

# 2.2 Reflections



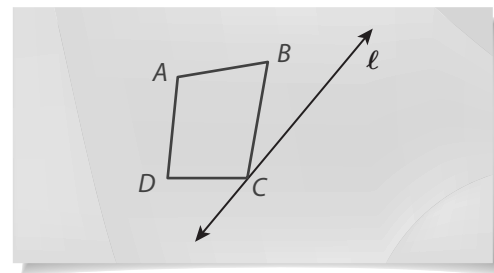
Resource Locker

**Essential Question:** How do you draw the image of a figure under a reflection?

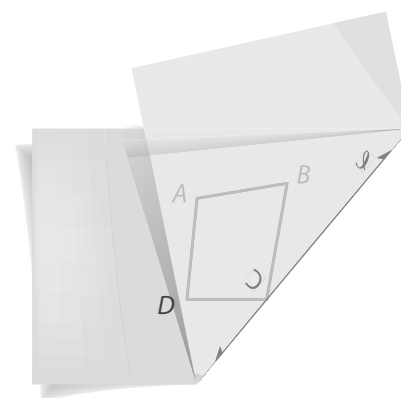
## Explore Exploring Reflections

Use tracing paper to explore reflections.

- A Draw and label a line  $\ell$  on tracing paper. Then draw and label a quadrilateral  $ABCD$  with vertex  $C$  on line  $\ell$ .



- B Fold the tracing paper along line  $\ell$ . Trace the quadrilateral. Then unfold the paper and draw the image of the quadrilateral. Label it  $A' B' C' D'$ .



- C Draw segments to connect each vertex of quadrilateral  $ABCD$  with its image. Use a protractor to measure the angle formed by each segment and line  $\ell$ . What do you notice?

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- D Use a ruler to measure each segment and the two shorter segments formed by its intersection with line  $\ell$ . What do you notice?

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### Reflect

1. In this activity, the fold line (line  $\ell$ ) is the line of reflection. What happens when a point is located on the line of reflection?

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2. **Discussion** A student claims that a figure and its reflected image always lie on opposite sides of the line of reflection. Do you agree? Why or why not?

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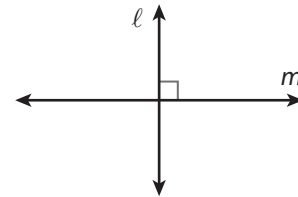
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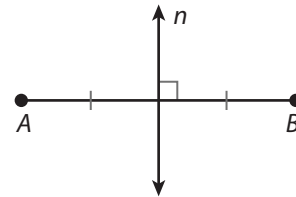
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## Explain 1 Reflecting Figures Using Graph Paper

**Perpendicular lines** are lines that intersect at right angles. In the figure, line  $\ell$  is perpendicular to line  $m$ . The right angle mark in the figure indicates that the lines are perpendicular.

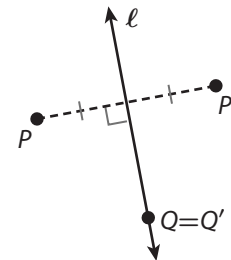


The **perpendicular bisector** of a line segment is a line perpendicular to the segment at the segment's midpoint. In the figure, line  $n$  is the perpendicular bisector of  $\overline{AB}$ .



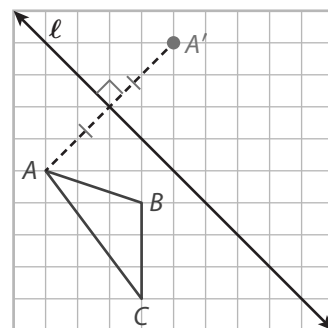
A **reflection** across line  $\ell$  maps a point  $P$  to its image  $P'$ .

- If  $P$  is not on line  $\ell$ , then line  $\ell$  is the perpendicular bisector of  $\overline{PP'}$ .
- If  $P$  is on line  $\ell$ , then  $P = P'$ .



