Name _____ Class _____ Date _____

2.3 Rotations

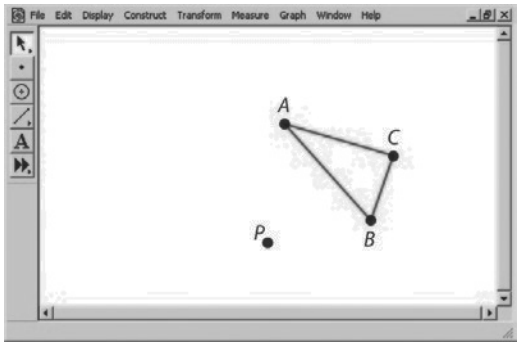
Essential Question: **How do you draw the image of a figure under a rotation?**

Resource Locker

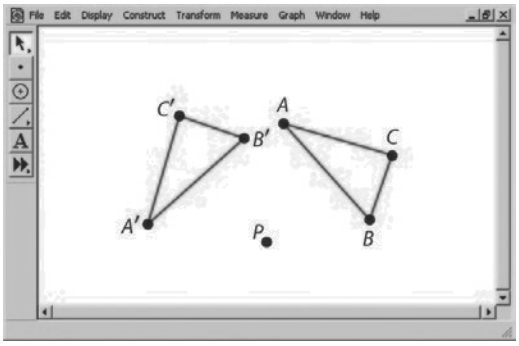
Explore Exploring Rotations

You can use geometry software or an online tool to explore rotations.

A Draw a triangle and label the vertices A, B, and C. Then draw a point P. Mark P as a center. This will allow you to rotate figures around point P.



B Select $\triangle ABC$ and rotate it 90° around point P. Label the image of $\triangle ABC$ as $\triangle A'B'C'$. Change the shape, size, or location of $\triangle ABC$ and notice how $\triangle A'B'C'$ changes.



C Draw $\angle APA'$, $\angle BPB'$, and $\angle CPC'$. Measure these angles. What do you notice? Does this relationship remain true as you move point P? What happens if you change the size and shape of $\triangle ABC$?

_____

_____

D Measure the distance from A to P and the distance from A' to P. What do you notice? Does this relationship remain true as you move point P? What happens if you change the size and shape of $\triangle ABC$?

_____

_____

Reflect

1. What can you conclude about the distance of a point and its image from the center of rotation?

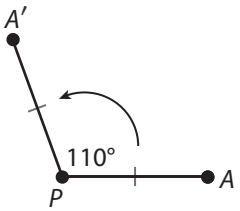
2. What are the advantages of using geometry software or an online tool rather than tracing paper or a protractor and ruler to investigate rotations?

Explain 1 Rotating Figures Using a Ruler and Protractor

A **rotation** is a transformation around point *P*, the **center of rotation**, such that the following is true.

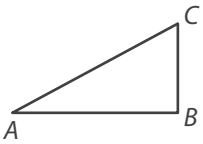
- Every point and its image are the same distance from *P*.
- All angles with vertex *P* formed by a point and its image have the same measure. This angle measure is the **angle of rotation**.

In the figure, the center of rotation is point *P* and the angle of rotation is 110°.

