

3.3 Corresponding Parts of Congruent Figures Are Congruent



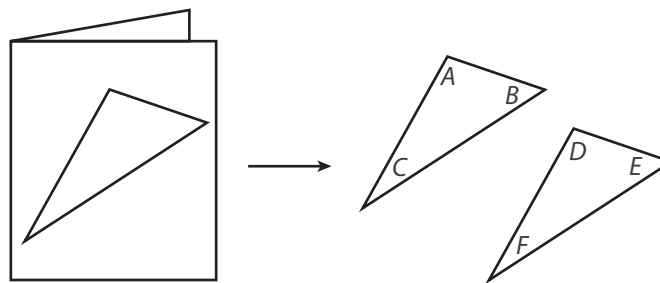
Resource Locker

Essential Question: What can you conclude about two figures that are congruent?

Explore Exploring Congruence of Parts of Transformed Figures

You will investigate some conclusions you can make when you know that two figures are congruent.

- (A) Fold a sheet of paper in half. Use a straightedge to draw a triangle on the folded sheet. Then cut out the triangle, cutting through both layers of paper to produce two congruent triangles. Label them $\triangle ABC$ and $\triangle DEF$, as shown.



- (B) Place the triangles next to each other on a desktop. Since the triangles are congruent, there must be a sequence of rigid motions that maps $\triangle ABC$ to $\triangle DEF$. Describe the sequence of rigid motions.

- (C) The same sequence of rigid motions that maps $\triangle ABC$ to $\triangle DEF$ maps parts of $\triangle ABC$ to parts of $\triangle DEF$. Complete the following.

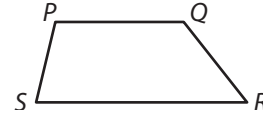
$\overline{AB} \rightarrow$
 $\overline{BC} \rightarrow$
 $\overline{AC} \rightarrow$
 $A \rightarrow$
 $B \rightarrow$
 $C \rightarrow$

- (D) What does Step C tell you about the corresponding parts of the two triangles? Why?

Reflect

1. If you know that $\triangle ABC \cong \triangle DEF$, what six congruence statements about segments and angles can you write? Why?

2. Do your findings in this Explore apply to figures other than triangles? For instance, if you know that quadrilaterals $JKLM$ and $PQRS$ are congruent, can you make any conclusions about corresponding parts? Why or why not?





Explain 1 Corresponding Parts of Congruent Figures Are Congruent

The following true statement summarizes what you discovered in the Explore.

Corresponding Parts of Congruent Figures Are Congruent

If two figures are congruent, then corresponding sides are congruent and corresponding angles are congruent.

Example 1 $\triangle ABC \cong \triangle DEF$. Find the given side length or angle measure.

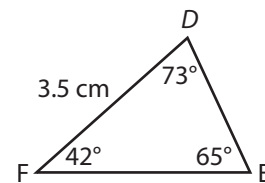
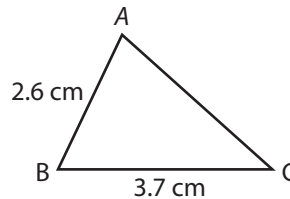
(A) DE

Step 1 Find the side that corresponds to \overline{DE} .

Since $\triangle ABC \cong \triangle DEF$, $\overline{AB} \cong \overline{DE}$.

Step 2 Find the unknown length.

$DE = AB$, and $AB = 2.6$ cm,
so $DE = 2.6$ cm.



(B) $m\angle B$

Step 1 Find the angle that corresponds to $\angle B$.

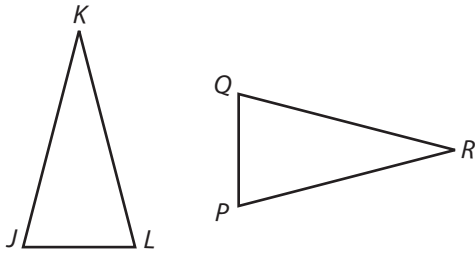
Since $\triangle ABC \cong \triangle DEF$, $\angle B \cong \angle \square$.

