Name ________________ Class ________ Date ______

20.3 Dependent Events

Essential Question: How do you find the probability of dependent events?

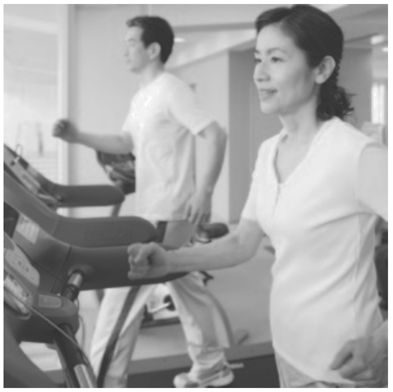
Resource Locker

Explore Finding a Way to Calculate the Probability of Dependent Events

You know two tests for the independence of events A and B:

1. If $P(A|B) = P(A)$, then A and B are independent.
2. If $P(A \cap B) = P(A) \cdot P(B)$, then A and B are independent.

Two events that fail either of these tests are **dependent events** because the occurrence of one event affects the occurrence of the other event.



(A) The two-way frequency table shows the results of a survey of 100 people who regularly walk for exercise. Let O be the event that a person prefers walking outdoors. Let M be the event that a person is male. Find $P(O)$, $P(M)$, and $P(O \cap M)$ as fractions. Then determine whether events O and M are independent or dependent.

	Prefers walking outdoors	Prefers walking on a treadmill	Total
Male	40	10	50
Female	20	30	50
Total	60	40	100

(B) Calculate the conditional probabilities $P(O|M)$ and $P(M|O)$.

$$P(O|M) = \frac{n(O \cap M)}{n(M)} = \frac{\square}{\square} = \frac{\square}{5}$$

$$P(M|O) = \frac{n(O \cap M)}{n(O)} = \frac{\square}{\square} = \frac{\square}{3}$$

Ⓒ Complete the multiplication table using the fractions for $P(O)$ and $P(M)$ from Step A and the fractions for $P(O|M)$ and $P(M|O)$ from Step B.

x	$P(O)$	$P(M)$
$P(O|M)$		
$P(M|O)$		

Ⓓ Do any of the four products in Step C equal $P(O \cap M)$, calculated in Step A? If so, which of the four products?

Reflect

1. In a previous lesson you learned the conditional probability formula $P(B|A) = \frac{P(A \cap B)}{P(A)}$. How does this formula explain the results you obtained in Step D?

2. Let F be the event that a person is female. Let T be the event that a person prefers walking on a treadmill. Write two formulas you can use to calculate $P(F \cap T)$. Use either one to find the value of $P(F \cap T)$, and then confirm the result by finding $P(F \cap T)$ directly from the two-way frequency table.

Explain 1 Finding the Probability of Two Dependent Events

You can use the Multiplication Rule to find the probability of dependent events.

Multiplication Rule

$P(A \cap B) = P(A) \cdot P(B|A)$ where $P(B|A)$ is the conditional probability of event B, given that event A has occurred.

Example 1 **There are 5 tiles with the letters A, B, C, D, and E in a bag. You choose a tile without looking, put it aside, and then choose another tile without looking. Use the Multiplication Rule to find the specified probability, writing it as a fraction.**

(A) Find the probability that you choose a vowel followed by a consonant.

Let V be the event that the first tile is a vowel. Let C be the event that the second tile is a consonant. Of the 5 tiles, there are 2 vowels, so $P(V) = \frac{2}{5}$.

Of the 4 remaining tiles, there are 3 consonants, so $P(C|V) = \frac{3}{4}$.

By the Multiplication Rule, $P(V \cap C) = P(V) \cdot P(V|C) = \frac{2}{5} \cdot \frac{3}{4} = \frac{6}{20} = \frac{3}{10}$.

(B) Find the probability that you choose a vowel followed by another vowel.

Let $V1$ be the event that the first tile is a vowel. Let $V2$ be the event that the second tile is also a vowel. Of the 5 tiles, there are ___ vowels, so $P(V1) = \frac{\square}{5}$.

Of the 4 remaining tiles, there is ___ vowel, so $P(V2|V1) = \frac{\square}{4}$.

By the Multiplication Rule, $P(V1 \cap V2) = P(V1) \cdot P(V2|V1) = \frac{\square}{5} \cdot \frac{\square}{4} = \frac{\square}{20} = \frac{\square}{10}$.

Your Turn

A bag holds 4 white marbles and 2 blue marbles. You choose a marble without looking, put it aside, and choose another marble without looking. Use the Multiplication Rule to find the specified probability, writing it as a fraction.

3. Find the probability that you choose a white marble followed by a blue marble.

4. Find the probability that you choose a white marble followed by another white marble.

Explain 2 Finding the Probability of Three or More Dependent Events

You can extend the Multiplication Rule to three or more events. For instance, for three events A, B, and C, the rule becomes $P(A \cap B \cap C) = P(A) \cdot P(B|A) \cdot P(C|A \cap B)$.

Example 2 **You have a key ring with 7 different keys. You're attempting to unlock a door in the dark, so you try keys one at a time and keep track of which ones you try.**

(A) Find the probability that the third key you try is the right one.