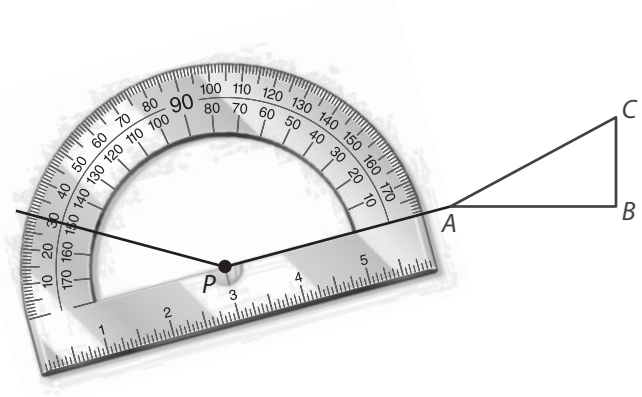


Example 1 **Draw the image of the triangle after the given rotation.**

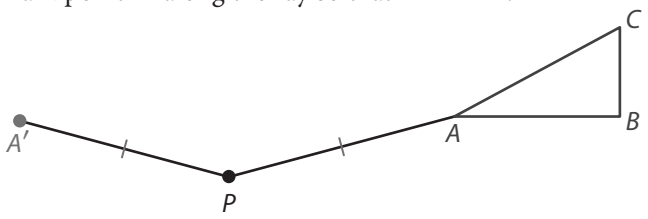
Ⓐ Counterclockwise rotation of 150° around point *P*



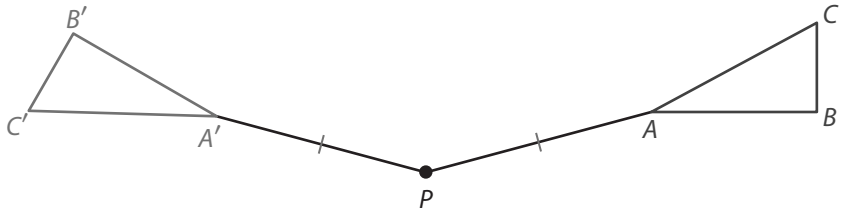
Step 1 Draw $\overline{PA}$. Then use a protractor to draw a ray that forms a 150° angle with $\overline{PA}$.



Step 2 Use a ruler to mark point A' along the ray so that $PA' = PA$.



Step 3 Repeat Steps 1 and 2 for points B and C to locate points B' and C'. Connect points A', B', and C' to draw $\triangle A'B'C'$.

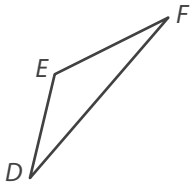


Ⓑ Clockwise rotation of 75° around point Q

Step 1 Draw $\overline{QD}$. Use a protractor to draw a ray forming a clockwise 75° angle with $\overline{QD}$.

Step 2 Use a ruler to mark point D' along the ray so that $QD' = QD$.

Step 3 Repeat Steps1 and 2 for points E and F to locate points E' and F'. Connect points D', E', and F' to draw $\triangle D'E'F'$.



Reflect

3. How could you use tracing paper to draw the image of $\triangle ABC$ in Part A?

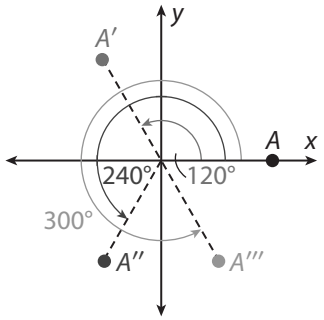
Your Turn

Draw the image of the triangle after the given rotation.

4. Counterclockwise rotation of 40° around point P **5.** Clockwise rotation of 125° around point Q

Explain 2 Drawing Rotations on a Coordinate Plane

You can rotate a figure by more than 180°. The diagram shows counterclockwise rotations of 120°, 240°, and 300°. Note that a rotation of 360° brings a figure back to its starting location.



When no direction is specified, you can assume that a rotation is counterclockwise. Also, a counterclockwise rotation of $x°$ is the same as a clockwise rotation of $(360 - x)°$.

The table summarizes rules for rotations on a coordinate plane.

Rules for Rotations Around the Origin on a Coordinate Plane	
90° rotation counterclockwise	$(x, y) \rightarrow (-y, x)$
180° rotation	$(x, y) \rightarrow (-x, -y)$
270° rotation counterclockwise	$(x, y) \rightarrow (y, -x)$
360° rotation	$(x, y) \rightarrow (x, y)$

Example 2 **Draw the image of the figure under the given rotation.**

(A) Quadrilateral $ABCD$; 270°

The rotation image of (x, y) is $(y, -x)$.