Step 2 Find the unknown angle measure.

$m\angle B = m\angle \square$, and $m\angle \square = \square^\circ$, so $m\angle B = \square^\circ$.

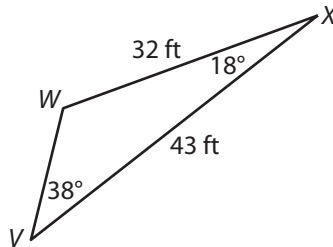
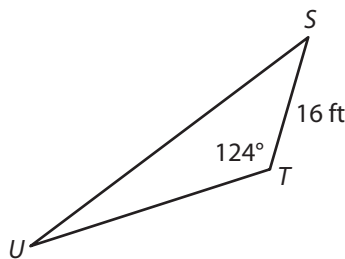
Reflect

3. **Discussion** The triangles shown in the figure are congruent. Can you conclude that $\overline{JK} \cong \overline{QR}$? Explain.



Your Turn

$\triangle STU \cong \triangle VWX$. Find the given side length or angle measure.



4. SU

5. $m\angle S$

Explain 2 Applying the Properties of Congruence

Rigid motions preserve length and angle measure. This means that congruent segments have the same length, so $\overline{UV} \cong \overline{XY}$ implies $UV = XY$ and vice versa. In the same way, congruent angles have the same measure, so $\angle J \cong \angle K$ implies $m\angle J = m\angle K$ and vice versa.

Properties of Congruence	
Reflexive Property of Congruence	$\overline{AB} \cong \overline{AB}$
Symmetric Property of Congruence	If $\overline{AB} \cong \overline{CD}$, then $\overline{CD} \cong \overline{AB}$.
Transitive Property of Congruence	If $\overline{AB} \cong \overline{CD}$ and $\overline{CD} \cong \overline{EF}$, then $\overline{AB} \cong \overline{EF}$.

Example 2 $\triangle ABC \cong \triangle DEF$. Find the given side length or angle measure.

(A) AB

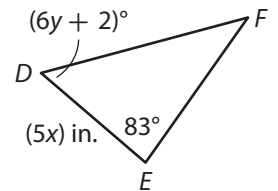
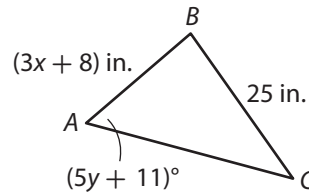
Since $\triangle ABC \cong \triangle DEF$, $\overline{AB} \cong \overline{DE}$.
Therefore, $AB = DE$.

Write an equation. $3x + 8 = 5x$

Subtract $3x$ from each side. $8 = 2x$

Divide each side by 2. $4 = x$

So, $AB = 3x + 8 = 3(4) + 8 = 12 + 8 = 20$ in.



B $m\angle D$

Since $\triangle ABC \cong \triangle DEF$, $\angle \square \cong \angle D$. Therefore, $m\angle \square = m\angle D$.

Write an equation. $5y + \square = \square + 2$

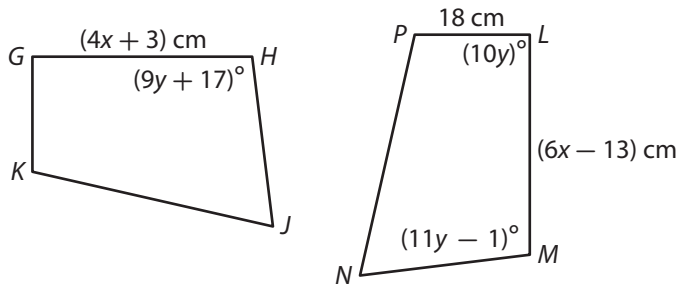
Subtract $5y$ from each side. $11 = \square + 2$

Subtract 2 from each side. $\square = \square$

So, $m\angle D = (6y + 2)^\circ = (6 \cdot \square + 2)^\circ = \square^\circ$.

Your Turn

Quadrilateral $GHJK \cong$ quadrilateral $LMNP$. Find the given side length or angle measure.