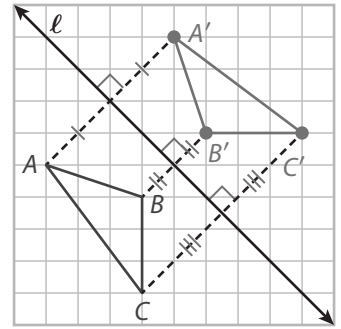
**Example 1** Draw the image of  $\triangle ABC$  after a reflection across line  $\ell$ .

- (A) Step 1** Draw a segment with an endpoint at vertex  $A$  so that the segment is perpendicular to line  $\ell$  and is bisected by line  $\ell$ . Label the other endpoint of the segment  $A'$ .



**Step 2** Repeat Step 1 at vertices  $B$  and  $C$ .

**Step 3** Connect points  $A'$ ,  $B'$ , and  $C'$ .  
 $\triangle A'B'C'$  is the image of  $\triangle ABC$ .

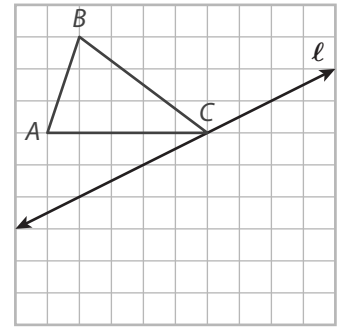


**B** Draw the image of  $\triangle ABC$  after a reflection across line  $\ell$ .

**Step 1** Draw a segment with an endpoint at vertex  $A$  so that the segment is perpendicular to line  $\ell$  and is bisected by line  $\ell$ . Label the other endpoint of the segment  $A'$ .

**Step 2** Repeat Step 1 at vertex  $B$ .  
 Notice that  $C$  and  $C'$  are the same point because  $C$  is on the line of reflection.

**Step 3** Connect points  $A'$ ,  $B'$ , and  $C'$ .  $\triangle A'B'C'$  is the image of  $\triangle ABC$ .



**Reflect**

**3.** How can you check that you drew the image of the triangle correctly?

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**4.** In Part A, how can you tell that  $\overline{AA'}$  is perpendicular to line  $\ell$ ?

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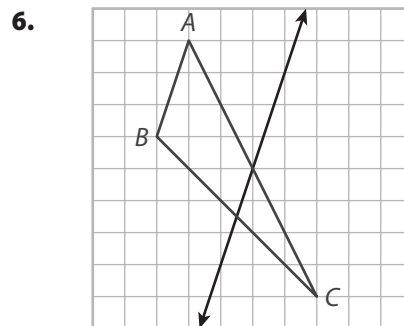
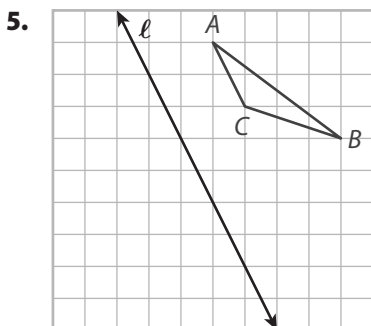
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**Your Turn**

Draw the image of  $\triangle ABC$  after a reflection across line  $\ell$ .



## Explain 2 Drawing Reflections on a Coordinate Plane

The table summarizes coordinate notation for reflections on a coordinate plane.

Rules for Reflections on a Coordinate Plane	
Reflection across the $x$ -axis	$(x, y) \rightarrow (x, -y)$
Reflection across the $y$ -axis	$(x, y) \rightarrow (-x, y)$
Reflection across the line $y = x$	$(x, y) \rightarrow (y, x)$
Reflection across the line $y = -x$	$(x, y) \rightarrow (-y, -x)$

**Example 2** Reflect the figure with the given vertices across the given line.

- A**  $M(1, 2), N(1, 4), P(3, 3); y$ -axis

**Step 1** Find the coordinates of the vertices of the image.

$$A(x, y) \rightarrow A'(-x, y).$$

$$M(1, 2) \rightarrow M'(-1, 2)$$

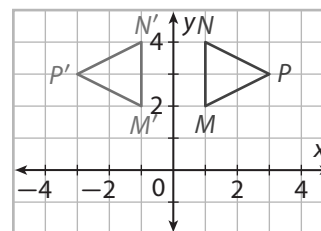
$$N(1, 4) \rightarrow N'(-1, 4)$$

$$P(3, 3) \rightarrow P'(-3, 3)$$

**Step 2** Graph the preimage.

**Step 3** Predict the quadrant in which the image will lie. Since  $\triangle MNP$  lies in Quadrant I and the triangle is reflected across the  $y$ -axis, the image will lie in Quadrant II.

