Name ______________________ Class ____________ Date ________

21.2 Analyzing Decisions

Essential Question: How can conditional probability help you make real-world decisions?

Resource Locker

Explore 1 Analyzing a Decision Using Probability

Suppose scientists have developed a test that can be used at birth to determine whether a baby is right-handed or left-handed. The test uses a drop of the baby's saliva and instantly gives the result. The test has been in development long enough for the scientists to track the babies as they grow into toddlers and to see whether their test is accurate. About 10% of babies turn out to be left-handed.

The scientists have learned that when children are left-handed, the test correctly identifies them as left-handed 92% of the time. Also, when children are right-handed, the test correctly identifies them as right-handed 95% of the time.

(A) In the first year on the market, the test is used on 1,000,000 babies. Complete the table. Begin with the grand total (lower-right table cell) and the row totals, and use those to help you fill in the four "Actually" versus "Tests" cells and then the two column totals.

	Tests Left-handed	Tests Right-handed	Total
Actually Left-handed			
Actually Right-handed			
Total			

(B) What is the probability that a baby who tests left-handed actually is left-handed? ____________

(C) What is the probability that a baby who tests right-handed actually is right-handed? ____________

Reflect

1. Is the test a good test of right-handedness?

____

2. A baby is tested, and the test shows the baby will be left-handed. The parents decide to buy a left-handed baseball glove for when the baby is old enough to play baseball. Is this a reasonable decision?

____

____

3. **Discussion** Describe two ways in which the test can become a more reliable indicator of left-handedness.

Explore 2 Deriving Bayes' Theorem

Bayes' Theorem gives a formula for calculating an unknown conditional probability from other known probabilities. You can use Bayes' Theorem to calculate conditional probabilities like those in Steps B and C in Explore 1.

A Complete the steps to derive Bayes' Theorem.

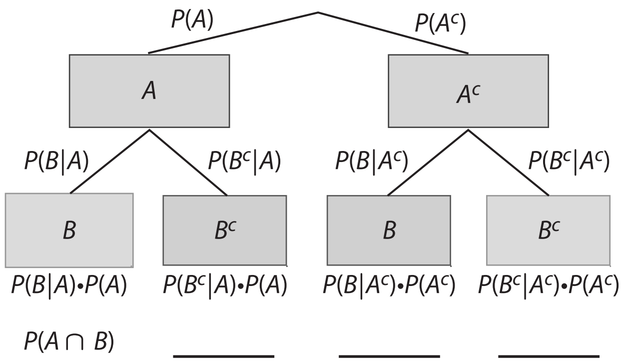
(1) Write the formula for $P(B|A)$. ______

(2) Solve for $P(A \cap B)$. ______

(3) Write the formula for $P(A|B)$. ______

(4) Substitute the expression for $P(A \cap B)$ in step (2) into the formula for $P(A|B)$ in step (3). ______

B Complete the equations at the bottom of the tree diagram. Use the fact that the probability shown along each branch is the probability of the single event to which that branch leads. Each product of probabilities listed at the bottom of the tree diagram is the probability that two events occur together.



C Complete the explanation for finding $P(B)$ using the tree diagram in Step B.

Because events A and A^c are mutually exclusive, the events $A \cap B$ and ______ are mutually exclusive. Since the union of $A \cap B$ and ______ is ______,

$P(B) = P(A \cap B) +$ ______ $= P(A) \cdot P(B|A) +$ ______.

D Use your result from Step C to rewrite your final expression from Step A to get another form of Bayes' Theorem.

Reflect

4. Explain in words what each expression means in the context of Explore 1.

$P(A)$ is the probability of actually being left-handed.

$P(B)$ is the probability of testing left-handed.

$P(A|B)$ is ______________________________.

$P(B|A)$ is ______________________________.

5. Use Bayes' Theorem to calculate the probability that a baby actually is left-handed, given that the baby tests left-handed. Explain what this probability means.

____

____

____

Explain 1 Using Bayes' Theorem

Bayes' Theorem is a useful tool when you need to make or analyze decisions.

Bayes' Theorem

Given two events A and B with $P(B) \neq 0$, $P(A|B) = \frac{P(A) \cdot P(B|A)}{P(B)}$.

Another form is $P(A|B) = \frac{P(A) \cdot P(B|A)}{P(A) \cdot P(B|A) + P(A^c) \cdot P(B|A^c)}$.