6. LM

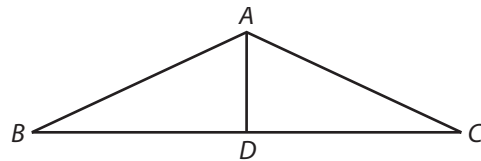
7. $m\angle H$

Explain 3 Using Congruent Corresponding Parts in a Proof

Example 3 Write each proof.

A Given: $\triangle ABD \cong \triangle ACD$

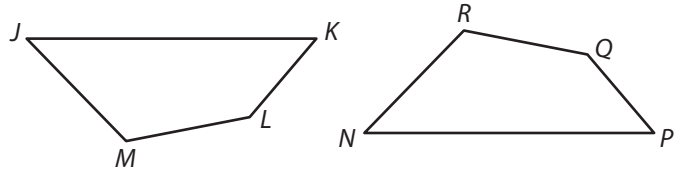
Prove: D is the midpoint of \overline{BC} .



Statements	Reasons
1. $\triangle ABD \cong \triangle ACD$	1. Given
2. $\overline{BD} \cong \overline{CD}$	2. Corresponding parts of congruent figures are congruent.
3. D is the midpoint of \overline{BC} .	3. Definition of midpoint.

- B** Given: Quadrilateral $JKLM \cong$ quadrilateral $NPQR$; $\angle J \cong \angle K$

Prove: $\angle J \cong \angle P$

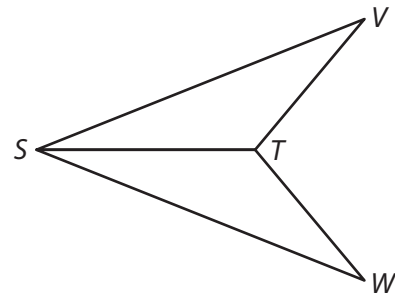


Statements	Reasons
1. Quadrilateral $JKLM \cong$ quadrilateral $NPQR$	1.
2. $\angle J \cong \angle K$	2.
3. $\angle K \cong \angle P$	3.
4. $\angle J \cong \angle P$	4.

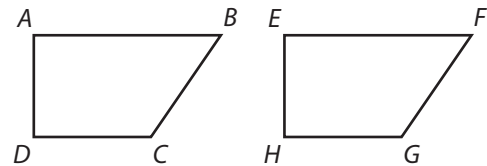
Your Turn

Write each proof.

- 8.** Given: $\triangle SVT \cong \triangle SWT$
Prove: \overline{ST} bisects $\angle VSW$.



- 9.** Given: Quadrilateral $ABCD \cong$ quadrilateral $EFGH$;
 $\overline{AD} \cong \overline{CD}$
Prove: $\overline{AD} \cong \overline{GH}$



Elaborate

10. A student claims that any two congruent triangles must have the same perimeter. Do you agree? Explain.

11. If $\triangle PQR$ is a right triangle and $\triangle PQR \cong \triangle XYZ$, does $\triangle XYZ$ have to be a right triangle? Why or why not?

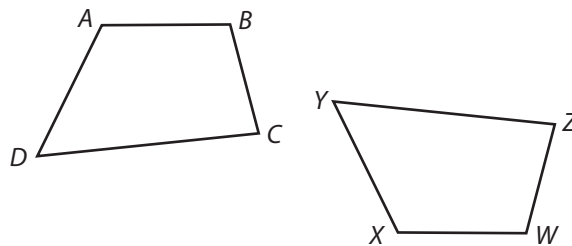
12. **Essential Question Check-In** Suppose you know that pentagon $ABCDE$ is congruent to pentagon $FGHJK$. How many additional congruence statements can you write using corresponding parts of the pentagons? Explain.

