Name _____ Class _____ Date _____

3.2 Proving Figures are Congruent Using Rigid Motions

Essential Question: How can you determine whether two figures are congruent?

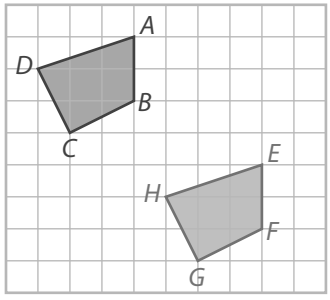
Resource Locker

Explore Confirming Congruence

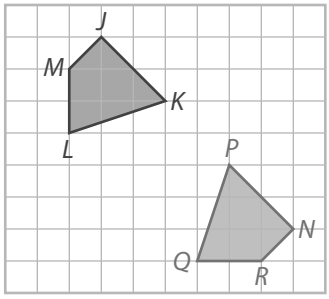
Two plane figures are congruent if and only if one can be obtained from the other by a sequence of rigid motions (that is, by a sequence of reflections, translations, and/or rotations).

A landscape architect uses a grid to design the landscape around a mall. Use tracing paper to confirm that the landscape elements are congruent.

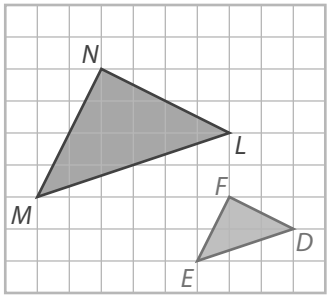
A Trace planter *ABCD*. Describe a transformation you can use to move the tracing paper so that planter *ABCD* is mapped onto planter *EFGH*. What does this confirm about the planters?



B Trace pools *JKLM* and *NPQR*. Fold the paper so that pool *JKLM* is mapped onto pool *NPQR*. Describe the transformation. What does this confirm about the pools?



C Determine whether the lawns are congruent. Is there a rigid transformation that maps $\triangle LMN$ to $\triangle DEF$? What does this confirm about the lawns?

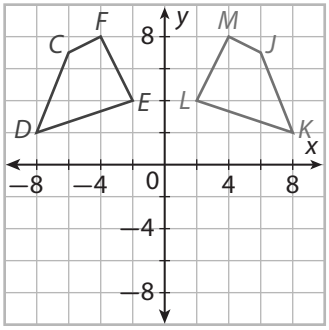


Reflect

1. How do the sizes of the pairs of figures help determine if they are congruent?

Explain 1 Determining if Figures are Congruent

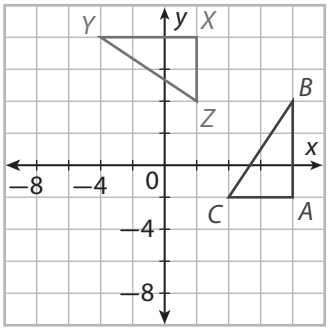
Example 1 **Use the definition of congruence to decide whether the two figures are congruent. Explain your answer.**

Ⓐ 

The two figures appear to be the same size and shape, so look for a rigid transformation that will map one to the other.

You can map *CDEF* onto *JKLM* by reflecting *CDEF* over the *y*-axis. This reflection is a rigid motion that maps *CDEF* to *JKLM*, so the two figures are congruent.

The coordinate notation for the reflection is $(x, y) \rightarrow (-x, y)$.

Ⓑ 

The two figures appear to be the same/different.

You can map $\triangle ABC$ to $\triangle XYZ$

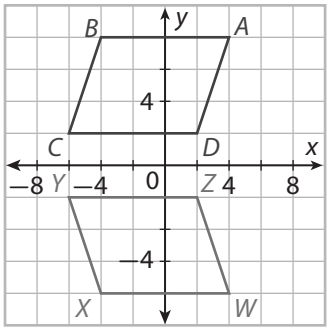
by _____.

