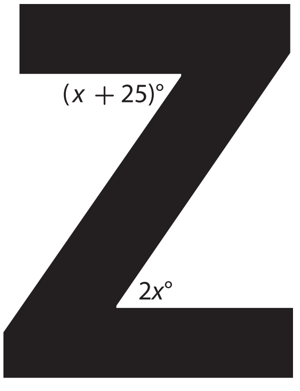
6. What must be true about $\angle 6$ and $\angle 3$ for the lines to be parallel? Name the postulate or theorem.

7. Suppose $m\angle 4 = (3x + 5)°$ and $m\angle 5 = (x + 95)°$, where $x = 20$. Are the lines parallel? Explain.

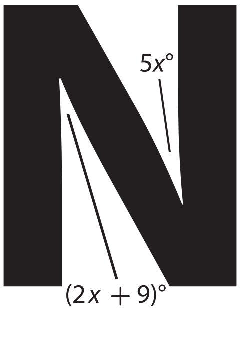
8. Suppose $m\angle 3 = (4x + 12)°$ and $m\angle 7 = (80 - x)°$, where $x = 15$. Are the lines parallel? Explain.

Use a converse to answer each question.

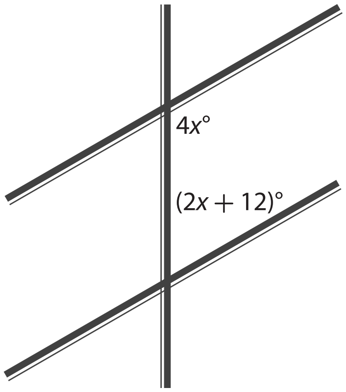
9. What value of x makes the horizontal parts of the letter Z parallel?



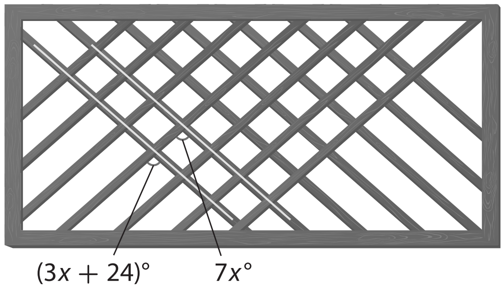
10. What value of x makes the vertical parts of the letter N parallel?



11. **Engineering** An overpass intersects two lanes of a highway. What must the value of x be to ensure the two lanes are parallel?



12. A trellis consists of overlapping wooden slats. What must the value of x be in order for the two slats to be parallel?



13. Construct a line parallel to ℓ that passes through P.

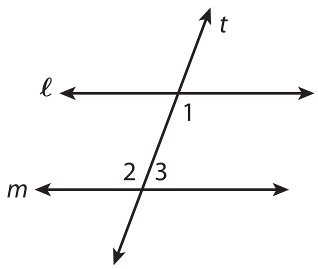
14. **Communicate Mathematical Ideas** In Exercise 13, how many parallel lines can you draw through P that are parallel to ℓ? Explain.

H.O.T. Focus on Higher Order Thinking

15. **Justify Reasoning** Write a two-column proof of the Converse of the Alternate Interior Angles Theorem.

Given: lines ℓ and m are cut by a transversal t; $\angle 1 \cong \angle 2$

Prove: $\ell \parallel m$

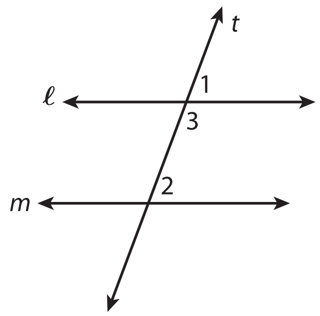


Statements	Reasons

16. **Justify Reasoning** Write a two-column proof of the Converse of the Corresponding Angles Theorem.

Given: lines ℓ and m are cut by a transversal t; $\angle 1 \cong \angle 2$

Prove: $\ell \parallel m$



Statements	Reasons

Lesson Performance Task

A simplified street map of a section of Harlem in New York City is shown at right. Draw a sketch of the rectangle bounded by West 110th Street and West 121st Street in one direction and Eighth Avenue and Lenox Avenue in the other. Include all the streets and avenues that run between sides of the rectangle. Show St. Nicholas Avenue as a diagonal of the rectangle.

Now imagine that you have been given the job of laying out these streets and avenues on a bare plot of land. Explain in detail how you would do it.