Name ______________________ Class ____________ Date ________

4.1 Angles Formed by Intersecting Lines

Essential Question: How can you find the measures of angles formed by intersecting lines?

Resource Locker

Explore 1 Exploring Angle Pairs Formed by Intersecting Lines

When two lines intersect, like the blades of a pair of scissors, a number of angle pairs are formed. You can find relationships between the measures of the angles in each pair.

(A) Using a straightedge, draw a pair of intersecting lines like the open scissors. Label the angles formed as 1, 2, 3, and 4.

(B) Use a protractor to find each measure.

Angle	Measure of Angle
$m\angle 1$	
$m\angle 2$	
$m\angle 3$	
$m\angle 4$	
$m\angle 1 + m\angle 2$	
$m\angle 2 + m\angle 3$	
$m\angle 3 + m\angle 4$	
$m\angle 1 + m\angle 4$	

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You have been measuring *vertical angles* and *linear pairs* of angles. When two lines intersect, the angles that are opposite each other are **vertical angles**. Recall that a *linear pair* is a pair of adjacent angles whose non-common sides are opposite rays. So, when two lines intersect, the angles that are on the same side of a line form a linear pair.

Reflect

1. Name a pair of vertical angles and a linear pair of angles in your diagram in Step A.

2. Make a conjecture about the measures of a pair of vertical angles.

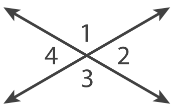
3. Use the Linear Pair Theorem to tell what you know about the measures of angles that form a linear pair.

Explore 2 Proving the Vertical Angles Theorem

The conjecture from the Explore about vertical angles can be proven so it can be stated as a theorem.

The Vertical Angles Theorem

If two angles are vertical angles, then the angles are congruent.



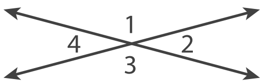
$\angle 1 \cong \angle 3$ and $\angle 2 \cong \angle 4$

You have written proofs in two-column and paragraph proof formats. Another type of proof is called a *flow proof*. A **flow proof** uses boxes and arrows to show the structure of the proof. The steps in a flow proof move from left to right or from top to bottom, shown by the arrows connecting each box. The justification for each step is written below the box. You can use a flow proof to prove the Vertical Angles Theorem.

Follow the steps to write a Plan for Proof and a flow proof to prove the Vertical Angles Theorem.

Given: $\angle 1$ and $\angle 3$ are vertical angles.

Prove: $\angle 1 \cong \angle 3$



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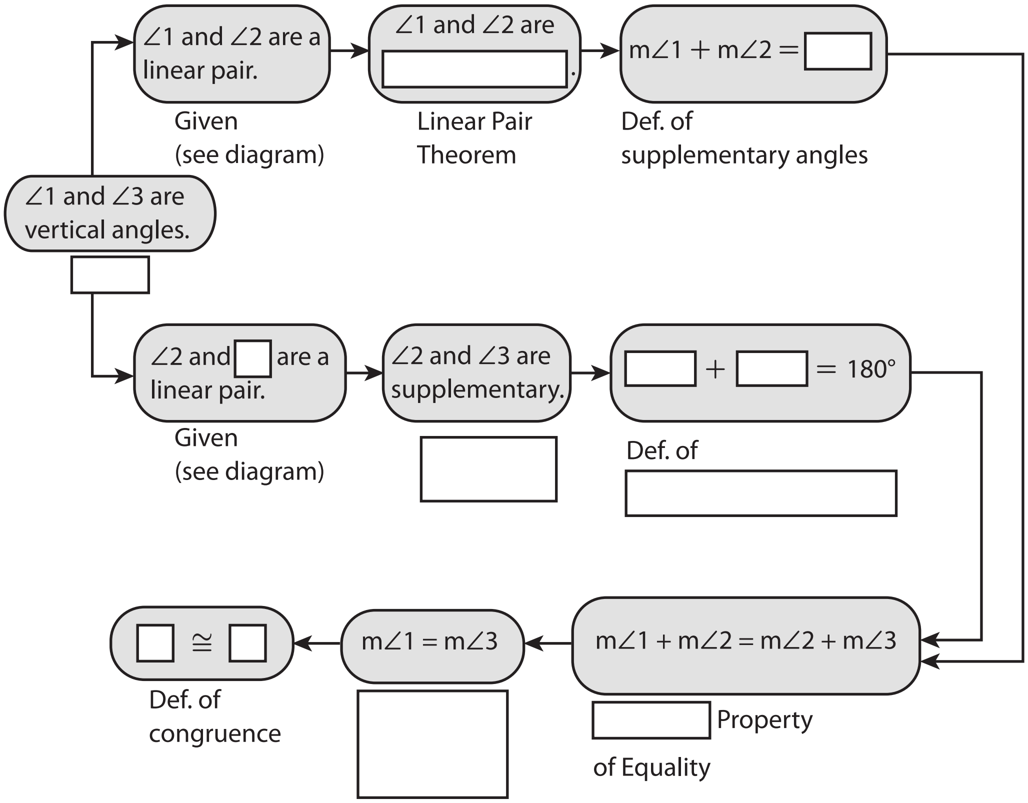
Ⓐ Complete the final steps of a Plan for Proof:

Because ∠1 and ∠2 are a linear pair and ∠2 and ∠3 are a linear pair, these pairs of angles are supplementary. This means that m∠1 + m∠2 = 180° and m∠2 + m∠3 = 180°. By the Transitive Property, m∠1 + m∠2 = m∠2 + m∠3. Next:

Ⓑ Use the Plan for Proof to complete the flow proof. Begin with what you know is true from the Given or the diagram. Use arrows to show the path of the reasoning. Fill in the missing statement or reason in each step.



Reflect

4. **Discussion** Using the other pair of angles in the diagram, ∠2 and ∠4, would a proof that ∠2 ≅ ∠4 also show that the Vertical Angles Theorem is true? Explain why or why not.

5. Draw two intersecting lines to form vertical angles. Label your lines and tell which angles are congruent. Measure the angles to check that they are congruent.

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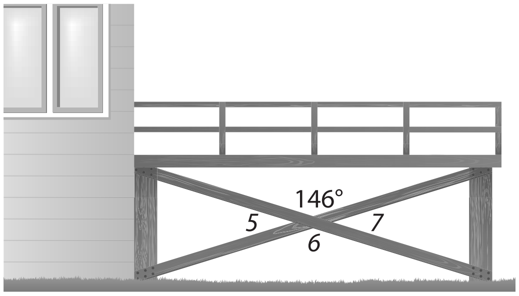
Explain 1 Using Vertical Angles

You can use the Vertical Angles Theorem to find missing angle measures in situations involving intersecting lines.

Example 1 **Cross braces help keep the deck posts straight. Find the measure of each angle.**



(A) $\angle 6$

Because vertical angles are congruent, $m\angle 6 = 146°$.

(B) $\angle 5$ and $\angle 7$

From Part A, $m\angle 6 = 146°$. Because $\angle 5$ and $\angle 6$ form a ______________, they are supplementary and $m\angle 5 = 180° - 146° =$ ☐. $m\angle$ ☐ $=$ ☐ because $\angle$ ☐ also forms a linear pair with $\angle 6$, or because it is a ________________ with $\angle 5$.

Your Turn

6. The measures of two vertical angles are 58° and $(3x + 4)°$. Find the value of x.

7. The measures of two vertical angles are given by the expressions $(x + 3)°$ and $(2x - 7)°$. Find the value of x. What is the measure of each angle?

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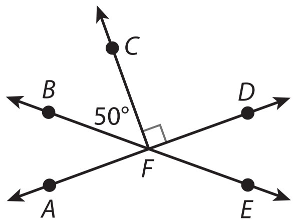
Explain 2 Using Supplementary and Complementary Angles

Recall what you know about complementary and supplementary angles. **Complementary angles** are two angles whose measures have a sum of 90°. **Supplementary angles** are two angles whose measures have a sum of 180°. You have seen that two angles that form a linear pair are supplementary.

Example 2 **Use the diagram below to find the missing angle measures. Explain your reasoning.**



Ⓐ Find the measures of $\angle AFC$ and $\angle AFB$.