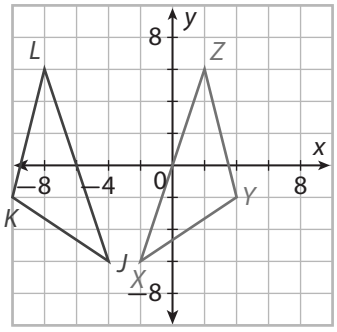
This is/is not a rigid motion that maps $\triangle ABC$ to $\triangle XYZ$, so the two figures are/are not congruent.

The coordinate notation for the rotation is _____.

Your Turn

Use the definition of congruence to decide whether the two figures are congruent. Explain your answer.

2. 

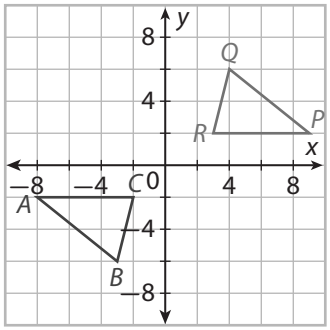
3. 

Explain 2 Finding a Sequence of Rigid Motions

The definition of congruence tells you that when two figures are known to be congruent, there must be some sequence of rigid motions that maps one to the other.

Example 2 **The figures shown are congruent. Find a sequence of rigid motions that maps one figure to the other. Give coordinate notation for the transformations you use.**

(A) $\triangle ABC \cong \triangle PQR$

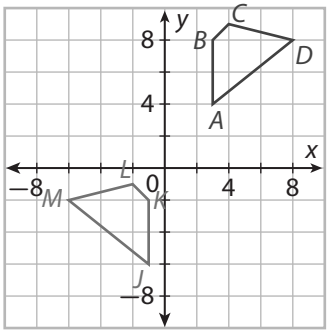


Map $\triangle ABC$ to $\triangle PQR$ with a rotation of 180° around the origin, followed by a horizontal translation.

Rotation: $(x, y) \rightarrow (-x, -y)$

Translation: $(x, y) \rightarrow (x + 1, y)$

(B) $ABCD \cong JKLM$



Map *ABCD* to *JKLM* with a

_____,

followed by a _____.

_____: $(x, y) \rightarrow$ _____

_____: $(x, y) \rightarrow$ _____

Reflect

4. How is the orientation of the figure affected by a sequence of transformations?

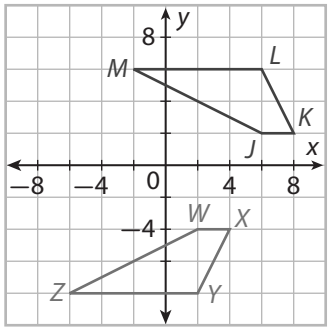
_____

_____

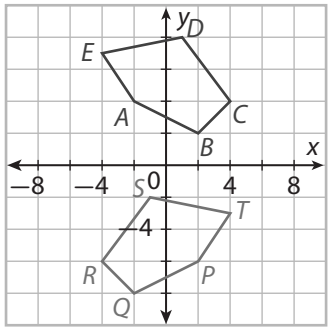
Your Turn

The figures shown are congruent. Find a sequence of rigid motions that maps one figure to the other. Give coordinate notation for the transformations you use.

5. *JKLM* ≅ *WXYZ*



6. *ABCDE* ≅ *PQRST*

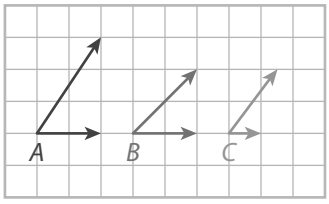


Explain 3 Investigating Congruent Segments and Angles

Congruence can refer to parts of figures as well as whole figures. Two angles are congruent if and only if one can be obtained from the other by rigid motions (that is, by a sequence of reflections, translations, and/or rotations.) The same conditions are required for two segments to be congruent to each other.

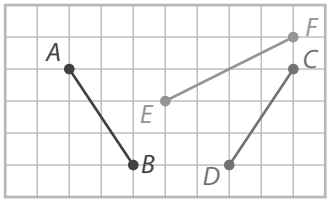
Example 3 **Determine which angles or segments are congruent. Describe transformations that can be used to verify congruence.**

A



$\angle A$ and $\angle C$ are congruent. The transformation is a translation. There is no transformation that maps $\angle B$ to either of the other angles.