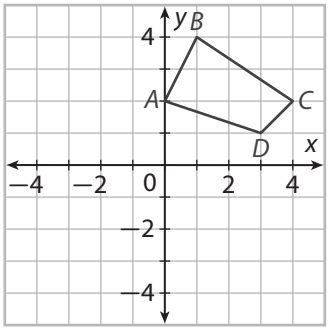
Find the coordinates of the vertices of the image.

$A(0, 2) \rightarrow A'(2, 0)$

$B(1, 4) \rightarrow B'(4, -1)$

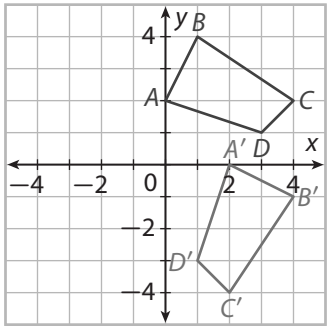
$C(4, 2) \rightarrow C'(2, -4)$

$D(3, 1) \rightarrow D'(1, -3)$



Predict the quadrant in which the image will lie. Since quadrilateral $ABCD$ lies in Quadrant I and the quadrilateral is rotated counterclockwise by 270°, the image will lie in Quadrant IV.

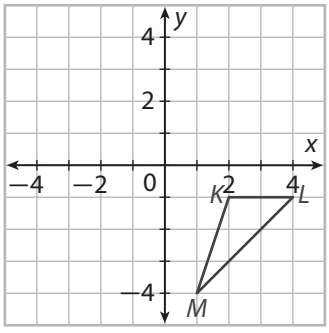
Plot A', B', C', and D' to graph the image.



B $\triangle KLM$; 180°

The rotation image of (x, y) is $\left(\square, \square\right)$.

Find the coordinates of the vertices of the image.



$K(2, -1) \rightarrow K'\left(\square, \square\right)$

$L(4, -1) \rightarrow L'\left(\square, \square\right)$

$M(1, -4) \rightarrow M'\left(\square, \square\right)$

Predict the quadrant in which the image will lie. Since $\triangle KLM$ lies in Quadrant ____ and the triangle is rotated by 180°, the image will lie in Quadrant ____.

Plot K', L', and M' to graph the image.

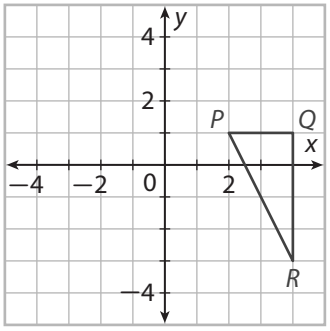
Reflect

6. **Discussion** Suppose you rotate quadrilateral $ABCD$ in Part A by 810°. In which quadrant will the image lie? Explain.

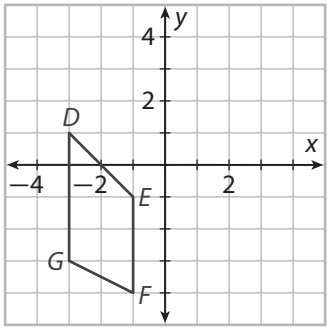
Your Turn

Draw the image of the figure under the given rotation.

7. $\triangle PQR$; 90°



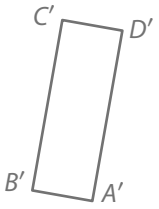
8. Quadrilateral *DEFG*; 270°

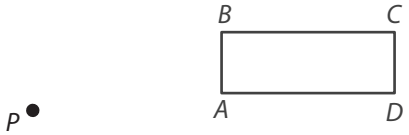


Explain 3 Specifying Rotation Angles

Example 3 **Find the angle of rotation and direction of rotation in the given figure. Point *P* is the center of rotation.**