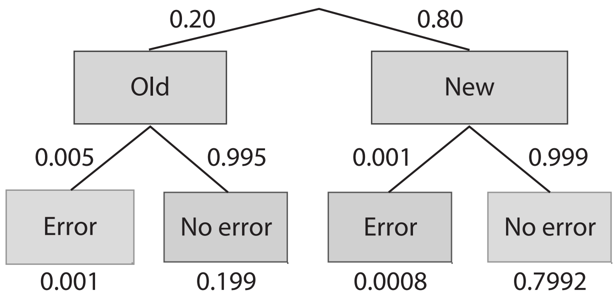
Example 1 **Suppose Walter operates an order-filling machine that has an error rate of 0.5%. He installs a new order-filling machine that has an error rate of only 0.1%. The new machine takes over 80% of the order-filling tasks.**

Ⓐ One day, Walter gets a call from a customer complaining that her order wasn't filled properly. Walter blames the problem on the old machine. Was he correct in doing so?

First, find the probability that the order was filled by the old machine given that there was an error in filling the order, $P(\text{old} \mid \text{error})$.

$$P(\text{old}|\text{error}) = \frac{P(\text{old}) \cdot P(\text{error}|\text{old})}{P(\text{error})}$$

$$= \frac{(0.20) \cdot 0.005}{0.001 + 0.0008} = \frac{0.001}{0.0018} = \frac{5}{9} \approx 0.56$$



Given that there is a mistake, the probability is about 56% that the old machine filled the order. The probability that the new machine filled the order is $1 - 0.56 = 44\%$. The old machine is only slightly more likely than the new machine to have filled the order. Walter shouldn't blame the old machine.

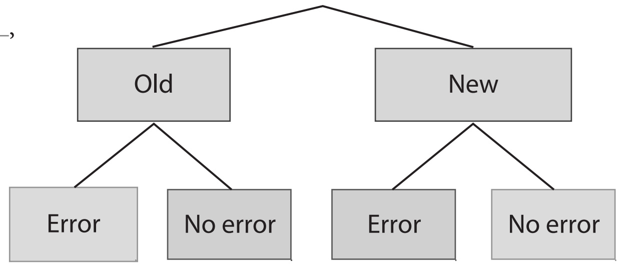
(B) Walter needs to increase capacity for filling orders so he increases the number of orders being filled by the old machine to 30% of the total orders. What percent of errors in filled orders are made by the old machine? Is Walter unreasonably increasing the risk of shipping incorrectly filled orders?

Find the probability that ______________________________

given that ______________________________,

$P(\_\_\_ \mid \_\_\_\_)$.

Use Bayes' Theorem.



____

____

Describe the result of making this change.

____

____

____

____

Reflect

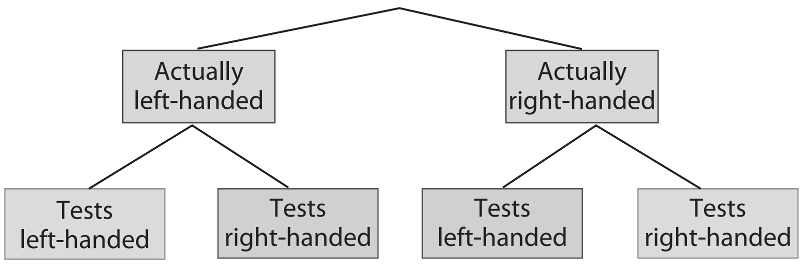
6. Given that the old machine fills so few orders, how can it be responsible for more than half of the errors?

____

Your Turn

In the situation described in Explore 1, suppose the scientists have changed the test so that now it correctly identifies left-handed babies 100% of the time, and still correctly identifies right-handed babies 95% of the time.

7. Complete the tree diagram.



8. With the new test, what is the probability that a child who tests left-handed will be left-handed? How does this compare to the original test?

9. With the new test, what is the probability that a child who tests right-handed will be right-handed? How does this compare to the original test?

Elaborate

10. Discussion Compare the probabilities you found in Explore 1 and Your Turn 8 and 9. Why did the probability that a baby who tests right-handed actually is right-handed become 100%?

11. Essential Question Check-In How can you use probability to help you analyze decisions?

Evaluate: Homework and Practice

Personal Math Trainer

- Online Homework
- Hints and Help
- Extra Practice

1. A factory manager is assessing the work of two assembly-line workers. Helen has been on the job longer than Kyle. Their production rates for the last month are in the table. Based on comparing the number of defective products, the manager is considering putting Helen on probation. Is this a good decision? Why or why not?