B

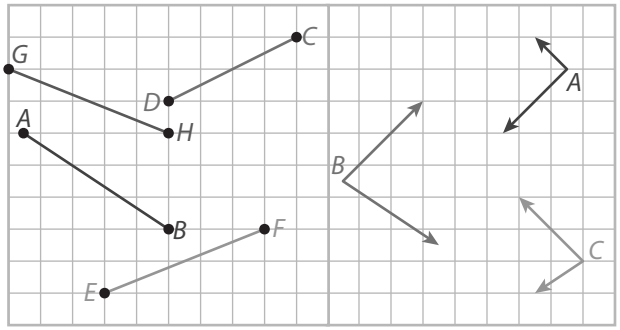


$\overline{AB}$ and _____ are congruent. A sequence of transformations is a _____ and a translation.

There is no transformation that maps _____ to either of the other segments.

Your Turn

7. Determine which segments and which angles are congruent. Describe transformations that can be used to show the congruence.



Elaborate

8. Can you say two angles are congruent if they have the same measure but the segments that identify the rays that form the angle are different lengths?

9. **Discussion** Can figures have congruent angles but not be congruent figures?

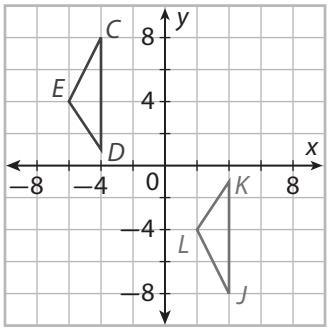
10. **Essential Question Check-In** Can you use transformations to prove that two figures are not congruent?

Evaluate: Homework and Practice

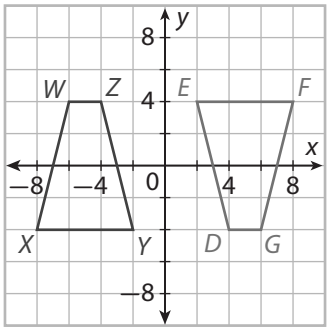
- Online Homework
- Hints and Help
- Extra Practice

Use the definition of congruence to decide whether the two figures are congruent. Explain your answer. Give coordinate notation for the transformations you use.

1.



2.



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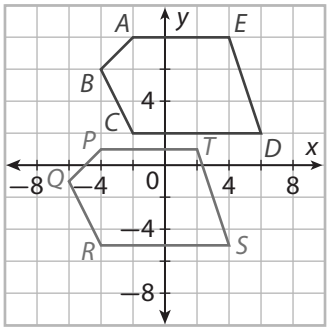

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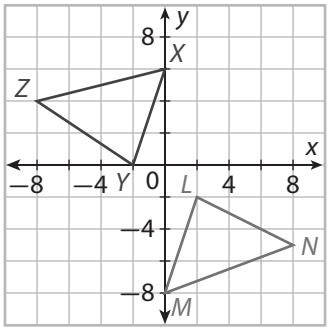

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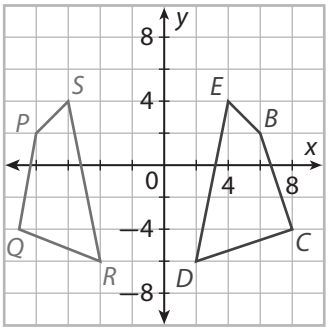
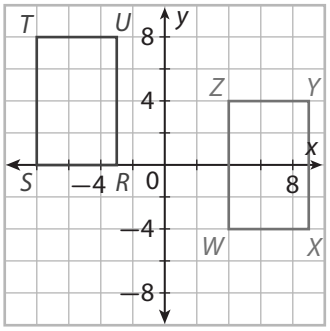
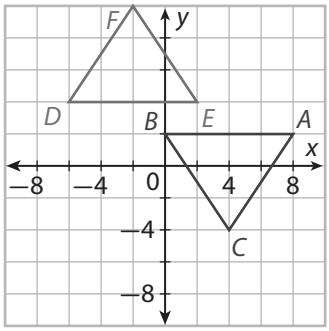


The figures shown are congruent. Find a sequence of rigid motions that maps one figure to the other. Give coordinate notation for the transformations you use.