Graph the image.



- B**  $D(2, 0), E(2, 2), F(5, 2), G(5, 1); y = x$

**Step 1** Find the coordinates of the vertices of the image.

$$A(x, y) \rightarrow A'(\quad, \quad)$$

$$D(2, 0) \rightarrow D'(\quad, \quad)$$

$$E(2, 2) \rightarrow E'(\quad, \quad)$$

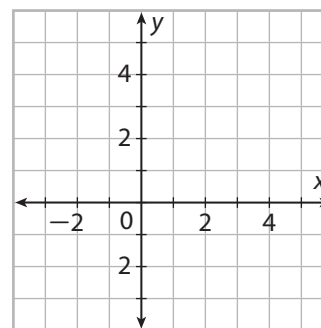
$$F(5, 2) \rightarrow F'(\quad, \quad)$$

$$G(5, 1) \rightarrow G'(\quad, \quad)$$

**Step 2** Graph the preimage.

**Step 3** Since  $DEFG$  lies in Quadrant I and the quadrilateral is reflected across the line  $y = x$ , the image will lie in Quadrant \_\_\_\_\_.

Graph the image.



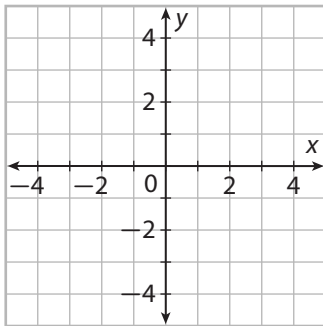
## Reflect

7. How would the image of  $\triangle MNP$  be similar to and different from the one you drew in Part A if the triangle were reflected across the  $x$ -axis?
8. A classmate claims that the rule  $(x, y) \rightarrow (-x, y)$  for reflecting a figure across the  $y$ -axis only works if all the vertices are in the first quadrant because the values of  $x$  and  $y$  must be positive. Explain why this reasoning is not correct.

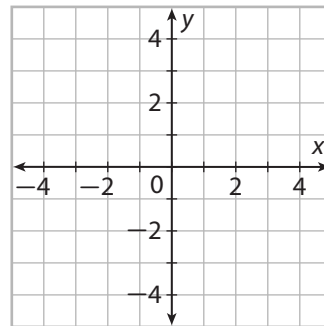
## Your Turn

Reflect the figure with the given vertices across the given line.

9.  $S(3, 4)$ ,  $T(3, 1)$ ,  $U(-2, 1)$ ,  $V(-2, 4)$ ;  $x$ -axis



10.  $A(-4, -2)$ ,  $B(-1, -1)$ ,  $C(-1, -4)$ ;  $y = -x$



## Explain 3 Specifying Lines of Reflection

**Example 3** Given that  $\triangle A'B'C'$  is the image of  $\triangle ABC$  under a reflection, draw the line of reflection.

- (A) Draw the segments  $\overline{AA'}$ ,  $\overline{BB'}$ , and  $\overline{CC'}$ .

Find the midpoint of each segment.

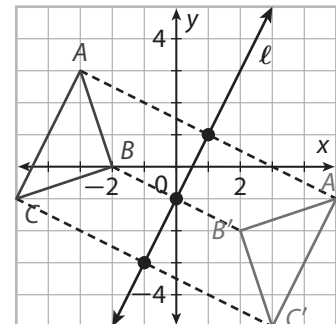
$$\text{The midpoint of } \overline{AA'} \text{ is } \left( \frac{-3 + 5}{2}, \frac{3 + (-1)}{2} \right) = (1, 1).$$

$$\text{The midpoint of } \overline{BB'} \text{ is } \left( \frac{-2 + 2}{2}, \frac{0 + (-2)}{2} \right) = (0, -1).$$

$$\text{The midpoint of } \overline{CC'} \text{ is } \left( \frac{-5 + 3}{2}, \frac{-1 + (-5)}{2} \right) = (-1, -3).$$

Plot the midpoints. Draw line  $\ell$  through the midpoints.