$\angle AFC$ and $\angle CFD$ are a linear pair formed by an intersecting line and ray, $\overleftrightarrow{AD}$ and $\overrightarrow{FC}$, so they are supplementary and the sum of their measures is 180°. By the diagram, $m\angle CFD = 90°$, so $m\angle AFC = 180° - 90° = 90°$ and $\angle AFC$ is also a right angle.

Because together they form the right angle $\angle AFC$, $\angle AFB$ and $\angle BFC$ are complementary and the sum of their measures is 90°. So, $m\angle AFB = 90° - m\angle BFC = 90° - 50° = 40°$.

Ⓑ Find the measures of $\angle DFE$ and $\angle AFE$.

$\angle BFA$ and $\angle DFE$ are formed by two ______________ and are opposite each other,

so the angles are ___________ angles. So, the angles are congruent. From Part A

$m\angle AFB = 40°$, so $m\angle DFE = \boxed{}$ also.

Because $\angle BFA$ and $\angle AFE$ form a linear pair, the angles are ______________ and the sum

of their measures is $\boxed{}$. So, $m\angle AFE = \boxed{} - m\angle BFA = \boxed{} - \boxed{} = \boxed{}$.

Reflect

8. In Part A, what do you notice about right angles $\angle AFC$ and $\angle CFD$? Make a conjecture about right angles.

____

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Your Turn

You can represent the measures of an angle and its complement as $x°$ and $(90 - x)°$. Similarly, you can represent the measures of an angle and its supplement as $x°$ and $(180 - x)°$. Use these expressions to find the measures of the angles described.

9. The measure of an angle is equal to the measure of its complement.

10. The measure of an angle is twice the measure of its supplement.

Elaborate

11. Describe how proving a theorem is different than solving a problem and describe how they are the same.

12. **Discussion** The proof of the Vertical Angles Theorem in the lesson includes a Plan for Proof. How are a Plan for Proof and the proof itself the same and how are they different?

13. Draw two intersecting lines. Label points on the lines and tell what angles you know are congruent and which are supplementary.

14. **Essential Question Check-In** If you know that the measure of one angle in a linear pair is 75°, how can you find the measure of the other angle?

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Evaluate: Homework and Practice

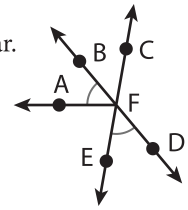
Personal Math Trainer

- Online Homework
- Hints and Help
- Extra Practice

Use this diagram and information for Exercises 1–4.

Given: m∠*AFB* = m∠*EFD* = 50°

Points *B*, *F*, *D* and points *E*, *F*, *C* are collinear.



1. Determine whether each pair of angles is a pair of vertical angles, a linear pair of angles, or neither. Select the correct answer for each lettered part.

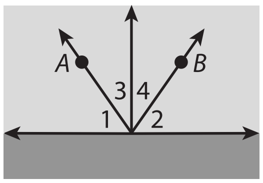
A. ∠*BFC* and ∠*DFE*	◯ Vertical	◯ Linear Pair	◯ Neither
B. ∠*BFA* and ∠*DFE*	◯ Vertical	◯ Linear Pair	◯ Neither
C. ∠*BFC* and ∠*CFD*	◯ Vertical	◯ Linear Pair	◯ Neither
D. ∠*AFE* and ∠*AFC*	◯ Vertical	◯ Linear Pair	◯ Neither
E. ∠*BFE* and ∠*CFD*	◯ Vertical	◯ Linear Pair	◯ Neither
F. ∠*AFE* and ∠*BFC*	◯ Vertical	◯ Linear Pair	◯ Neither

2. Find m∠*AFE*.

3. Find m∠*DFC*.

4. Find m∠*BFC*.

5. **Represent Real-World Problems** A sprinkler swings back and forth between *A* and *B* in such a way that ∠1 ≅ ∠2, ∠1 and ∠3 are complementary, and ∠2 and ∠4 are complementary. If m∠1 = 47.5°, find m∠2, m∠3, and m∠4.