	Helen	Kyle	Total
Defective	50	20	70
Not defective	965	350	1,315
Total	1,015	370	1,385

2. Multiple Step A reporter asked 150 voters if they plan to vote in favor of a new library and a new arena. The table shows the results. If you are given that a voter plans to vote no for the new arena, what is the probability that the voter also plans to vote no for the new library?

	Yes for Library	No for Library	Total
Yes for Arena	21	30	51
No for Arena	57	42	99
Total	78	72	150

3. You want to hand out coupons for a local restaurant to students who live off campus at a rural college with a population of 10,000 students. You know that 10% of the students live off campus and that 98% of those students ride a bike. Also, 62% of the students who live on campus do not have a bike. You decide to give a coupon to any student you see who is riding a bike. Complete the table. Then explain whether this a good decision.

	Bike	No bike	Total
On campus			
Off campus			
Total			

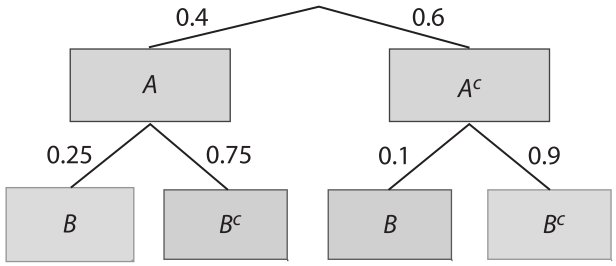
4. A test for a virus correctly identifies someone who has the virus (by returning a positive result) 99% of the time. The test correctly identifies someone who does not have the virus (by returning a negative result) 99% of the time. It is known that 0.5% of the population has the virus. A doctor decides to treat anyone who tests positive for the virus. Complete the two-way table assuming a total population of 1,000,000 people have been tested. Is this a good decision?

	Tests Positive	Tests Negative	Total
Virus			
No virus			
Total			

5. It is known that 2% of the population has a certain allergy. A test correctly identifies people who have the allergy 98% of the time. The test correctly identifies people who do not have the allergy 95% of the time. A website recommends that anyone who tests positive for the allergy should begin taking anti-allergy medication. Complete the two-way table. Do you think this is a good recommendation? Why or why not?

	Test Positive	Test Negative	Total
Total			10,000

6. Use the tree diagram shown.

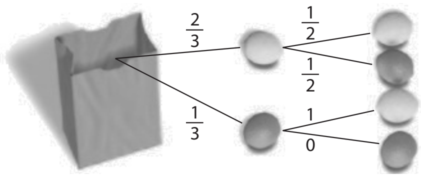


a. Find · $P(A^c)$. $P(B|A^c)$

b. Find $P(B)$.

c. Use Bayes's Theorem to find $P(A^c|B)$.

7. The probabilities of drawing lemons and limes from a bag are shown in the tree diagram. Find the probability of drawing the two pieces of fruit randomly from the bag.



a. two lemons

b. two limes

c. lime then lemon

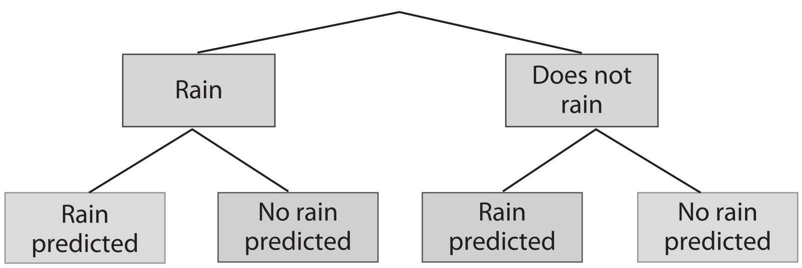
d. lemon then lime OR lime then lemon

8. **Multiple Step** A school principal plans a school picnic for June 2. A few days before the event, the weather forecast predicts rain for June 2, so the principal decides to cancel the picnic. Consider the following information.

- In the school's town, the probability that it rains on any day in June is 3%.
- When it rains, the forecast correctly predicts rain 90% of the time.
- When it does not rain, the forecast incorrectly predicts rain 5% of the time.

a. Find $P(\text{prediction of rain}\,|\,\text{rains})$ and $P(\text{rains})$.

b. Complete the tree diagram, and find $P(\text{prediction of rain})$.