A



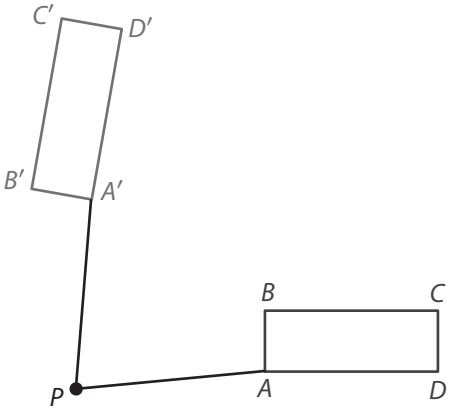


Draw segments from the center of rotation to a vertex and to the image of the vertex.

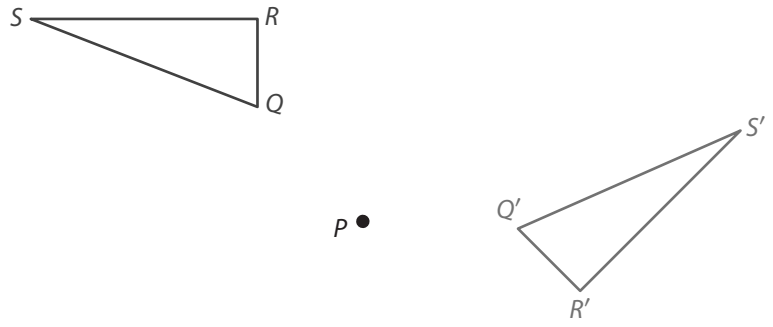
Measure the angle formed by the segments. The angle measure is 80°.

Compare the locations of the preimage and image to find the direction of the rotation.

The rotation is 80° counterclockwise.



B



Draw segments from the center of rotation to a vertex and to the image of the vertex.

Measure the angle formed by the segments.

The angle measure is ____°.

The rotation is ____° (clockwise/counterclockwise).

Reflect

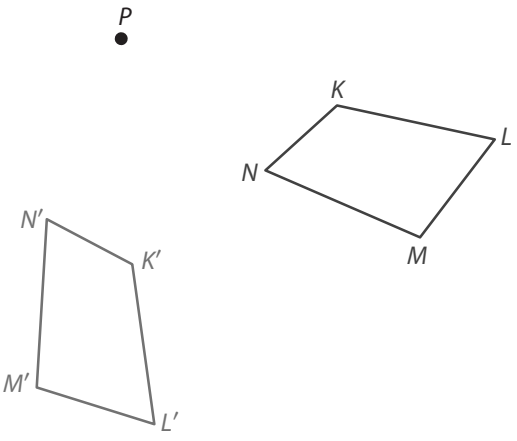
9. **Discussion** Does it matter which points you choose when you draw segments from the center of rotation to points of the preimage and image? Explain.

10. In Part A, is a different angle of rotation and direction possible? Explain.

Your Turn

Find the angle of rotation and direction of rotation in the given figure. Point *P* is the center of rotation.

11.



Elaborate

12. If you are given a figure, a center of rotation, and an angle of rotation, what steps can you use to draw the image of the figure under the rotation?

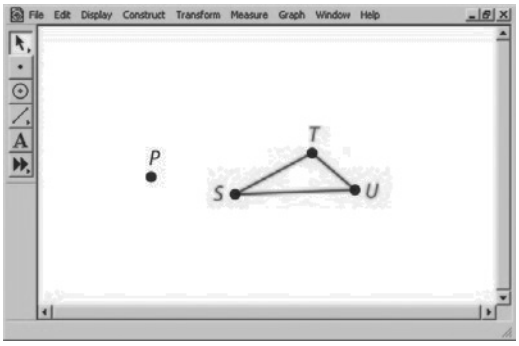
13. Suppose you are given $\triangle DEF$, $\triangle D'E'F'$, and point P. What are two different ways to prove that a rotation around point P cannot be used to map $\triangle DEF$ to $\triangle D'E'F'$?

