Name ______________________ Class ____________ Date ________

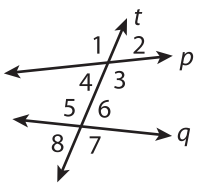
4.2 Transversals and Parallel Lines

Essential Question: **How can you prove and use theorems about angles formed by transversals that intersect parallel lines?**

Resource Locker

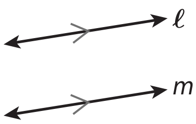
Explore — Exploring Parallel Lines and Transversals

A **transversal** is a line that intersects two coplanar lines at two different points. In the figure, line *t* is a transversal. The table summarizes the names of angle pairs formed by a transversal.



Angle Pair	Example
Corresponding angles lie on the same side of the transversal and on the same sides of the intersected lines.	$\angle 1$ and $\angle 5$
Same-side interior angles lie on the same side of the transversal and between the intersected lines.	$\angle 3$ and $\angle 6$
Alternate interior angles are nonadjacent angles that lie on opposite sides of the transversal between the intersected lines.	$\angle 3$ and $\angle 5$
Alternate exterior angles lie on opposite sides of the transversal and outside the intersected lines.	$\angle 1$ and $\angle 7$

Recall that parallel lines lie in the same plane and never intersect. In the figure, line ℓ is parallel to line m, written $\ell \| m$. The arrows on the lines also indicate that they are parallel.



$\ell \parallel m$

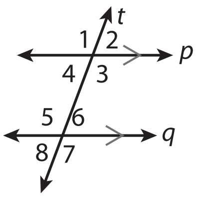
When parallel lines are cut by a transversal, the angle pairs formed are either congruent or supplementary. The following postulate is the starting point for proving theorems about parallel lines that are intersected by a transversal.

Same-Side Interior Angles Postulate

If two parallel lines are cut by a transversal, then the pairs of same-side interior angles are supplementary.

Follow the steps to illustrate the postulate and use it to find angle measures.

(A) Draw two parallel lines and a transversal, and number the angles formed from 1 to 8.



(B) Identify the pairs of same-side interior angles.

____

(C) What does the postulate tell you about these same-side interior angle pairs?

____

(D) If m∠4 = 70°, what is m∠5? Explain.

____

____

Reflect

1. Explain how you can find m∠3 in the diagram if $p \parallel q$ and m∠6 = 61°.

____

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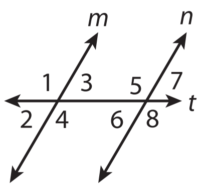
2. **What If?** If $m \parallel n$, how many pairs of same-side interior angles are shown in the figure? What are the pairs?

____



Explain 1 Proving that Alternate Interior Angles are Congruent

Other pairs of angles formed by parallel lines cut by a transversal are alternate interior angles.

Alternate Interior Angles Theorem

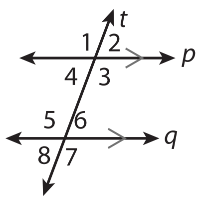
If two parallel lines are cut by a transversal, then the pairs of alternate interior angles have the same measure.

To prove something to be true, you use definitions, properties, postulates, and theorems that you already know.

Example 1 **Prove the Alternate Interior Angles Theorem.**

Given: $p \parallel q$

Prove: $m\angle 3 = m\angle 5$



Complete the proof by writing the missing reasons. Choose from the following reasons. You may use a reason more than once.

- Same-Side Interior Angles Postulate
- Given
- Definition of supplementary angles
- Subtraction Property of Equality
- Substitution Property of Equality
- Linear Pair Theorem

Statements	Reasons
1. $p \parallel q$	
2. $\angle 3$ and $\angle 6$ are supplementary.	
3. $m\angle 3 + m\angle 6 = 180°$	
4. $\angle 5$ and $\angle 6$ are a linear pair.	
5. $\angle 5$ and $\angle 6$ are supplementary.	
6. $m\angle 5 + m\angle 6 = 180°$	
7. $m\angle 3 + m\angle 6 = m\angle 5 + m\angle 6$	
8. $m\angle 3 = m\angle 5$	

Reflect

3. In the figure, explain why $\angle 1$, $\angle 3$, $\angle 5$, and $\angle 7$ all have the same measure.

4. Suppose m∠4 = 57° in the figure shown. Describe two different ways to determine *m*∠6.

Explain 2 Proving that Corresponding Angles are Congruent

Two parallel lines cut by a transversal also form angle pairs called corresponding angles.