Evaluate: Homework and Practice

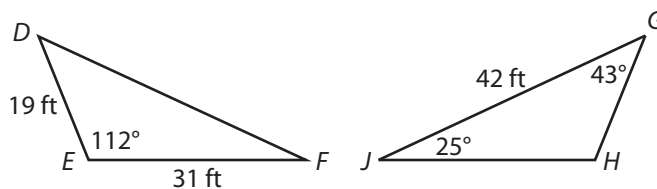


- Online Homework
- Hints and Help
- Extra Practice

1. Danielle finds that she can use a translation and a reflection to make quadrilateral $ABCD$ fit perfectly on top of quadrilateral $WXYZ$. What congruence statements can Danielle write using the sides and angles of the quadrilaterals? Why?



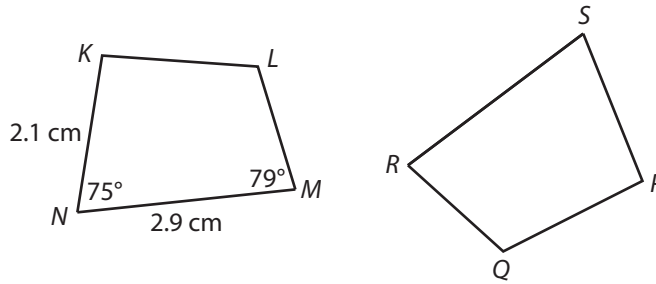
$\triangle DEF \cong \triangle GHJ$. Find the given side length or angle measure.



2. JH

3. $m\angle D$

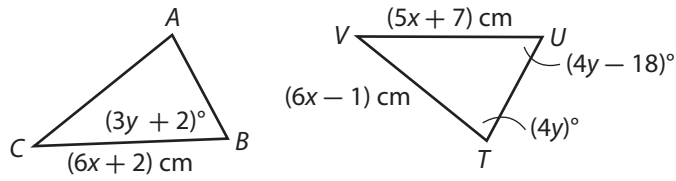
$KLMN \cong PQRS$. Find the given side length or angle measure.



4. $m\angle R$

5. PS

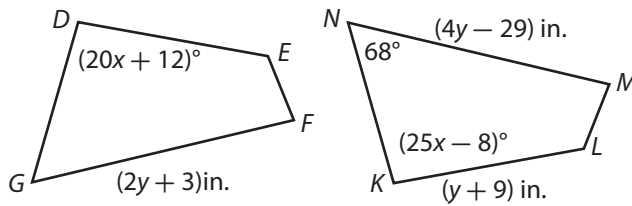
$\triangle ABC \cong \triangle TUV$. Find the given side length or angle measure.



6. BC

7. $m\angle U$

$DEFG \cong KLMN$. Find the given side length or angle measure.



8. FG

9. $m\angle D$

$\triangle GHJ \cong \triangle PQR$ and $\triangle PQR \cong \triangle STU$. Complete the following using a side or angle of $\triangle STU$. Justify your answers.

10. $\overline{GH} \cong$ _____

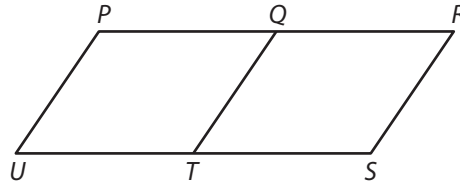
11. $\angle J \cong$ _____

12. $GJ =$ _____

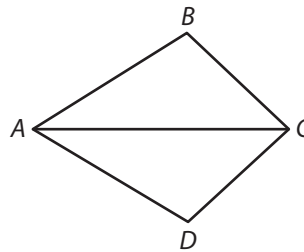
13. $m\angle G =$ _____

Write each proof.

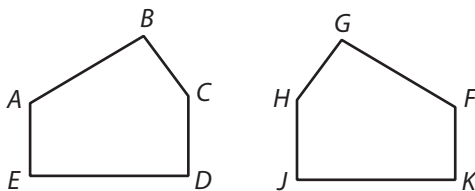
14. Given: Quadrilateral $PQTU \cong$ quadrilateral $QRST$
Prove: \overline{QT} bisects \overline{PR} .