14. **Essential Question Check-In** How do you draw the image of a figure under a counterclockwise rotation of 90° around the origin?

Evaluate: Homework and Practice

- Online Homework
- Hints and Help
- Extra Practice

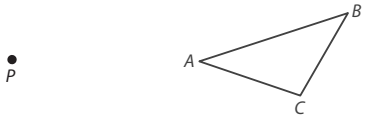
1. Alberto uses geometry software to draw $\triangle STU$ and point P, as shown. He marks P as a center and uses the software to rotate $\triangle STU$ 115° around point P. He labels the image of $\triangle STU$ as $\triangle S'T'U'$.



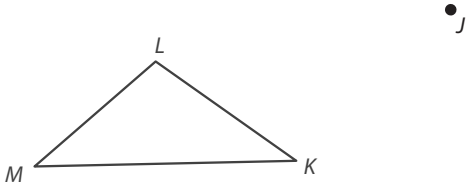
Which three angles must have the same measure? What is the measure of these angles?

Draw the image of the triangle after the given rotation.

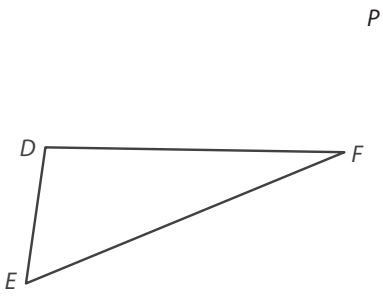
2. Counterclockwise rotation of 30° around point *P*



3. Clockwise rotation of 55° around point *J*

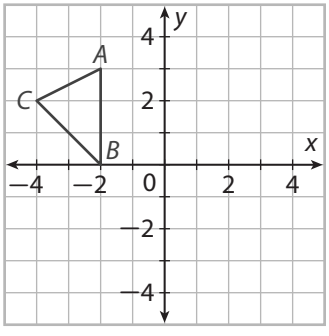


4. Counterclockwise rotation of 90° around point *P*

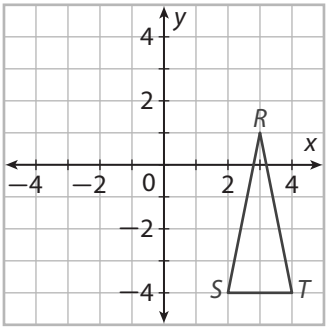


Draw the image of the figure under the given rotation.

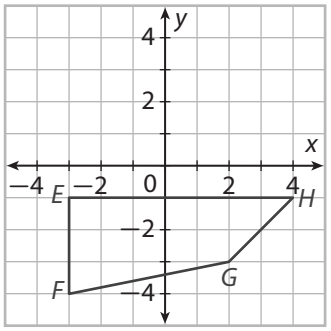
5. △*ABC*; 270°



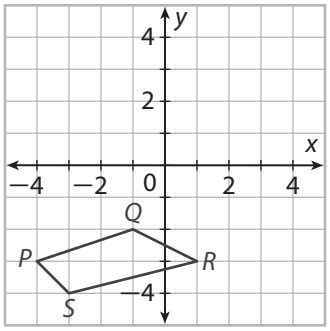
6. △*RST*; 90°



7. Quadrilateral *EFGH*; 180°

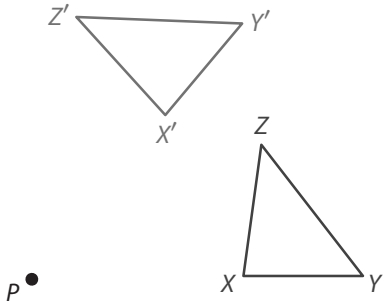


8. Quadrilateral *PQRS*; 270°

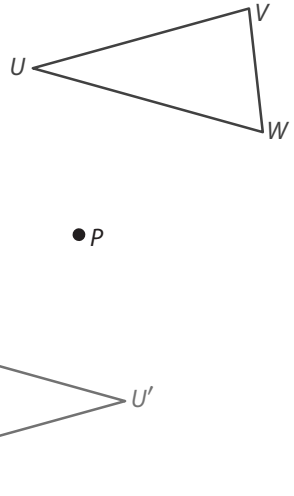


Find the angle of rotation and direction of rotation in the given figure. Point *P* is the center of rotation.

