Name ______________________ Class ______________ Date __________

21.1 Using Probability to Make Fair Decisions

Essential Question: How can you use probability to help you make fair decisions?

Resource Locker

Explore Using Probabilities When Drawing at Random

You are sharing a veggie supreme pizza with friends. There is one slice left and you and a friend both want it. Both of you have already had two slices. What is a fair way to solve this problem?

(A) Suppose you both decide to have the same amount of pizza. This means that the last slice will be cut into two pieces. Describe a fair way to split this last piece.

(B) Suppose instead you decide that one of you will get the whole slice. Complete the table so that the result of each option gives a fair chance for each of you to get the last slice. Why do each of these possibilities give a fair chance?

Option	Result (you get last slice)	Result (friend gets last slice)
Flip a coin	Heads	
Roll a standard die		1, 3, 5
Play Rock, Paper, Scissors	You win.	You ________.
Draw lots using two straws of different lengths		Short straw

Reflect

1. Suppose, when down to the last piece, you tell your friend, "I will cut the last piece, and I will choose which piece you get." Why is this method unfair?

2. Your friend suggests that you shoot free throws to decide who gets the last piece. Use probability to explain why this might not be a fair way to decide.

Explain 1 Awarding a Prize to a Random Winner

Suppose you have to decide how to award a prize to a person at an event. You might want every person attending to have the same chance of winning, or you might want people to do something to improve their chance of winning. How can you award the prize fairly?

Example 1 **Explain whether each method of awarding a prize is fair.**

(A) The sponsor of an event wants to award a door prize to one attendee. Each person in attendance is given a ticket with a unique number on it. All of the numbers are placed in a bowl, and one is drawn at random. The person with the matching number wins the prize.

The method of awarding a door prize is fair. Each number has the same chance of being chosen, so each attendee has an equal probability of winning the prize. If n attendees are at the event, then the probability of winning the prize is $\frac{1}{n}$ for each attendee.

(B) A fundraiser includes a raffle in which half of the money collected goes to a charity, and the other half goes to one winner. Tickets are sold for $5 each. Copies of all the tickets are placed in a box, and one ticket is drawn at random. The person with the matching ticket wins the raffle.

The method of choosing a raffle winner is fair/not fair because

each ________ has an equal probability of being drawn.

Reflect

3. In Example 1B, the probability may not be the same for each person to win the raffle. Explain why the method is still fair.

Your Turn

4. Each month, a company wants to award a special parking space to an employee at random. Describe a fair way to do this. Include a way to ensure that a person doesn't win a second time before each employee has won once.

Explain 2 Solving Real-World Problems Fairly

You can use a random number generator to choose a winner of a prize.

Example 2 **Use a problem solving plan.**

A class of 24 students sold 65 magazine subscriptions to raise money for a trip. The table shows three of the students and the number of subscriptions each sold. As an incentive to participate, you will award a prize to one student in the class. Describe a method of awarding the prize fairly. Use probabilities to explain why your method is fair for the students listed.

Student	Subscriptions Sold
Miri	5
Liam	2
Madison	0

Analyze Information

Identify the important information.

- There are ____ students.
- They sold ____ magazine subscriptions.
- There is one prize, so there will be one winner.

Formulate a Plan

To be fair, students who sold more subscriptions should have a better chance of winning the prize than the students who sold fewer.

Find a method of assigning outcomes so that the chance of winning is proportional to the number of subscriptions sold.

Solve

The class sold 65 subscriptions, so assign the numbers 1–65 to the students. Each student gets as many numbers as the number of subscriptions he or she sold.

Student	Subscriptions Sold	Numbers Assigned	Probability of Winning
Miri	5	1–5	$\frac{\square}{65} \approx 7.7\%$
Liam	2	6, 7	$\frac{\square}{65} \approx 3.1\%$
Madison	0	none	$\frac{\square}{65} = 0\%$

Then use a calculator to find a random integer from 1 to 65 (randInt(1, 65)). Then, for instance, if the result is 7, Liam wins the prize.