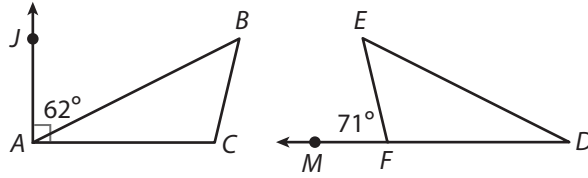
15. Given: $\triangle ABC \cong \triangle ADC$
Prove: \overline{AC} bisects $\angle BAD$ and \overline{AC} bisects $\angle BCD$.



16. Given: Pentagon $ABCDE \cong$ pentagon $FGHJK$; $\angle D \cong \angle E$
Prove: $\angle D \cong \angle K$



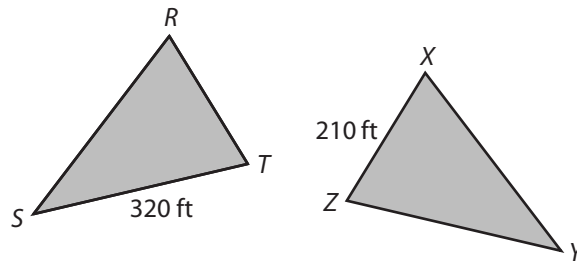
$\triangle ABC \cong \triangle DEF$. Find the given side length or angle measure.



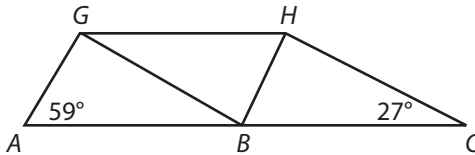
17. $m\angle D$

18. $m\angle C$

19. The figure shows the dimensions of two city parks, where $\triangle RST \cong \triangle XYZ$ and $\overline{YX} \cong \overline{YZ}$. A city employee wants to order new fences to surround both parks. What is the total length of the fences required to surround the parks?

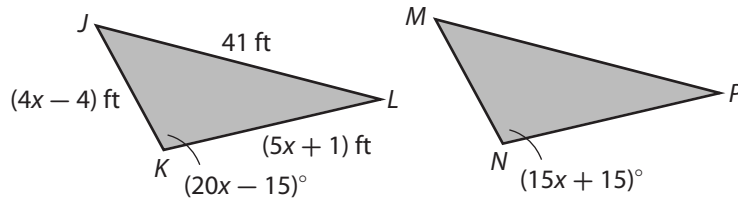


20. A tower crane is used to lift steel, concrete, and building materials at construction sites. The figure shows part of the horizontal beam of a tower crane, in which $\triangle ABG \cong \triangle BCH \cong \triangle HGB$

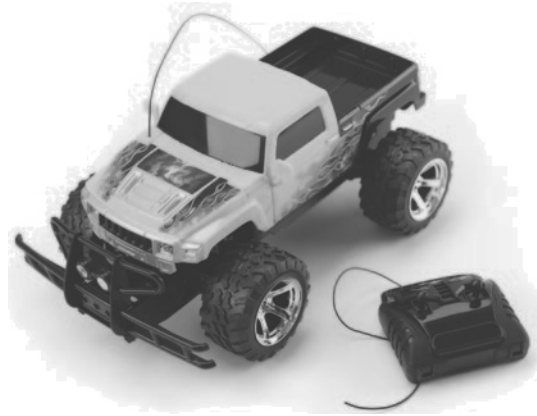
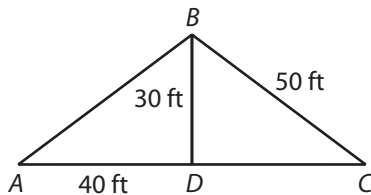


- Is it possible to determine $m\angle GBH$? If so, how? If not, why not?
- A member of the construction crew claims that \overline{AC} is twice as long as \overline{AB} . Do you agree? Explain.