9.

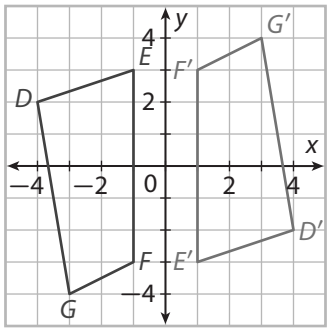


10.

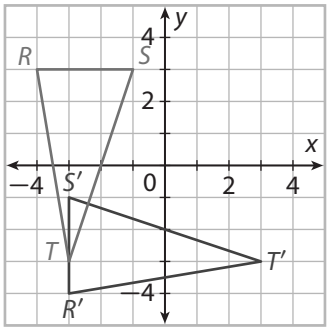


Write an algebraic rule for the rotation shown. Then describe the transformation in words.

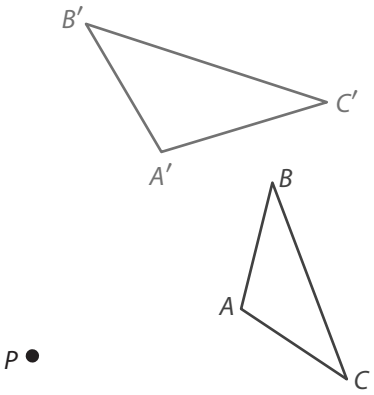
11.



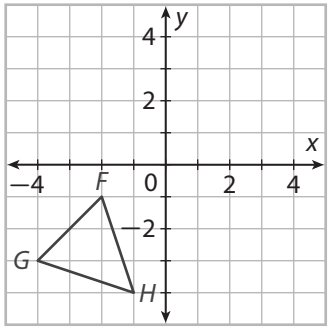
12.



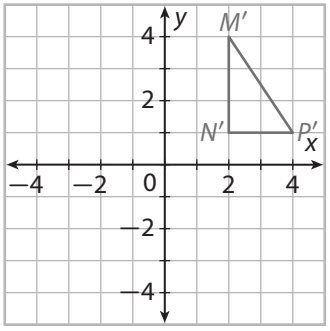
13. Vanessa used geometry software to apply a transformation to $\triangle ABC$, as shown. According to the software, $m\angle APA' = m\angle BPB' = m\angle CPC'$. Vanessa said this means the transformation must be a rotation. Do you agree? Explain.



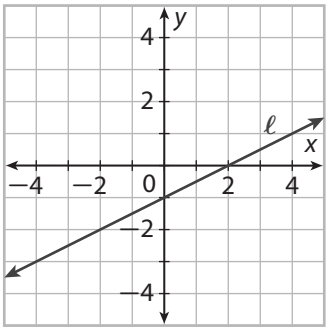
14. **Make a Prediction** In which quadrant will the image of $\triangle FGH$ lie after a counterclockwise rotation of 1980°? Explain how you made your prediction.



15. **Critical Thinking** The figure shows the image of $\triangle MNP$ after a counterclockwise rotation of 270°. Draw and label $\triangle MNP$.



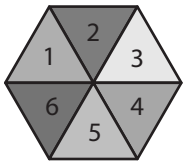
16. **Multi-Step** Write the equation of the image of line ℓ after a clockwise rotation of 90°. (*Hint*: To find the image of line ℓ, choose two or more points on the line and find the images of the points.)



