Name ______________________ Class ____________ Date ________

2.4 Investigating Symmetry

Essential Question: How do you determine whether a figure has line symmetry or rotational symmetry?

Resource Locker

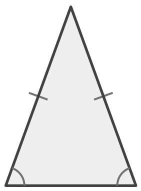
Explore 1 Identifying Line Symmetry

A figure has **symmetry** if a rigid motion exists that maps the figure onto itself. A figure has **line symmetry** (or *reflectional symmetry*) if a reflection maps the figure onto itself. Each of these lines of reflection is called a **line of symmetry**.

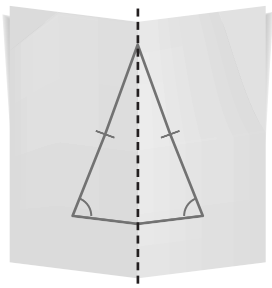
Line of symmetry

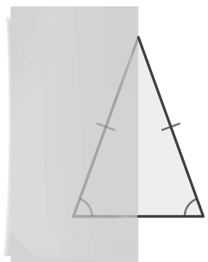
You can use paper folding to determine whether a figure has line symmetry.

(A) Trace the figure on a piece of tracing paper.



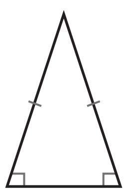
(B) If the figure can be folded along a straight line so that one half of the figure exactly matches the other half, the figure has line symmetry. The crease is the line of symmetry. Place your shape against the original figure to check that each crease is a line of symmetry.



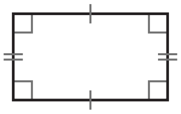
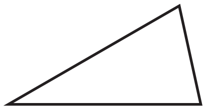
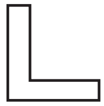


(C) Sketch any lines of symmetry on the figure.

The figure has ________ line of symmetry.



Ⓓ Draw the lines of symmetry, if any, on each figure and tell the total number of lines of symmetry each figure has.

Figure			
How many lines of symmetry?			

Reflect

1. What do you have to know about any segments and angles in a figure to decide whether the figure has line symmetry?

____

____

____

2. What figure has an infinite number of lines of symmetry? ____________________

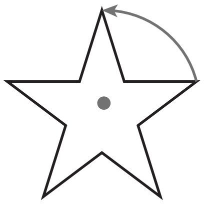
3. **Discussion** A figure undergoes a rigid motion, such as a rotation. If the figure has line symmetry, does the image of the figure have line symmetry as well? Give an example.

____

____

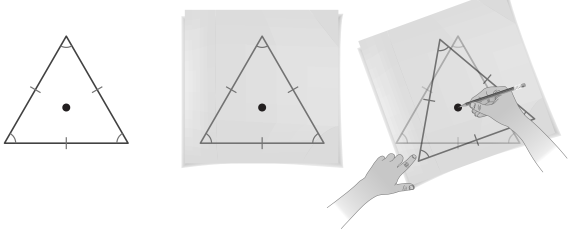
Explore 2 Identifying Rotational Symmetry

A figure has **rotational symmetry** if a rotation maps the figure onto itself. The **angle of rotational symmetry**, which is greater than 0° but less than or equal to 180°, is the smallest angle of rotation that maps a figure onto itself.



An angle of rotational symmetry is a fractional part of 360°. Notice that every time the 5-pointed star rotates $\frac{360°}{5} = 72°$, the star coincides with itself. The angles of rotation for the star are 72°, 144°, 216°, and 288°. If a copy of the figure rotates to exactly match the original, the figure has rotational symmetry.

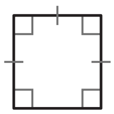
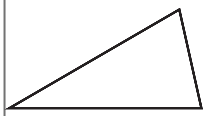
Ⓐ Trace the figure onto tracing paper. Hold the center of the traced figure against the original figure with your pencil. Rotate the traced figure counterclockwise until it coincides again with the original figure beneath.



By how many degrees did you rotate the figure? ____________

What are all the angles of rotation? ____________

(B) Determine whether each figure has rotational symmetry. If so, identify all the angles of rotation less than 360°.

Figure			
Angles of rotation less than 360°			

Reflect

4. What figure is mapped onto itself by a rotation of any angle? ______________________

5. **Discussion** A figure is formed by line *l* and line *m*, which intersect at an angle of 60°. Does the figure have an angle of rotational symmetry of 60°? If not, what is the angle of rotational symmetry?