Line  $\ell$  is the line of reflection.

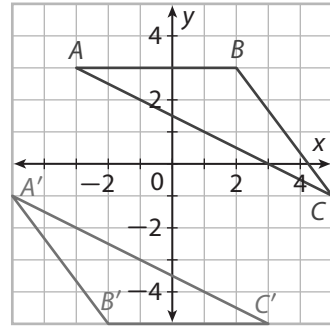


**B** Draw  $\overline{AA'}$ ,  $\overline{BB'}$ , and  $\overline{CC'}$ . Find the midpoint of each segment.

The midpoint of  $\overline{AA'}$  is  $\left(\frac{\square + \square}{2}, \frac{\square + \square}{2}\right) = (\square, \square)$ .

The midpoint of  $\overline{BB'}$  is  $\left(\frac{\square + \square}{2}, \frac{\square + \square}{2}\right) = (\square, \square)$ .

The midpoint of  $\overline{CC'}$  is  $\left(\frac{\square + \square}{2}, \frac{\square + \square}{2}\right) = (\square, \square)$ .



Plot the midpoints. Draw line  $\ell$  through the midpoints. Line  $\ell$  is the line of reflection.

**Reflect**

**11.** How can you use a ruler and protractor to check that line  $\ell$  is the line of reflection?

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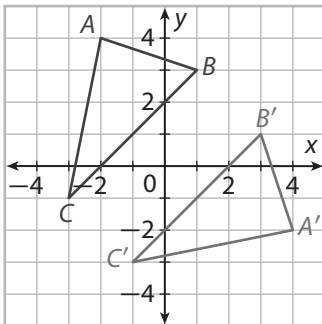


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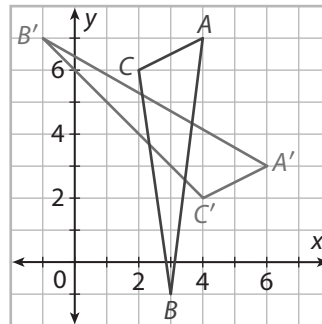
**Your Turn**

Given that  $\triangle A'B'C'$  is the image of  $\triangle ABC$  under a reflection, draw the line of reflection.

**12.**



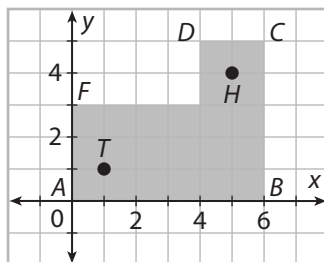
**13.**



**Explain 4 Applying Reflections**

**Example 4**

The figure shows one hole of a miniature golf course. It is not possible to hit the ball in a straight line from the tee  $T$  to the hole  $H$ . At what point should a player aim in order to make a hole in one?





### Understand the Problem

The problem asks you to locate point  $X$  on the wall of the miniature golf hole so that the ball can travel in a straight line from  $T$  to  $X$  and from  $X$  to  $H$ .

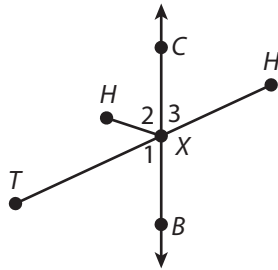


### Make a Plan

In order for the ball to travel directly from  $T$  to  $X$  to  $H$ , the angle of the ball's path as it hits the wall must equal the angle of the ball's path as it leaves the wall. In the figure,  $m\angle 1$  must equal  $m\angle 2$ .

Let  $H'$  be the reflection of point  $H$  across  $\overline{BC}$ .

Reflections preserve angle measure, so  $m\angle 2 = m\angle \square$ . Therefore,  $m\angle 1$  is equal to  $m\angle 2$  when  $m\angle 1$  is equal to  $m\angle 3$ . This occurs when  $T$ ,  $\square$ , and  $H'$  are collinear.