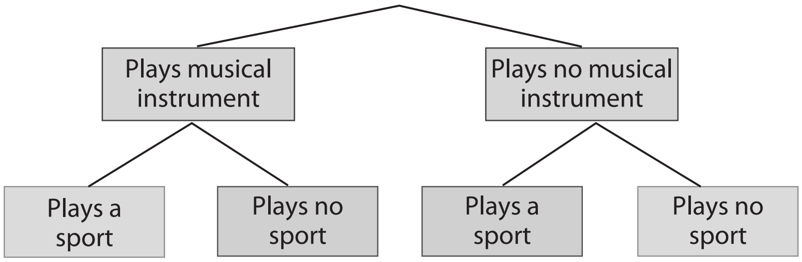
c. Find $P(\text{rains}|\ \text{prediction of rain})$.

d. Is the decision to cancel the picnic reasonable?

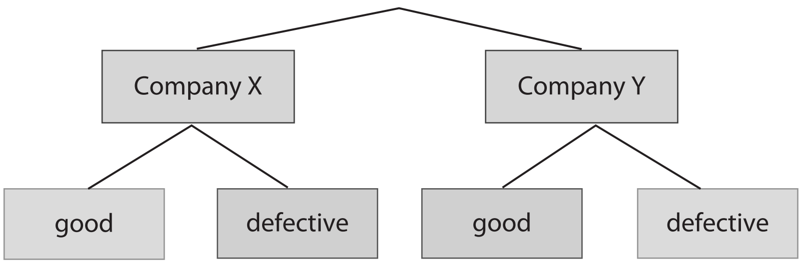
9. Pamela has collected data on the number of students in the sophomore class who play a sport or play a musical instrument. She has learned the following.

- 42.5% of all students in her school play a musical instrument.
- 20% of those who play a musical instrument also play a sport.
- 40% of those who play no instrument also play no sport.

Complete the tree diagram. Would it be reasonable to conclude that a student who doesn't play a sport plays a musical instrument?



10. **Interpret the Answer** Company X supplies 35% of the phones to an electronics store and Company Y supplies the remainder. The manager of the store knows that 25% of the phones in the last shipment from Company X were defective, while only 5% of the phones from Company Y were defective. The manager chooses a phone at random and finds that it is defective. The manager decides that the phone must have come from Company X. Do you think this is a reasonable conclusion? Why or why not?



11. Suppose that strep throat affects 2% of the population and a test to detect it produces an accurate result 99% of the time. Create a tree diagram and use Bayes' Theorem to find the probability that someone who tests positive actually has strep throat.

12. **Fundraising** A hand-made quilt is first prize in a fund-raiser raffle. The table shows information about all the ticket buyers. Given that the winner of the quilt is a man, what is the probability that he resides in Sharonville?

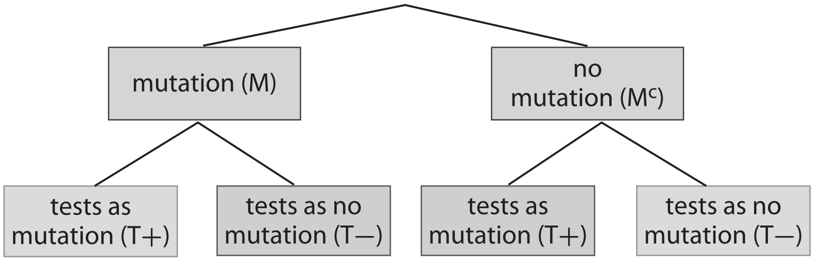
	Men	Women	Total
Forestview	35	45	80
Sharonville	15	25	40

13. Recall that the Multiplication Rule says that $P(A \cap B) = P(A) \cdot P(B|A)$. If you switch the order of events A and B, then the rule becomes $P(B \cap A) = P(B) \cdot P(A|B)$. Use the Multiplication Rule and the fact that $P(B \cap A) = P(A \cap B)$ to prove Bayes' Theorem. (Hint: Divide each side by P(B).)

14. **Sociology** A sociologist collected data on the types of pets in 100 randomly selected households. Suppose you want to offer a service to households that own both a cat and a dog. Complete the table. Based on the data in the table, would it be more effective to hand information to people walking dogs or to people buying cat food?