Let $W1$ be the event that the first key you try is wrong. Let $W2$ be the event that the second key you try is also wrong. Let R be the event that the third key you try is right.

On the first try, there are 6 wrong keys among the 7 keys, so $P(W1) = \frac{6}{7}$.

On the second try, there are 5 wrong keys among the 6 remaining keys, so $P(W2|W1) = \frac{5}{6}$.

On the third try, there is 1 right key among the 5 remaining keys, so $P(R|W2 \cap W1) = \frac{1}{5}$.

By the Multiplication Rule, $P(W1 \cap W2 \cap R) = P(W1) \cdot P(W2|W1) \cdot P(R|W1 \cap W2) = \frac{6}{7} \cdot \frac{5}{6} \cdot \frac{1}{5} = \frac{1}{7}$.

(B) Find the probability that one of the first three keys you try is right.

There are two ways to approach this problem:

1. You can break the problem into three cases: (1) the first key you try is right; (2) the first key is wrong, but the second key is right; and (3) the first two keys are wrong, but the third key is right.

2. You can use the complement: The complement of the event that one of the first three keys is right is the event that *none* of the first three keys is right.

Use the second approach.

Let $W1$, $W2$, and $W3$ be the events that the first, second, and third keys, respectively, are wrong.

From Part A, you already know that $P(W1) = \frac{\square}{7}$ and $P(W2|W1) = \frac{\square}{6}$.

On the third try, there are 4 wrong keys among the 5 remaining keys, so $P(W3|W2 \cap W1) = \frac{\square}{5}$.

By the Multiplication Rule,

$P(W1 \cap W2 \cap W3) = P(W1) \cdot P(W2|W1) \cdot P(W3|W1 \cap W2) = \frac{\square}{7} \cdot \frac{\square}{6} \cdot \frac{\square}{5} = \frac{\square}{\square}$.

The event $W1 \cap W2 \cap W3$ is the complement of the one you want. So, the probability that one of the first three keys you try is right is $1 - P(W1 \cap W2 \cap W3) = 1 - \frac{\square}{\square} = \frac{\square}{\square}$.

Reflect

5. In Part B, show that the first approach to solving the problem gives the same result.

6. In Part A, suppose you don't keep track of the keys as you try them. How does the probability change? Explain.

Your Turn

Three people are standing in line at a car rental agency at an airport. Each person is willing to take whatever rental car is offered. The agency has 4 white cars and 2 silver ones available and offers them to customers on a random basis.

7. Find the probability that all three customers get white cars.

8. Find the probability that two of the customers get the silver cars and one gets a white car.

Elaborate

9. When are two events dependent?

10. Suppose you are given a bag with 3 blue marbles and 2 red marbles, and you are asked to find the probability of drawing 2 blue marbles by drawing one marble at a time and not replacing the first marble drawn. Why does not replacing the first marble make these events dependent? What would make these events independent? Explain.

11. **Essential Question Check-In** According to the Multiplication Rule, when finding $P(A \cap B)$ for dependent events A and B, you multiply $P(A)$ by what?

Evaluate: Homework and Practice

Personal Math Trainer

- Online Homework
- Hints and Help
- Extra Practice

1. Town officials are considering a property tax increase to finance the building of a new school. The two-way frequency table shows the results of a survey of 110 town residents.

	Supports a property tax increase	Does not support a property tax increase	Total
Lives in a household with children	50	20	70
Lives in a household without children	10	30	40
Total	60	50	110

a. Let C be the event that a person lives in a household with children. Let S be the event that a person supports a property tax increase. Are the events C and S independent or dependent? Explain.

b. Find $P(C|S)$ and $P(S|C)$. Which of these two conditional probabilities can you multiply with $P(C)$ to get $P(C \cap S)$? Which of the two can you multiply with $P(S)$ to get $P(C \cap S)$?

2. A mall surveyed 120 shoppers to find out whether they typically wait for a sale to get a better price or make purchases on the spur of the moment regardless of price. The two-way frequency table shows the results of the survey.

	Waits for a Sale	Buys on Impulse	Total
Woman	40	10	50
Man	50	20	70
Total	90	30	120

a. Let W be the event that a shopper is a woman. Let S be the event that a shopper typically waits for a sale. Are the events W and S independent or dependent? Explain.

b. Find $P(W|S)$ and $P(S|W)$. Which of these two conditional probabilities can you multiply with $P(W)$ to get $P(W \cap S)$? Which of the two can you multiply with $P(S)$ to get $P(W \cap S)$?

There are 4 green, 10 red, and 6 yellow marbles in a bag. Each time you randomly choose a marble, you put it aside before choosing another marble at random. Use the Multiplication Rule to find the specified probability, writing it as a fraction.

3. Find the probability that you choose a red marble followed by a yellow marble.

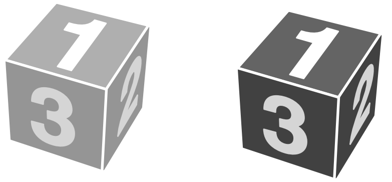
4. Find the probabilty that you choose one yellow marble followed by another yellow marble.