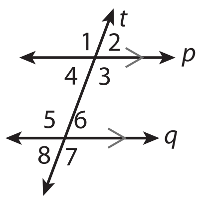
Corresponding Angles Theorem

If two parallel lines are cut by a transversal, then the pairs of corresponding angles have the same measure.

Example 2 Complete a proof in paragraph form for the Corresponding Angles Theorem.

Given: $p \| q$

Prove: m∠4 = m∠8



By the given statement, $p \| q$. ∠4 and ∠6 form a pair of ______________.

So, using the Alternate Interior Angles Theorem, ______________.

∠6 and ∠8 form a pair of vertical angles. So, using the Vertical Angles Theorem,

______________. Using the ______________

in m∠4 = m∠6, substitute ______ for m∠6. The result is ______________.

Reflect

5. Use the diagram in Example 2 to explain how you can prove the Corresponding Angles Theorem using the Same-Side Interior Angles Postulate and a linear pair of angles.

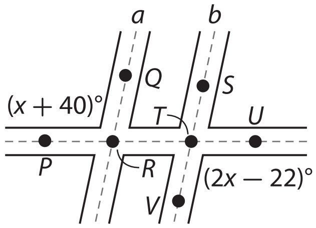
6. Suppose m∠4 = 36°. Find m∠5. Explain.

Explain 3 Using Parallel Lines to Find Angle Pair Relationships

You can apply the theorems and postulates about parallel lines cut by a transversal to solve problems.

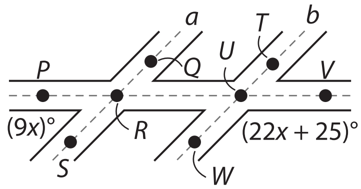
Example 3 **Find each value. Explain how to find the values using postulates, theorems, and algebraic reasoning.**

Ⓐ In the diagram, roads a and b are parallel. Explain how to find the measure of $\angle VTU$.



It is given that $m\angle PRQ = (x + 40)°$ and $m\angle VTU = (2x - 22)°$.
$m\angle PRQ = m\angle RTS$ by the Corresponding Angles Theorem and
$m\angle RTS = m\angle VTU$ by the Vertical Angles Theorem.
So, $m\angle PRQ = m\angle VTU$, and $x + 40 = 2x - 22$. Solving for x,
$x + 62 = 2x$, and $x = 62$. Substitute the value of x to find $m\angle VTU$:
$m\angle VTU = \left(2(62) - 22\right)° = 102°$.

Ⓑ In the diagram, roads a and b are parallel. Explain how to find the measure of $m\angle WUV$.



It is given that $m\angle PRS = (9x)°$ and $m\angle WUV = (22x + 25)°$.

$m\angle PRS = m\angle RUW$ by the ______________________.

$\angle RUW$ and ________ are supplementary angles.

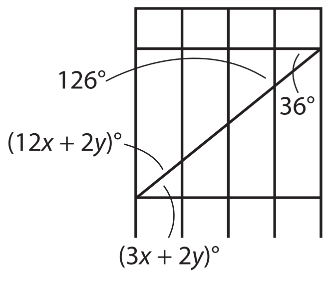
So, $m\angle RUW + m\angle WUV =$ ________. Solving for x, $31x + 25 = 180$,

and ____________ .Substitute the value of x to find __________;

$m\angle WUV = \left(22(5) + 25\right)°$ __________.

Your Turn

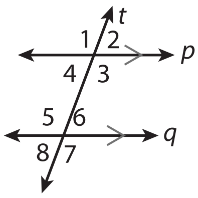
7. In the diagram of a gate, the horizontal bars are parallel and the vertical bars are parallel. Find x and y. Name the postulates and/or theorems that you used to find the values.



Elaborate

8. How is the Same-Side Interior Angles Postulate different from the two theorems in the lesson (Alternate Interior Angles Theorem and Corresponding Angles Theorem)?

9. **Discussion** Look at the figure below. If you know that p and q are parallel, and are given one angle measure, can you find all the other angle measures? Explain.

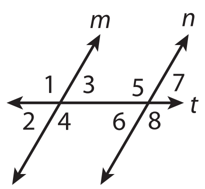


10. **Essential Question Check-In** Why is it important to establish the Same-Side Interior Angles Postulate before proving the other theorems?