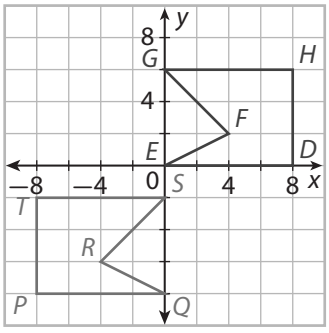
6. $RSTU \cong WXYZ$



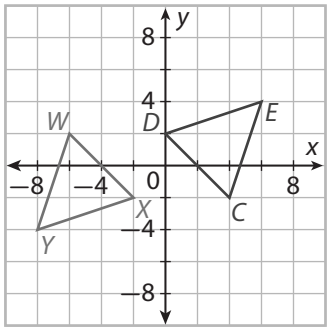
7. $\triangle ABC \cong \triangle DEF$



8. $DEFGH \cong PQRST$

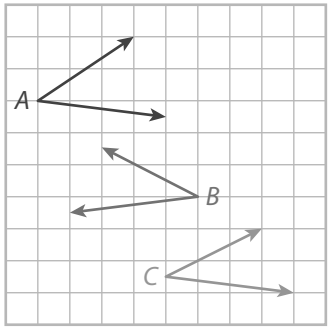


9. $\triangle CDE \cong \triangle WXY$



Determine which of the angles are congruent. Which transformations can be used to verify the congruence?

10.



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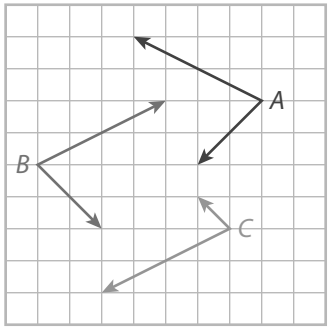
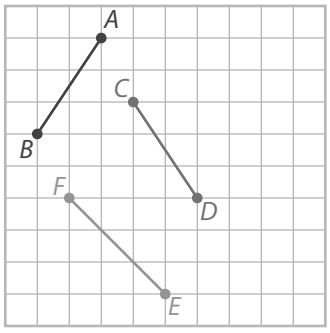
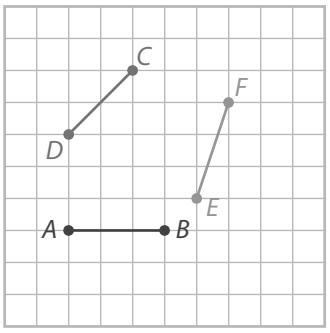


Determine which of the segments are congruent. Which transformations can be used to verify the congruence?

12.

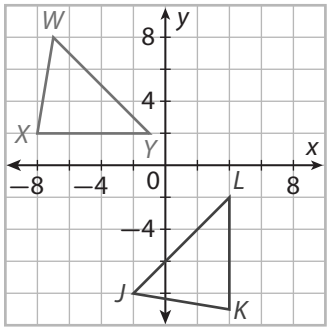


13.

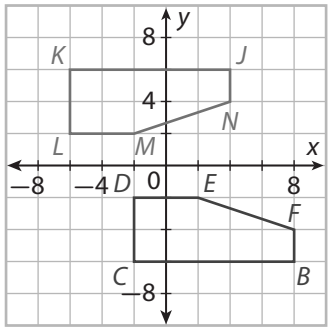


Use the definition of congruence to decide whether the two figures are congruent. Explain your answer. Give coordinate notation for the transformations you use.

14.



15.