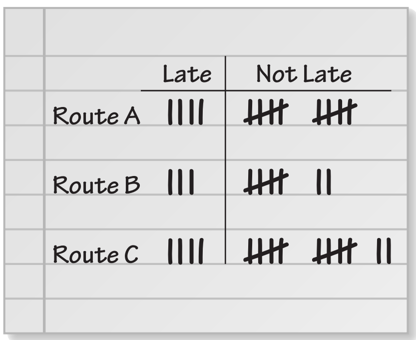
		Owns a Cat		
		Yes	**No**	
Owns a Dog	**Yes**	15	24	
	No	18	43	

15. Interpret the Answer It is known that 1% of all mice in a laboratory have a genetic mutation. A test for the mutation correctly identifies mice that have the mutation 98% of the time. The test correctly identifies mice that do not have the mutation 96% of the time. A lab assistant tests a mouse and finds that the mouse tests positive for the mutation. The lab assistant decides that the mouse must have the mutation. Is this a good decision? Complete the tree diagram and explain your answer.



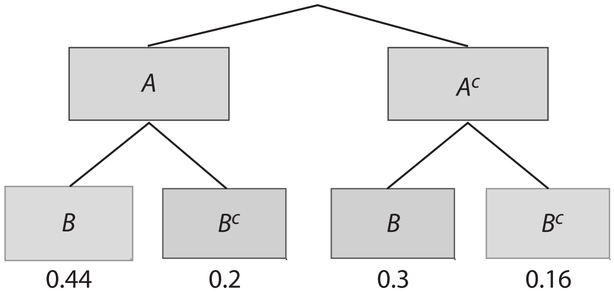
16. Interpret the Answer It is known that 96% of all dogs do not get trained. One professional trainer claims that 54% of trained dogs will sit on one of the first four commands to sit and that no other dogs will sit on command. A condominium community wants to impose a restriction on dogs that are not trained. They want each dog owner to show that his or her dog will sit on one of the first four commands. Assuming that the professional trainer's claim is correct, is this a fair way to identify dogs that have not been trained? Explain.

17. Multiple Steps Tomas has a choice of three possible routes to work. On each day, he randomly selects a route and keeps track of whether he is late. Based on this 40-day trial, which route makes Tomas least likely to be late for work?

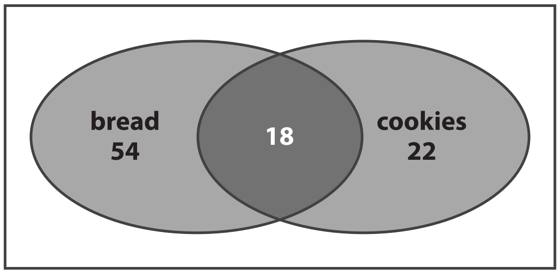


H.O.T. Focus on Higher Order Thinking

18. Critique Reasoning When Elisabeth saw this tree diagram, she said that the calculations must be incorrect. Do you agree? Justify your answer.



19. Multiple Representations The Venn diagram shows how many of the first 100 customers of a new bakery bought either bread or cookies, both, or neither. Taryn claims that the data indicate that a customer who bought cookies is more likely to have bought bread than a customer who bought bread is likely to have bought cookies. Is she correct?

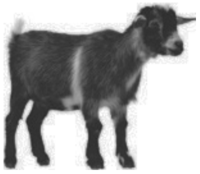


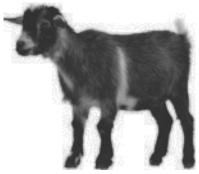
20. Persevere in Problem Solving At one high school, the probability that a student is absent today, given that the student was absent yesterday, is 0.12. The probability that a student is absent today, given that the student was present yesterday, is 0.05. The probability that a student was absent yesterday is 0.1. A teacher forgot to take attendance in several classes yesterday, so he assumed that attendance in his class today is the same as yesterday. If there were 40 students in these classes, how many errors would you expect by doing this?

Lesson Performance Task

You're a contestant on a TV quiz show. Before you are three doors. Behind two of the doors, there's a goat. Behind one of the doors, there's a new car. You are asked to pick a door. After you make your choice, the quizmaster opens one of the doors you *didn't* choose, revealing a goat.


Door 1


Door 2


Door 3

Now there are only two doors. You can stick with your original choice or you can switch to the one remaining door. Should you switch?

Intuition tells most people that, with two doors left, there's a 50% probability that they're right and a 50% probability that they're wrong. They conclude that it doesn't matter whether they switch or not.

Does it? Using Bayes' Theorem, it can be shown mathematically that you're much better off switching! You can reach the same conclusion using logical reasoning skills. Assume that the car is behind Door 1. (The same reasoning can be applied if the car is behind one of the other doors.) You've decided to switch your choice after the first goat is revealed. There are three possibilities based on which door you choose first. Describe each possibility and what happens when you switch. Based on the three possibilities, what is the probability that you win the car? Explain how you obtain the probability..