Justify and Evaluate

This method seems fair/unfair because it gives everyone who sold subscriptions a chance of winning. You could award a prize to the student who sold the most subscriptions, but this might not be possible if multiple students all sold the same number, and it might not seem fair if some students have better access to buyers than others.

Reflect

5. A student suggests that it would be better to assign the numbers to students randomly rather than in numerical order. Would doing this affect the probability of winning?

6. A charity is giving a movie ticket for every 10 coats donated. Jacob collected 8 coats, Ben collected 6, and Ryan and Zak each collected 3. They decide to donate the coats together so that they will get 2 movie tickets. Describe how to use a random number generator to decide which 2 boys get a ticket.

Explain 3 Solving the Problem of Points

The decision-making process that you will apply in this example is based on the “Problem of Points” that was studied by the French mathematicians Blaise Pascal and Pierre de Fermat in the 17th century. Their work on the problem launched the branch of mathematics now known as probability.

Example 3 **Two students, Lee and Rory, find a box containing 100 baseball cards. To determine who should get the cards, they decide to play a game with the rules shown.**

Game Rules

- One of the students repeatedly tosses a coin.
- When the coin lands heads up, Lee gets a point.
- When the coin lands tails up, Rory gets a point.
- The first student to reach 20 points wins the game and gets the baseball cards.

As Lee and Rory are playing the game, they are interrupted and unable to continue. How should the 100 baseball cards be divided between the students given that the game was interrupted at the described moment?

(A) When they are interrupted, Lee has 19 points and Rory has 17 points.

At most, 3 coin tosses would have been needed for someone to win the game.

Make a list of all possible results using H for heads and T for tails. Draw boxes around the outcomes in which Lee wins the game.

0T, 3H	1T, 2H	2T, 1H	3T, 0H
$\boxed{\text{HHH}}$	$\boxed{\text{THH}}$	$\boxed{\text{TTH}}$	TTT
	$\boxed{\text{HTH}}$	$\boxed{\text{THT}}$	
	$\boxed{\text{HHT}}$	$\boxed{\text{HTT}}$	

There are 8 possible results. Lee wins in 7 of them and Rory wins in 1 of them.

The probability of Lee winning is $\frac{7}{8}$, so he should get $\frac{7}{8}$ of the cards which is 87.5 cards.
The probability of Rory winning is $\frac{1}{8}$, so he should get $\frac{1}{8}$ of the cards which is 12.5 cards.
Rather than split a card into two, they might decide to flip a coin for that card or let Lee have it because he was more likely to win it.

(B) When they are interrupted, Lee has 18 points and Rory has 17 points.

At most, ________ more coin tosses would have been needed.

List all possible results. Draw boxes around the outcomes in which Lee wins.

0T, 4H   1T, 3H   2T, 2H   3T, 1H   4T, 0H

There are _____ possible results. Lee wins in _____ of them and Rory wins in _____ of them.

The probability of Lee winning is _____ , so he should get _____ cards.

The probability of Rory winning is _____ , so he should get _____ cards.

Reflect

7. **Discussion** A student suggests that a better way to divide the cards in Example 3B would be to split the cards based on the number of points earned so far. Which method do you think is better?

Your Turn

8. Describe a situation where the game is interrupted, resulting in the cards needing to be divided evenly between the two players.

Elaborate

9. **Discussion** In the situation described in the Explore, suppose you like the crust and your friend does not. Is there a fair way to cut the slice of pizza that might not result in two pieces with the same area?

10. How would the solution to Example 2 need to change if there were two prizes to award? Assume that you do not want one student to win both prizes.

11. **Essential Question Check-In** Describe a way to use probability to make a fair choice of a raffle winner.

Evaluate: Homework and Practice

- Online Homework
- Hints and Help
- Extra Practice

1. You and a friend split the cost of a package of five passes to a climbing gym. Describe a way that you could fairly decide who gets to use the fifth pass.

2. In addition to prizes for first, second, and third place, the organizers of a race have a prize that they want each participant to have an equal chance of winning. Describe a fair method of choosing a winner for this prize.

Decide whether each method is a fair way to choose a winner based on whether each person has an equal chance of winning. Explain your answer by calculating each person's probability of winning.