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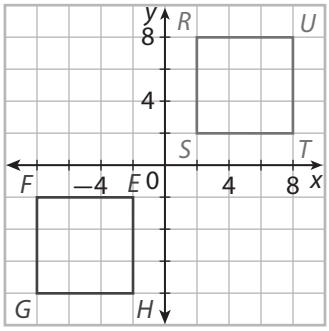

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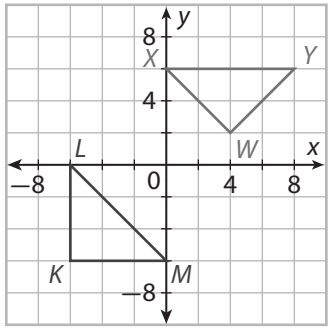
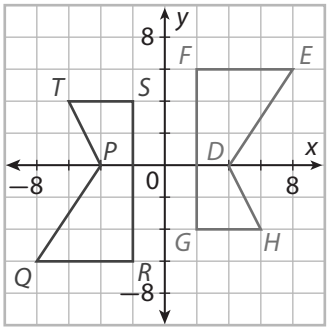
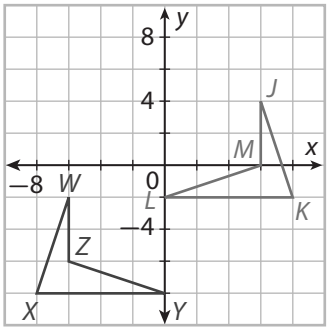


The figures shown are congruent. Find a sequence of transformations for the indicated mapping. Give coordinate notation for the transformations you use.

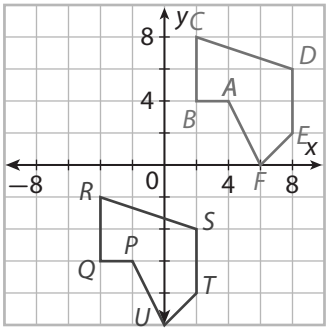
18. Map *PQRST* to *DEFGH*.



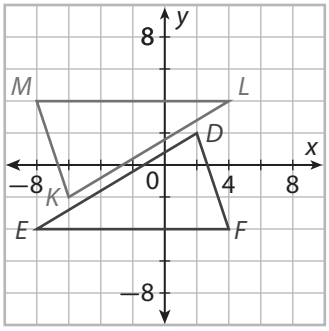
19. Map *WXYZ* to *JKLM*.



20. Map *PQRSTU* to *ABCDEF*.

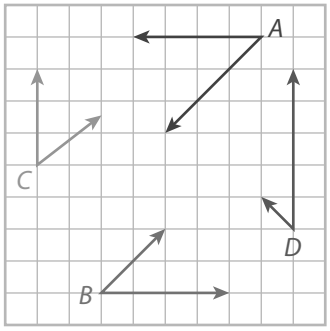


21. Map $\triangle DEF$ to $\triangle KLM$.

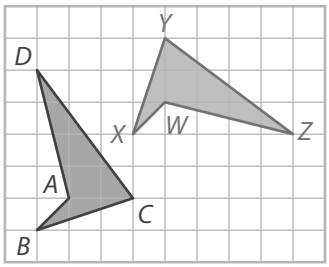


22. Determine whether each pair of angles is congruent or not congruent. Select the correct answer for each lettered part.

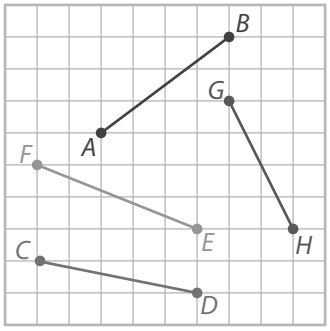
a.	$\angle A$ and $\angle B$	◯ Congruent	◯ Not congruent
b.	$\angle A$ and $\angle C$	◯ Congruent	◯ Not congruent
c.	$\angle B$ and $\angle C$	◯ Congruent	◯ Not congruent
d.	$\angle B$ and $\angle D$	◯ Congruent	◯ Not congruent
e.	$\angle C$ and $\angle D$	◯ Congruent	◯ Not congruent



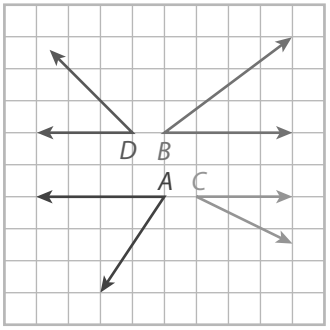
23. If *ABCD* and *WXYZ* are congruent, then *ABCD* can be mapped to *WXYZ* using a rotation and a translation. Determine whether the statement is true or false. Then explain your reasoning.