____

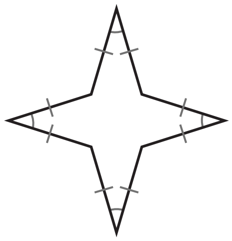
____

Explain 1 Describing Symmetries

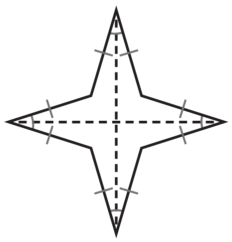
A figure may have line symmetry, rotational symmetry, both types of symmetry, or no symmetry.

Example 1 **Describe the symmetry of each figure. Draw the lines of symmetry, name the angles of rotation, or both if the figure has both.**

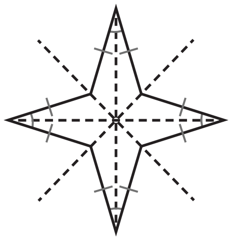
(A)



Step 1 Begin by finding the line symmetry of the figure. Look for matching halves of the figure. For example, you could fold the left half over the right half, and fold the top half over the bottom half. Draw one line of symmetry for each fold. Notice that the lines intersect at the center of the figure.

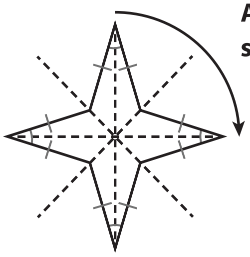


Step 2 Now look for other lines of symmetry. The two diagonals also describe matching halves. The figure has a total of 4 lines of symmetry.



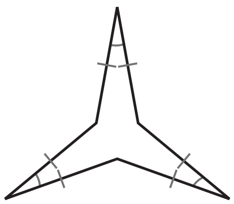
Step 3 Next, look for rotational symmetry. Think of the figure rotated about its center until it matches its original position. The angle of rotational symmetry of this figure is $\frac{1}{4}$ of 360°, or 90°.

The other angles of rotation for the figure are the multiples of 90° that are less than 360°. So the angles of rotation are 90°, 180°, and 270°.



Number of lines of symmetry: 4 Angles of rotation: 90°, 180°, 270° ___________

(B)



Step 1 Look for lines of symmetry. One line divides the figure into left and right halves. Draw this line on the figure. Then draw similar lines that begin at the other vertices of the figure.

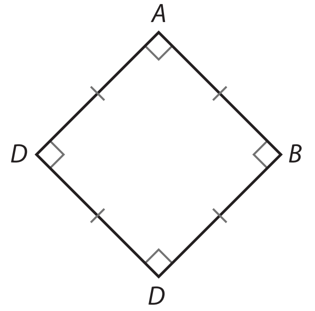
Step 2 Now look for rotational symmetry. Think of the figure rotating about its center until it matches the original figure. It rotates around the circle by a fraction of ___________. Multiply by 360° to find the angle of rotation, which is ___________. Find multiples of this angle to find other angles of rotation.

Number of lines of symmetry: ___________ Angles of rotation: ___________

Your Turn

Describe the type of symmetry for each figure. Draw the lines of symmetry, name the angles of rotation, or both if the figure has both.

6. Figure *ABCD*



Types of symmetry: ______________________

Number of lines of symmetry: ______________

Angles of rotation: _______________________

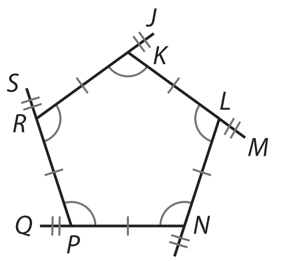
7. Figure *EFGHI*

Types of symmetry: ______________________

Number of lines of symmetry: ______________

Angles of rotation: _______________________

8. Figure *KLNPR*

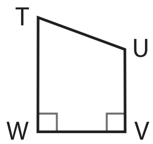


Types of symmetry: ______________________

Number of lines of symmetry: ______________

Angles of rotation: _______________________

9. Figure *TUVW*



Types of symmetry: ______________________

Number of lines of symmetry: ______________

Angles of rotation: _______________________

Elaborate

10. How are the two types of symmetry alike? How are they different?

____

____

____

11. **Essential Question Check-In** How do you determine whether a figure has line symmetry or rotational symmetry?

____

____

____

____

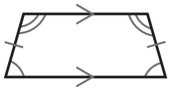
Evaluate: Homework and Practice

Personal Math Trainer

- Online Homework
- Hints and Help
- Extra Practice

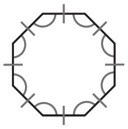
Draw all the lines of symmetry for the figure, and give the number of lines of symmetry. If the figure has no line symmetry, write zero.