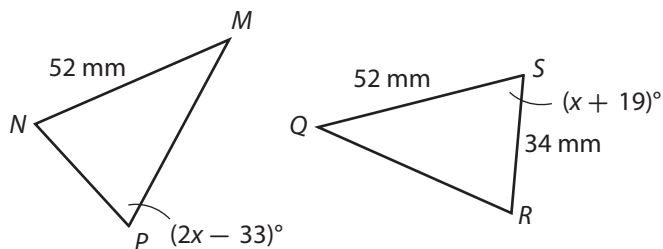
- 21. Multi-Step** A company installs triangular pools at hotels. All of the pools are congruent and $\triangle JKL \cong \triangle MNP$ in the figure. What is the perimeter of each pool?



- 22.** Kendall and Ava lay out the course shown below for their radio-controlled trucks. In the figure, $\triangle ABD \cong \triangle CBD$. The trucks travel at a constant speed of 15 feet per second. How long does it take a truck to travel on the course from A to B to C to D? Round to the nearest tenth of a second.



- 23.** $\triangle MNP \cong \triangle QRS$. Determine whether each statement about the triangles is true or false. Select the correct answer for each lettered part.

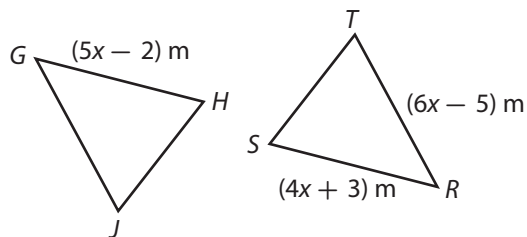


- a. $\triangle QRS$ is isosceles. True False
- b. \overline{MP} is longer than \overline{MN} . True False
- c. $m\angle P = 52^\circ$ True False
- d. The perimeter of $\triangle QRS$ is 120 mm. True False
- e. $\angle M \cong \angle Q$ True False

H.O.T. Focus on Higher Order Thinking

- 24. Justify Reasoning** Given that $\triangle ABC \cong \triangle DEF$, $AB = 2.7$ ft, and $AC = 3.4$ ft, is it possible to determine the length of \overline{EF} ? If so, find the length and justify your steps. If not, explain why not.

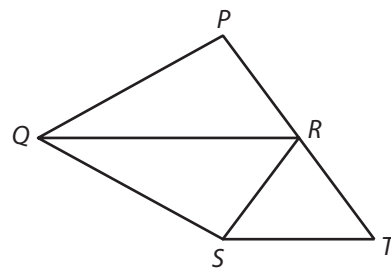
- 25. Explain the Error** A student was told that $\triangle GHJ \cong \triangle RST$ and was asked to find GH . The student's work is shown below. Explain the error and find the correct answer.



Student's Work
$5x - 2 = 6x - 5$
$-2 = x - 5$
$3 = x$
$GH = 5x - 2 = 5(3) - 2 = 13$ m

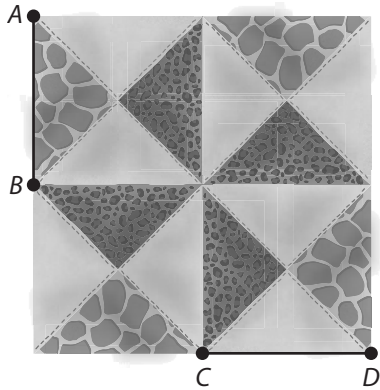
- 26. Critical Thinking** In $\triangle ABC$, $m\angle A = 55^\circ$, $m\angle B = 50^\circ$, and $m\angle C = 75^\circ$. In $\triangle DEF$, $m\angle E = 50^\circ$, and $m\angle F = 65^\circ$. Is it possible for the triangles to be congruent? Explain.

- 27. Analyze Relationships** $\triangle PQR \cong \triangle SQR$ and $\overline{RS} \cong \overline{RT}$. A student said that point R appears to be the midpoint of \overline{PT} . Is it possible to prove this? If so, write the proof. If not, explain why not.



Lesson Performance Task

The illustration shows a “Yankee Puzzle” quilt.



- Use the idea of congruent shapes to describe the design of the quilt.
- Explain how the triangle with base \overline{AB} can be transformed to the position of the triangle with base \overline{CD} .
- Explain how you know that $CD = AB$.