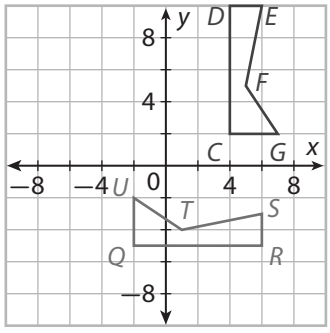
24. Which segments are congruent? Which are not congruent? Explain.



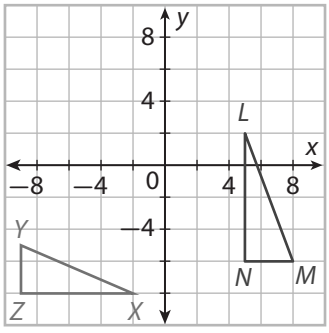
25. Which angles are congruent? Which are not congruent? Explain.



26. The figures shown are congruent. Find a sequence of transformations that will map *CDEFG* to *QRSTU*. Give coordinate notation for the transformations you use.



27. The figures shown are congruent. Find a sequence of transformations that will map $\triangle LMN$ to $\triangle XYZ$. Give coordinate notation for the transformations you use.



28. Which sequence of transformations does not map a figure onto a congruent figure? Explain.

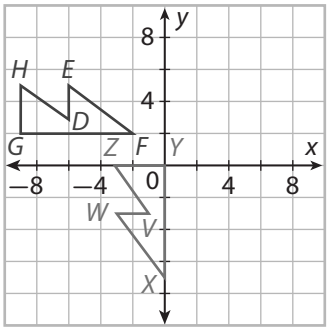
A. Rotation of 180° about the origin, reflection across the *x*-axis, horizontal translation $(x, y) \rightarrow (x + 4, y)$

B. Reflection across the *y*-axis, combined translation $(x, y) \rightarrow (x - 5, y + 2)$

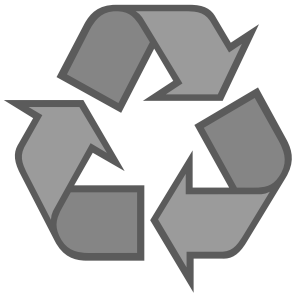
C. Rotation of 180° about the origin, reflection across the *y*-axis, dilation $(x, y) \rightarrow (2x, 2y)$

D. Counterclockwise rotation of 90° about the origin, reflection across the *y*-axis, combined translation $(x, y) \rightarrow (x - 11, y - 12)$

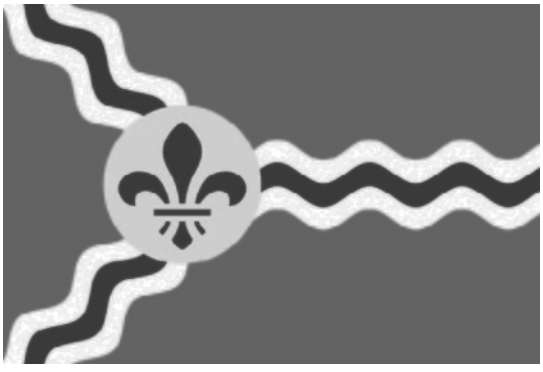
29. The figures shown are congruent. Find a sequence of transformations that will map *DEFGH* to *VWXYZ*. Give coordinate notation for the transformations you use.



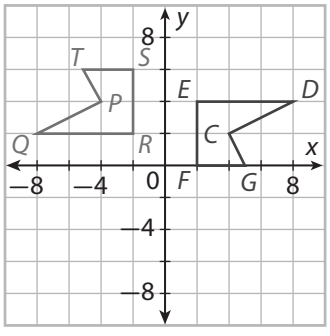
30. How can you prove that two arrows in the recycling symbol are congruent to each other?