5. Find the probability that you choose a red marble, followed by a yellow marble, followed by a green marble.

6. Find the probability that you choose three red marbles.

The table shows the sums that are possible when you roll two number cubes and add the numbers. Use this information to answer the questions.

+	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12



7. Let A be the event that you roll a 2 on the number cube represented by the row labeled 2. Let B be the event that the sum of the numbers on the cubes is 7.

a. Are these events independent or dependent? Explain.

b. What is $P(A \cap B)$?

8. Let A be the event that you roll a 3 on the number cube represented by the row labeled 3. Let B be the event that the sum of the numbers on the cubes is 5.

a. Are these events independent or dependent? Explain.

b. What is $P(A \cap B)$?

9. A cooler contains 6 bottles of apple juice and 8 bottles of grape juice. You choose a bottle without looking, put it aside, and then choose another bottle without looking. Match each situation with its probability. More than one situation can have the same probability.

a. Choose apple juice and then grape juice. ______ $\frac{4}{13}$

b. Choose apple juice and then apple juice. ______ $\frac{24}{91}$

c. Choose grape juice and then apple juice. ______ $\frac{15}{91}$

d. Choose grape juice and then grape juice.

10. Jorge plays all tracks on a playlist with no repeats. The playlist he's listening to has 12 songs, 4 of which are his favorites.

a. What is the probability that the first song played is one of his favorites, but the next two songs are not?

b. What is the probability that the first three songs played are all his favorites?

c. Jorge can also play the tracks on his playlist in a random order with repeats possible. If he does this, how does your answer to part b change? Explain why.

11. You are playing a game of bingo with friends. In this game, balls are labeled with one of the letters of the word BINGO and a number. Some of these letter-number combinations are written on a bingo card in a 5 × 5 array, and as balls are randomly drawn and announced, players mark their cards if the ball's letter-number combination appears on the cards. The first player to complete a row, column, or diagonal on a card says "Bingo!" and wins the game. In the game you're playing, there are 20 balls left. To complete a row on your card, you need N-32 called. To complete a column, you need G-51 called. To complete a diagonal, you need B-6 called.

a. What is the probability that the next two balls drawn do not have a letter-number combination you need, but the third ball does?

b. What is the probability that none of the letter-number combinations you need is called from the next three balls?

H.O.T. Focus on Higher Order Thinking

12. You are talking with 3 friends, and the conversation turns to birthdays.

a. What is the probability that no two people in your group were born in the same month?

b. Is the probability that at least two people in your group were born in the same month greater or less than $\frac{1}{2}$? Explain.

c. How many people in a group would it take for the probability that at least two people were born in the same month to be greater than $\frac{1}{2}$? Explain.

13. Construct Arguments Show how to extend the Multiplication Rule to three events *A*, *B*, and *C*.

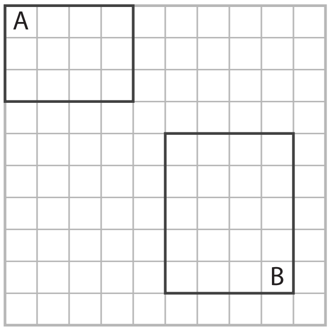
14. Make a Prediction A bag contains the same number of red marbles and blue marbles. You choose a marble without looking, put it aside, and then choose another marble. Is there a greater-than-50% chance or a less-than-50% chance that you choose two marbles with different colors? Explain.

Lesson Performance Task

To prepare for an accuracy landing competition, a team of skydivers has laid out targets in a large open field. During practice sessions, team members attempt to land inside a target.

Two rectangular targets are shown on each field. Assuming a skydiver lands at random in the field, find the probabilities that the skydiver lands inside the specified target(s).

1. Calculate the probabilities using the targets shown here.



a. $P(A)$

b. $P(B)$

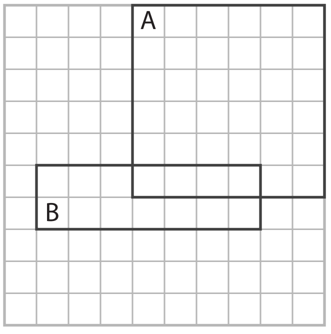
c. $P(A \cap B)$

d. $P(A \cup B)$

e. $P(A|B)$

f. $P(B|A)$

2. Calculate the probabilities using the targets shown here.



a. $P(A)$

b. $P(B)$

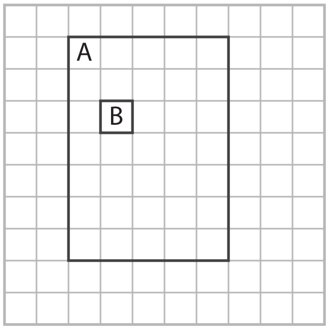
c. $P(A \cap B)$

d. $P(A \cup B)$

e. $P(A|B)$

f. $P(B|A)$

3. Calculate the probabilities using the targets shown here.



a. $P(A)$

b. $P(B)$

c. $P(A \cap B)$

d. $P(A \cup B)$

e. $P(A|B)$

f. $P(B|A)$