3. Roll a standard die. Meri wins if the result is less than 3. Riley wins if the result is greater than 3.

4. Draw a card from a standard deck of cards. Meri wins if the card is red. Riley wins if the card is black.

5. Flip a coin. Meri wins if it lands heads. Riley wins if it lands tails.

6. Meri and Riley both jump as high as they can. Whoever jumps higher wins.

7. Roll a standard die. Meri wins if the result is even. Riley wins if the result is odd.

8. Randomly draw a stone from a box that contains 5 black stones and 4 white stones. Meri wins if the stone is black. Riley wins if the stone is white.

9. A chess club has received a chess set to give to one of its members. The club decides that everyone should have a chance of winning the set based on how many games they have won this season. Describe a fair method to decide who wins the set. Find the probability that each member will win it.

Member	Games Won	Probability of Winning	Member	Games Won	Probability of Winning
Kayla	30		Hailey	12	
Noah	23		Gabe	12	
Ava	18		Concour	5	

10. Owen, Diego, and Cody often play a game of chance during lunch. When they can't finish, they calculate the probability that each will win given the current state of the game and assign partial wins. Today, when they had to stop, they calculated that there were 56 possibilities for how the game could be completed. Owen was the winner in 23 of the possibilities, Diego was the winner in 18 of them, and Cody was the winner in 15. To two decimal places, how should they assign partial wins?

Represent Real-World Problems **Twenty students, including Paige, volunteer to work at the school banquet. Each volunteer worked a nonzero whole number of hours. Paige worked 4 hours. The students worked a total of 45 hours. The organizers would like to award a prize to one of the volunteers.**

11. Describe a process for awarding the prize so that each volunteer has an equal chance of winning. Find the probability that Paige wins.

12. Describe a process for awarding the prize so that each volunteer's chance of winning is proportional to how many hours the volunteer worked. Find the probability that Paige wins.

There are 10,000 seats available in a sports stadium. Each seat has a package beneath it, and 20 of the seats have an additional prize-winning package with a family pass for the entire season.

13. Under what circumstances is this method of choosing winners of the family passes fair?

14. Assuming that the method of choosing winners is fair, what is the probability of winning a family pass if you attend the game?

15. What is the probability of not winning a family pass if you attend the game?

A teacher tells students, "For each puzzle problem you complete, I will assign you an entry number for a prize giveaway." In all, 10 students complete 53 puzzle problems. Leon completed 7. To award the prize, the teacher uses a calculator to generate a random integer from 1 to 53. Leon is assigned the numbers 18 to 24.

16. What is the probability that a particular number is chosen?

17. What is the probability that one of Leon's numbers will be chosen?

18. What is the probability that one of Leon's numbers will not be chosen?

19. Is this fair to Leon according to the original instructions? Explain.

H.O.T. Focus on Higher Order Thinking

20. **Make a Conjecture** Two teams are playing a game against one another in class to earn 10 extra points on an assignment. The teacher said that the points will be split fairly between the two teams, depending on the results of the game. If Team A earned 1300 points and Team B earned 2200 points, describe one way the teacher could split up the 10 extra points. Explain.

21. **Persevere in Problem Solving** Alexa and Sofia are at a yard sale, and they find a box of 20 collectible toys that they both want. They can't agree about who saw it first, so they flip a coin until Alexa gets 10 heads or Sofia gets 10 tails. When Alexa has 3 heads and Sofia has 6 tails, they decide to stop and divide the toys proportionally based on the probability each has of winning under the original rules. How should they divide the toys?

Lesson Performance Task

Three games are described below. For each game, tell whether it is fair (all players are equally likely to win) or unfair (one player has an advantage). Explain how you reached your decision, being sure to discuss how probability entered into your decision.

1. You and your friend each toss a quarter. If two heads turn up, you win. If a head and a tail turn up, your friend wins. If two tails turn up, you play again.
2. You and your friend each roll a number cube. If the sum of the numbers is odd, you get 1 point. If the sum is even, your friend gets 1 point.
3. You and your friend each roll a number cube. If the product of the numbers is odd, you get 1 point. If the product is even, your friend gets 1 point.