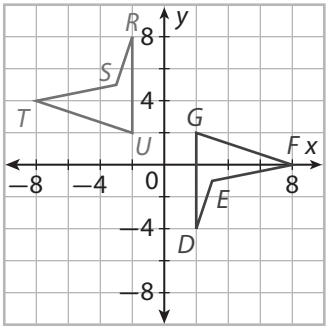
31. The city of St. Louis was settled by the French in the mid 1700s and joined the United States in 1803 as part of the Louisiana Purchase. The city flag reflects its French history by featuring the fleur-de-lis. How can you prove that the left and right petals are congruent to each other?



32. **Draw Conclusions** Two students are trying to show that the two figures are congruent. The first student decides to map *CDEFG* to *PQRST* using a rotation of 180° around the origin, followed by the translation $(x, y) \rightarrow (x, y + 6)$. The second student believes the correct transformations are a reflection across the *y*-axis, followed by the vertical translation $(x, y) \rightarrow (x, y - 2)$. Are both students correct, is only one student correct, or is neither student correct?



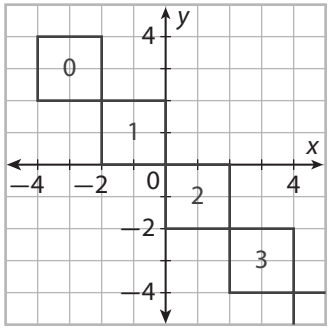
33. **Justify Reasoning** Two students are trying to show that the two figures are congruent. The first student decides to map *DEFG* to *RSTU* using a rotation of 180° about the origin, followed by the vertical translation $(x, y) \rightarrow (x, y + 4)$. The second student uses a reflection across the *x*-axis, followed by the vertical translation $(x, y) \rightarrow (x, y + 4)$, followed by a reflection across the *y*-axis. Are both students correct, is only one student correct, or is neither student correct?



H.O.T. Focus on Higher Order Thinking

34. **Look for a Pattern** Assume the pattern of congruent squares shown in the figure continues forever.

Write rules for rigid motions that map square 0 onto square 1, square 0 onto square 2, and square 0 onto square 3.



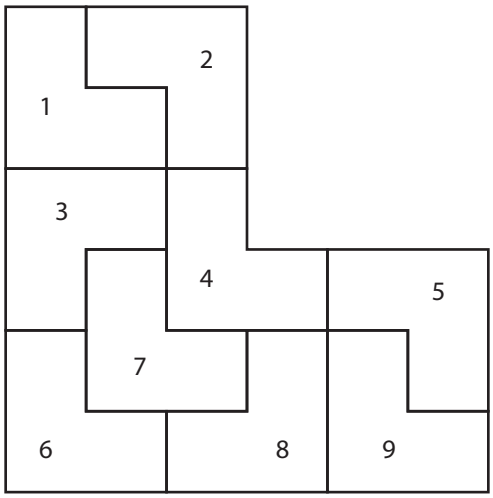
Write a rule for a rigid motion that maps square 0 onto square *n*.

35. **Analyze Relationships** Suppose you know that $\triangle ABC$ is congruent to $\triangle DEF$ and that $\triangle DEF$ is congruent to $\triangle GHJ$. Can you conclude that $\triangle ABC$ is congruent to $\triangle GHJ$? Explain.

36. **Communicate Mathematical Ideas** Ella plotted the points $A(0, 0)$, $B(4, 0)$, and $C(0, 4)$. Then she drew $\overline{AB}$ and $\overline{AC}$. Give two different arguments to explain why the segments are congruent.

Lesson Performance Task

The illustration shows how nine congruent shapes can be fitted together to form a larger shape. Each of the shapes can be formed from Shape #1 through a combination of translations, reflections, and/or rotations.



Describe how each of Shapes 2–9 can be formed from Shape #1 through a combination of translations, reflections, and/or rotations. Then design a figure like this one, using at least eight congruent shapes. Number the shapes. Then describe how each of them can be formed from Shape #1 through a combination of translations, reflections, and/or rotations.