Evaluate: Homework and Practice

- Online Homework
- Hints and Help
- Extra Practice

1. In the figure below, $m \| n$. Match the angle pairs with the correct label for the pairs. Indicate a match by writing the letter for the angle pairs on the line in front of the corresponding labels.



A. $\angle 4$ and $\angle 6$ ________ Corresponding Angles

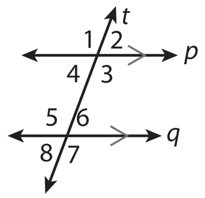
B. $\angle 5$ and $\angle 8$ ________ Same-Side Interior Angles

C. $\angle 2$ and $\angle 6$ ________ Alternate Interior Angles

D. $\angle 4$ and $\angle 5$ ________ Vertical Angles

2. Complete the definition: A ____________ is a line that intersects two coplanar lines at two different points.

Use the figure to find angle measures. In the figure, *p* || *q*.



3. Suppose m∠4 = 82°. Find m∠5.

4. Suppose m∠3 = 105°. Find m∠6.

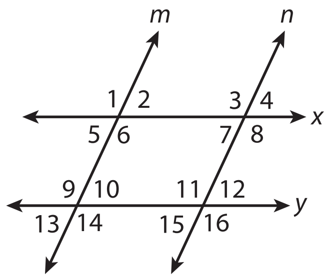
5. Suppose m∠3 = 122°. Find m∠5.

6. Suppose m∠4 = 76°. Find m∠6.

7. Suppose m∠5 = 109°. Find m∠1.

8. Suppose m∠6 = 74°. Find m∠2.

Use the figure to find angle measures. In the figure, *m* || *n* and *x* || *y*.



9. Suppose m∠5 = 69°. Find m∠10.

10. Suppose m∠9 = 115°. Find m∠6.

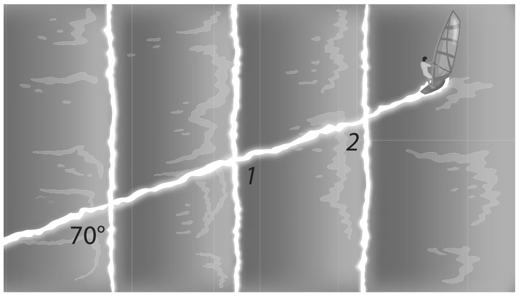
11. Suppose m∠12 = 118°. Find m∠7.

12. Suppose m∠4 = 72°. Find m∠11.

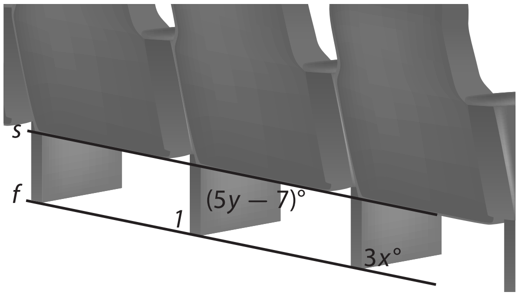
13. Suppose m∠4 = 114°. Find m∠14.

14. Suppose m∠5 = 86°. Find m∠12.

15. Ocean waves move in parallel lines toward the shore. The figure shows the path that a windsurfer takes across several waves. For this exercise, think of the windsurfer's wake as a line. If $m\angle 1 = (2x + 2y)°$ and $m\angle 2 = (2x + y)°$, find x and y. Explain your reasoning.



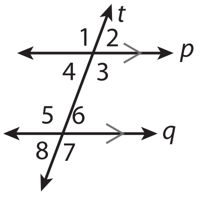
In the diagram of movie theater seats, the incline of the floor, *f*, is parallel to the seats, *s*.



16. If $m\angle 1 = 60°$, what is x?

17. If $m\angle 1 = 68°$, what is y?

18. Complete a proof in paragraph form for the Alternate Interior Angles Theorem.

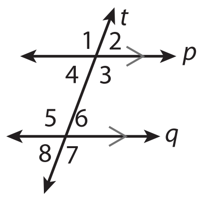


Given: $p \parallel q$

Prove: $m\angle 3 = m\angle 5$

It is given that $p \parallel q$, so using the Same-Side Interior Angles Postulate, $\angle 3$ and $\angle 6$ are __________________. So, the sum of their measures is _________ and $m\angle 3 + m\angle 6 = 180°$. You can see from the diagram that $\angle 5$ and $\angle 6$ form a line, so they are a __________, which makes them __________________. Then $m\angle 5 + m\angle 6 = 180°$. Using the Substitution Property of Equality, you can substitute _________ in $m\angle 3 + m\angle 6 = 180°$ with $m\angle 5 + m\angle 6$. This results in $m\angle 3 + m\angle 6 = m\angle 5 + m\angle 6$. Using the Subtraction Property of Equality, you can subtract _________ from both sides. So, __________________.