### Solve

Reflect  $H$  across  $\overline{BC}$  to locate  $H'$ .

The coordinates of  $H'$  are  $(\square, \square)$ .

Draw  $\overline{TH'}$  and locate point  $X$  where  $\overline{TH'}$  intersects  $\overline{BC}$ .

The coordinates of point  $X$  are  $(\square, \square)$ .

The player should aim at this point.



### Look Back

To check that the answer is reasonable, plot point  $X$  using the coordinates you found. Then use a protractor to check that the angle of the ball's path as it hits the wall at point  $X$  is equal to the angle of the ball's path as it leaves the wall from point  $X$ .

### Reflect

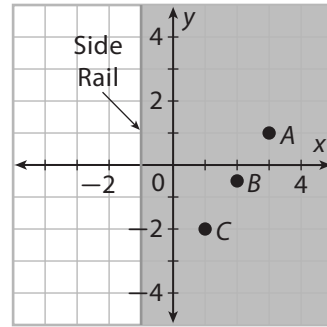
14. Is there another path the ball can take to hit a wall and then travel directly to the hole? Explain.

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**Your Turn**

15. Cara is playing pool. She wants to use the cue ball  $C$  to hit the ball at point  $A$  without hitting the ball at point  $B$ . To do so, she has to bounce the cue ball off the side rail and into the ball at point  $A$ . Find the coordinates of the exact point along the side rail that Cara should aim for.



**Elaborate**

16. Do any points in the plane have themselves as images under a reflection? Explain.

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17. If you are given a figure and its image under a reflection, how can you use paper folding to find the line of reflection?

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18. **Essential Question Check-In** How do you draw the image of a figure under a reflection across the  $x$ -axis?

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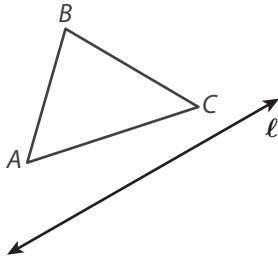
# Evaluate: Homework and Practice



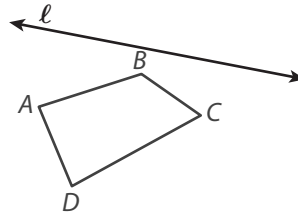
- Online Homework
- Hints and Help
- Extra Practice

Use tracing paper to copy each figure and line  $\ell$ . Then fold the paper to draw and label the image of the figure after a reflection across line  $\ell$ .

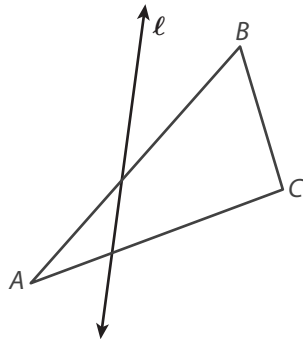
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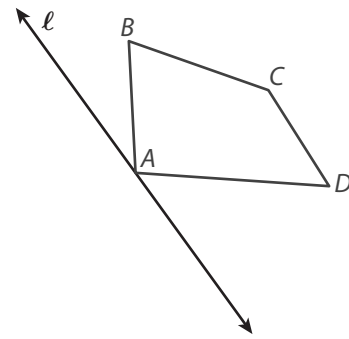
2.



3.

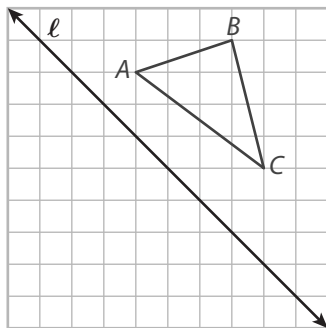


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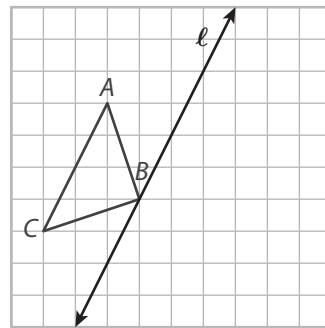


Draw the image of  $\triangle ABC$  after a reflection across line  $\ell$ .