1.



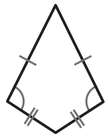
Lines of symmetry: ___________

2.



Lines of symmetry: ___________

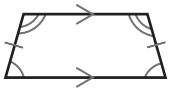
3.



Lines of symmetry: _________

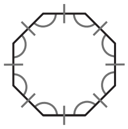
For the figures that have rotational symmetry, list the angles of rotation less than 360°. For figures without rotational symmetry, write "no rotational symmetry."

4.



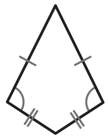
Angles of rotation: ______________________

5.



Angles of rotation: ______________________

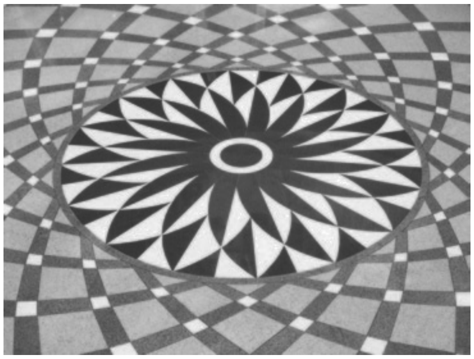
6.



Angles of rotation: ______________________

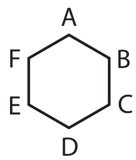
In the tile design shown, identify whether the pattern has line symmetry, rotational symmetry, both line and rotational symmetry, or no symmetry.

7.



8.

For figure *ABCDEF* shown here, identify the image after each transformation described. For example, a reflection across $\overline{AD}$ has an image of figure *AFEDCB*. In the figure, all the sides are the same length and all the angles are the same measure.



9. Reflection across $\overline{CF}$

Figure _________

10. rotation of 240° clockwise, or 120° counterclockwise

Figure _________

11. reflection across the line that connects the midpoint of $\overline{BC}$ and the midpoint of $\overline{EF}$

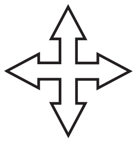
Figure _________

In the space provided, sketch an example of a figure with the given characteristics.

12. no line symmetry; angle of rotational symmetry: 180°

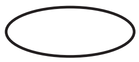
13. one line of symmetry; no rotational symmetry

14. Describe the line and rotational symmetry in this figure.



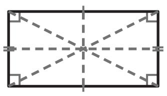
H.O.T. Focus on Higher Order Thinking

15. **Communicate Mathematical Ideas** How is a rectangle similar to an ellipse? Use concepts of symmetry in your answer.

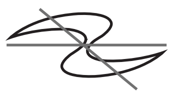


16. **Explain the Error** A student was asked to draw all of the lines of symmetry on each figure shown. Identify the student's work as correct or incorrect. If incorrect, explain why.

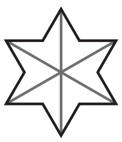
a.



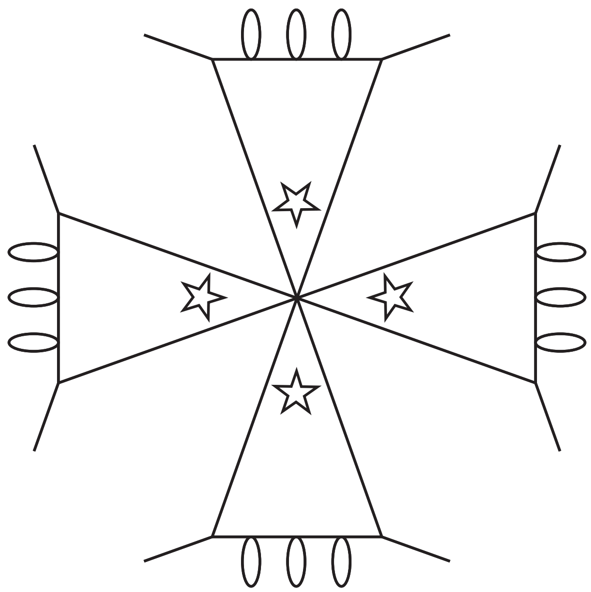
b.



c.



Lesson Performance Task



Use symmetry to design a work of art. Begin by drawing one simple geometric figure, such as a triangle, square, or rectangle, on a piece of construction paper. Then add other lines or two-dimensional shapes to the figure. Next, make identical copies of the figure, and then arrange them in a symmetric pattern.

Evaluate the symmetry of the work of art you created. Rotate it to identify an angle of rotational symmetry. Compare the line symmetry of the original figure with the line symmetry of the finished work.