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Determine whether each statement is true or false. If false, explain why.

6. If an angle is acute, then the measure of its complement must be greater than the measure of its supplement.

7. A pair of vertical angles may also form a linear pair.

8. If two angles are supplementary and congruent, the measure of each angle is 90°.

9. If a ray divides an angle into two complementary angles, then the original angle is a right angle.

You can represent the measures of an angle and its complement as $x°$ and $(90 - x)°$. Similarly, you can represent the measures of an angle and its supplement as $x°$ and $(180 - x)°$. Use these expressions to find the measures of the angles described.

10. The measure of an angle is three times the measure of its supplement.

11. The measure of the supplement of an angle is three times the measure of its complement.

12. The measure of an angle increased by 20° is equal to the measure of its complement.

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Write a plan for a proof for each theorem.

13. If two angles are congruent, then their complements are congruent.

Given: $\angle ABC \cong \angle DEF$

Prove: The complement of $\angle ABC \cong$ the complement of $\angle DEF$.

14. If two angles are congruent, then their supplements are congruent.

Given: $\angle ABC \cong \angle DEF$

Prove: The supplement of $\angle ABC \cong$ the supplement of $\angle DEF$.

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15. **Justify Reasoning** Complete the two-column proof for the theorem "If two angles are congruent, then their supplements are congruent."

Statements	Reasons
1. $\angle ABC \cong \angle DEF$	**1.** Given
2. The measure of the supplement of $\angle ABC = 180° - m\angle ABC$.	**2.** Definition of the ______________ of an angle
3. The measure of the supplement of $\angle DEF = 180° - m\angle DEF$.	**3.** ________________________________
4. ________________________________	**4.** If two angles are congruent, their measures are equal.
5. The measure of the supplement of $\angle DEF = 180° - m\angle ABC$.	**5.** Substitution Property of __________
6. The measure of the supplement of $\angle ABC$ = the measure of the supplement of $\angle DEF$.	**6.** ________________________________
7. The supplement of $\angle ABC \cong$ the supplement of ________.	**7.** If the measures of the supplements of two angles are equal, then supplements of the angles are congruent.

16. **Probability** The probability P of choosing an object at random from a group of objects is found by the fraction $P(\text{event}) = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}}$. Suppose the angle measures 30°, 60°, 120°, and 150° are written on slips of paper. You choose two slips of paper at random.

a. What is the probability that the measures you choose are complementary?

b. What is the probability that the measures you choose are supplementary?

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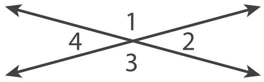
H.O.T. Focus on Higher Order Thinking

17. **Communicate Mathematical Ideas** Write a proof of the Vertical Angles Theorem in paragraph proof form.



Given: $\angle 2$ and $\angle 4$ are vertical angles.

Prove: $\angle 2 \cong \angle 4$

18. **Analyze Relationships** If one angle of a linear pair is acute, then the other angle must be obtuse. Explain why.

19. **Critique Reasoning** Your friend says that there is an angle whose measure is the same as the measure of the sum of its supplement and its complement. Is your friend correct? What is the measure of the angle? Explain your friend's reasoning.

20. **Critical Thinking** Two statements in a proof are:

$$m\angle A = m\angle B$$

$$m\angle B = m\angle C$$

What reason could you give for the statement $m\angle A = m\angle C$? Explain your reasoning.

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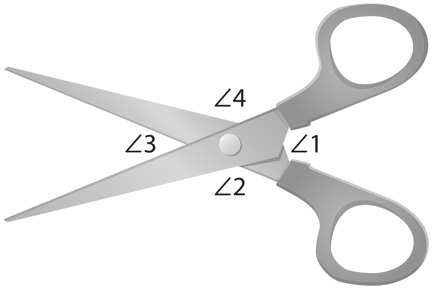
Lesson Performance Task

The image shows the angles formed by a pair of scissors. When the scissors are closed, m$\angle 1 = 0°$. As the scissors are opened, the measures of all four angles change in relation to each other. Describe how the measures change as m$\angle 1$ increases from 0° to 180°.



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