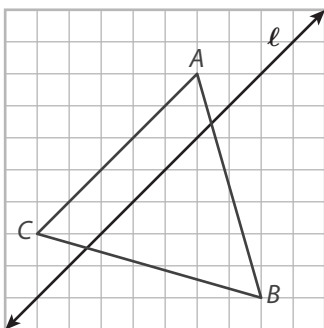
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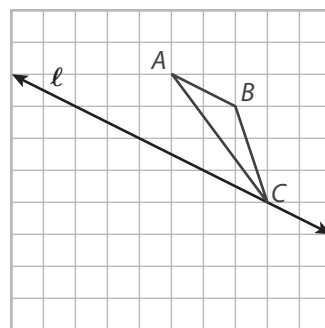
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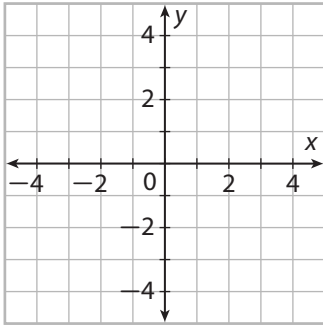


8.

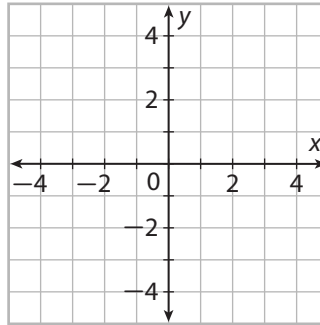


Reflect the figure with the given vertices across the given line.

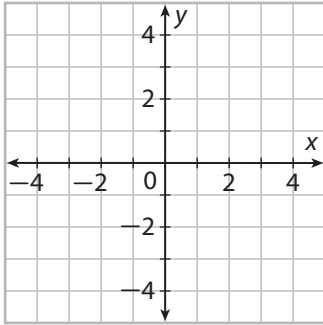
9.  $P(-2, 3), Q(4, 3), R(-1, 0), S(-4, 1)$ ;  $x$ -axis



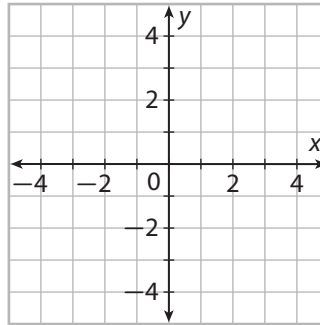
10.  $A(-3, -3), B(1, 3), C(3, -1)$ ;  $y$ -axis



11.  $J(-1, 2), K(2, 4), L(4, -1)$ ;  $y = -x$

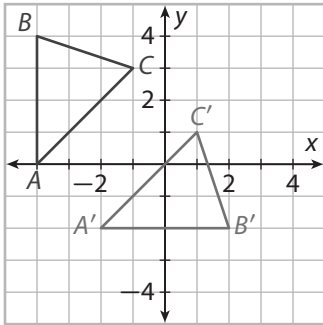


12.  $D(-1, 1), E(3, 2), F(4, -1), G(-1, -3)$ ;  $y = x$

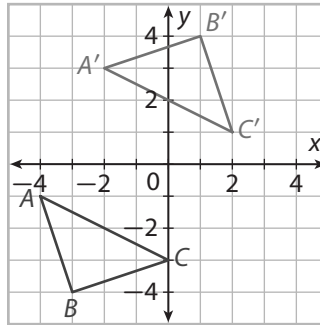


Given that  $\triangle A'B'C'$  is the image of  $\triangle ABC$  under a reflection, draw the line of reflection.

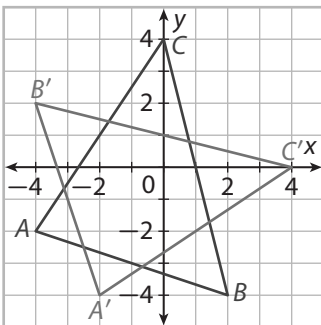
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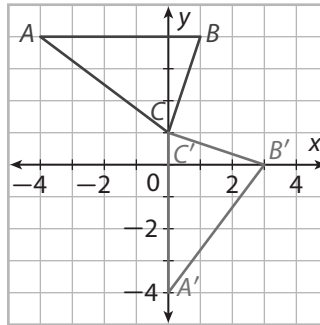
14.