19. Write a proof in two-column form for the Corresponding Angles Theorem.

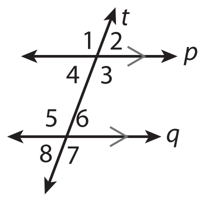


Given: $p \parallel q$

Prove: $m\angle 1 = m\angle 5$

Statements	Reasons

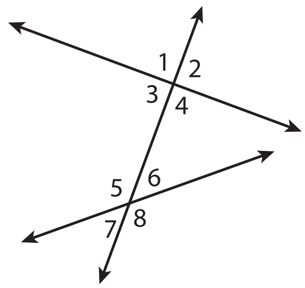
20. Explain the Error Angelina wrote a proof in paragraph form to prove that the measures of corresponding angles are congruent. Identify her error, and describe how to fix the error.



Angelina's proof:

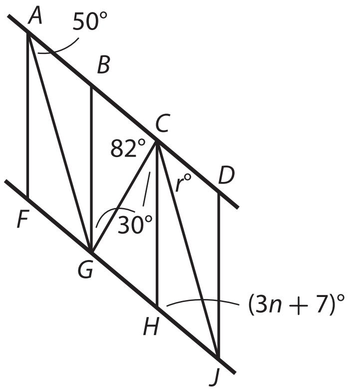
I am given that $p \parallel q$. $\angle 1$ and $\angle 4$ are supplementary angles because they form a linear pair, so $m\angle 1 + m\angle 4 = 180°$. $\angle 4$ and $\angle 8$ are also supplementary because of the Same-Side Interior Angles Postulate, so $m\angle 4 + m\angle 8 = 180°$. You can substitute $m\angle 4 + m\angle 8$ for 180° in the first equation above. The result is $m\angle 1 + m\angle 4 = m\angle 4 + m\angle 8$. After subtracting $m\angle 4$ from each side, I see that $\angle 1$ and $\angle 8$ are corresponding angles and $m\angle 1 = m\angle 8$.

21. Counterexample Ellen thinks that when two lines that are not parallel are cut by a transversal, the measures of the alternate interior angles are the same. Write a proof to show that she is correct or use a counterexample to show that she is incorrect.



H.O.T. Focus on Higher Order Thinking

Analyzing Mathematical Relationships **Use the diagram of a staircase railing for Exercises 22 and 23. $\overline{AG} \parallel \overline{CJ}$ and $\overline{AD} \parallel \overline{FJ}$. Choose the best answer.**



22. Which is a true statement about the measure of $\angle DCJ$?

A. It is 30°, by the Alternate Interior Angles Theorem.

B. It is 30°, by the Corresponding Angles Theorem.

C. It is 50°, by the Alternate Interior Angles Theorem.

D. It is 50°, by the Corresponding Angles Theorem.

23. Which is a true statement about the value of n?

A. It is 25, by the Alternate Interior Angles Theorem.

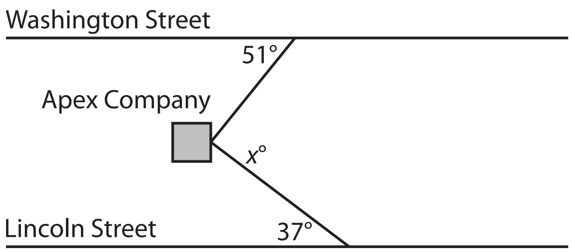
B. It is 25, by the Same-Side Interior Angles Postulate.

C. It is 35, by Alternate Interior Angles Theorem.

D. It is 35, by the Corresponding Angles Theorem.

Lesson Performance Task

Washington Street is parallel to Lincoln Street. The Apex Company's headquarters is located between the streets. From headquarters, a straight road leads to Washington Street, intersecting it at a 51° angle. Another straight road leads to Lincoln Street, intersecting it at a 37° angle.



a. Find x. Explain your method.

b. Suppose that another straight road leads from the opposite side of headquarters to Washington Street, intersecting it at a y° angle, and another straight road leads from headquarters to Lincoln Street, intersecting it at a z° angle. Find the measure of the angle w formed by the two roads. Explain how you found w.