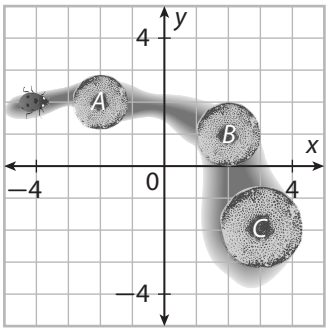
17. A Ferris wheel has 20 cars that are equally spaced around the circumference of the wheel. The wheel rotates so that the car at the bottom of the ride is replaced by the next car. By how many degrees does the wheel rotate?

18. The Skylon Tower, in Niagara Falls, Canada, has a revolving restaurant 775 feet above the falls. The restaurant makes a complete revolution once every hour. While a visitor was at the tower, the restaurant rotated through 135°. How long was the visitor at the tower?

19. Amani plans to use drawing software to make the design shown here. She starts by drawing Triangle 1. Explain how she can finish the design using rotations.



20. An animator is drawing a scene in which a ladybug moves around three mushrooms. The figure shows the starting position of the ladybug. The animator rotates the ladybug 180° around mushroom *A*, then 180° around mushroom *B*, and finally 180° around mushroom *C*. What are the final coordinates of the ladybug?



21. Determine whether each statement about the rotation $(x, y) \rightarrow (y, -x)$ is true or false. Select the correct answer for each lettered part.

a. Every point in Quadrant I is mapped to a point in Quadrant II. ◯ True ◯ False

b. Points on the x-axis are mapped to points on the y-axis. ◯ True ◯ False

c. The origin is a fixed point under the rotation. ◯ True ◯ False

d. The rotation has the same effect as a 90° clockwise rotation. ◯ True ◯ False

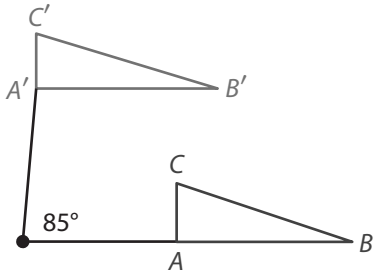
e. The angle of rotation is 180°. ◯ True ◯ False

f. A point on the line $y = x$ is mapped to another point on the line $y = x$. ◯ True ◯ False

H.O.T. Focus on Higher Order Thinking

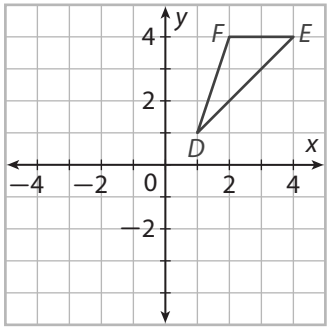
22. Communicate Mathematical Ideas Suppose you are given a figure and a center of rotation P. Describe two different ways you can use a ruler and protractor to draw the image of the figure after a 210° counterclockwise rotation around P.

23. Explain the Error Kevin drew the image of $\triangle ABC$ after a rotation of 85° around point P. Explain how you can tell from the figure that he made an error. Describe the error.



24. Critique Reasoning Isabella said that all points turn around the center of rotation by
the same angle, so all points move the same distance under a rotation. Do you agree with Isabella's statement? Explain.

25. Look for a Pattern Isaiah uses software to draw $\triangle DEF$ as shown. Each time he presses the left arrow key, the software rotates the figure on the screen 90° counterclockwise. Explain how Isaiah can determine which quadrant the triangle will lie in if he presses the left arrow key n times.



Lesson Performance Task

A tourist in London looks up at the clock in Big Ben tower and finds that it is exactly 8:00. When she looks up at the clock later, it is exactly 8:10.

a. Through what angle of rotation did the minute hand turn? Through what angle of rotation did the hour hand turn?

b. Make a table that shows different amounts of time, from 5 minutes to 60 minutes, in 5-minute increments. For each number of minutes, provide the angle of rotation for the minute hand of a clock and the angle of rotation for the hour hand of a clock.