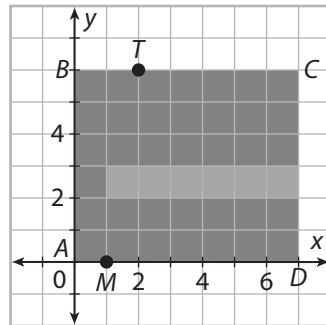
15.



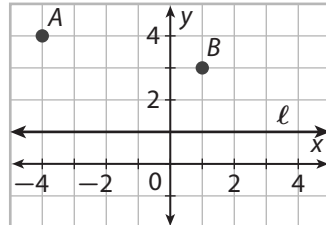
16.



17. Jamar is playing a video game. The object of the game is to roll a marble into a target. In the figure, the shaded rectangular area represents the video screen and the striped rectangle is a barrier. Because of the barrier, it is not possible to roll the marble  $M$  directly into the target  $T$ . At what point should Jamar aim the marble so that it will bounce off a wall and roll into the target?



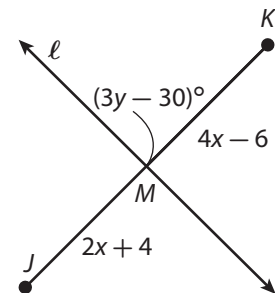
18. A trail designer is planning two trails that connect campsites  $A$  and  $B$  to a point on the river, line  $\ell$ . She wants the total length of the trails to be as short as possible. At what point should the trails meet the river?



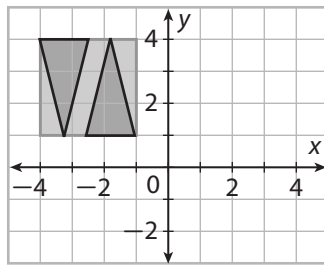
**Algebra** In the figure, point  $K$  is the image of point  $J$  under a reflection across line  $\ell$ . Find each of the following.

19.  $JM$

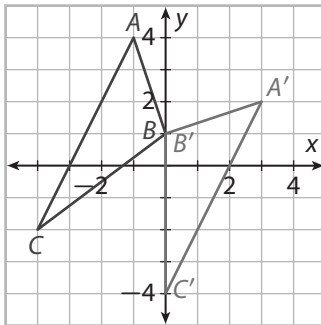
20.  $y$



- 21. Make a Prediction** Each time Jenny presses the tab key on her keyboard, the software reflects the logo she is designing across the  $x$ -axis. Jenny's cat steps on the keyboard and presses the tab key 25 times. In which quadrant does the logo end up? Explain.

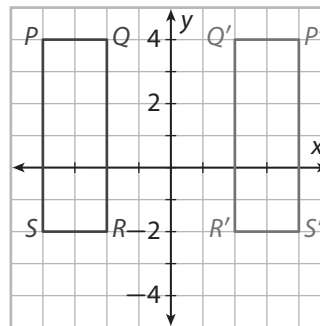


- 22. Multi-Step** Write the equation of the line of reflection.



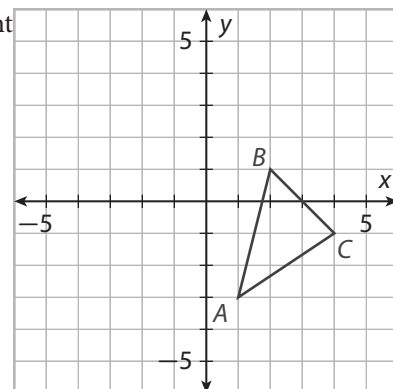
- 23. Communicate Mathematical Ideas**

The figure shows rectangle  $PQRS$  and its image after a reflection across the  $y$ -axis. A student said that  $PQRS$  could also be mapped to its image using the translation  $(x, y) \rightarrow (x + 6, y)$ . Do you agree? Explain why or why not.



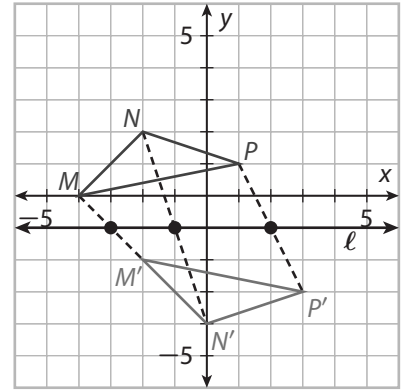
- 24.** Which of the following transformations map  $\triangle ABC$  to a triangle that intersects the  $x$ -axis? Select all that apply.

- A.  $(x, y) \rightarrow (-x, y)$       D.  $(x, y) \rightarrow (-y, -x)$   
 B.  $(x, y) \rightarrow (x, -y)$       E.  $(x, y) \rightarrow (x, y + 1)$   
 C.  $(x, y) \rightarrow (y, x)$

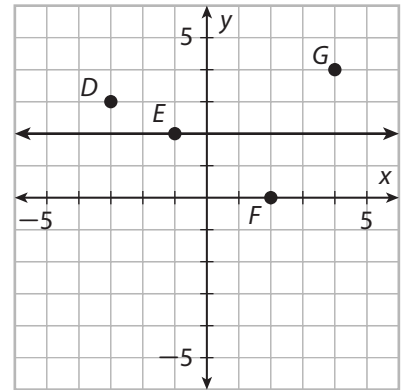


**H.O.T. Focus on Higher Order Thinking**

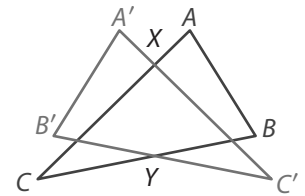
- 25. Explain the Error**  $\triangle M'N'P'$  is the image of  $\triangle MNP$ . Casey draws  $\overline{MM'}$ ,  $\overline{NN'}$ , and  $\overline{PP'}$ . Then she finds the midpoint of each segment and draws line  $\ell$  through the midpoints. She claims that line  $\ell$  is the line of reflection. Do you agree? Explain.



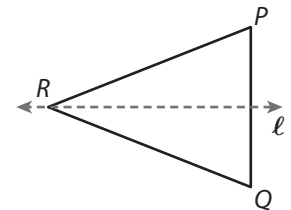
- 26. Draw Conclusions** Plot the images of points  $D$ ,  $E$ ,  $F$ , and  $G$  after a reflection across the line  $y = 2$ . Then write an algebraic rule for the reflection.



- 27. Critique Reasoning** Mayumi wants to draw the line of reflection for the reflection that maps  $\triangle ABC$  to  $\triangle A'B'C'$ . She claims that she just needs to draw the line through the points  $X$  and  $Y$ . Do you agree? Explain.



- 28. Justify Reasoning** Point  $Q$  is the image of point  $P$  under a reflection across line  $\ell$ . Point  $R$  lies on line  $\ell$ . What type of triangle is  $\triangle PQR$ ? Justify your answer.



# Lesson Performance Task

In order to see the entire length of your body in a mirror, do you need a mirror that is as tall as you are? If not, what is the length of the shortest mirror you can use, and how should you position it on a wall?

- a. Let the  $x$ -axis represent the floor and let the  $y$ -axis represent the wall on which the mirror hangs. Suppose the bottom of your feet are at  $F(3, 0)$ , your eyes are at  $E(3, 7)$ , and the top of your head is at  $H(3, 8)$ . Plot these points and the points that represent their reflection images. (*Hint:* When you look in a mirror, your reflection appears to be as far behind the mirror as you are in front of it.) Draw the lines of sight from your eyes to the reflection of the top of your head and to the reflection of the bottom of your feet. Determine where these lines of sight intersect the mirror.
- b. Experiment by changing your distance from the mirror, the height of your eyes, and/or the height of the top of your head. Use your results to determine the length of the shortest mirror you can use and where it should be positioned on the wall so that you can see the entire length of your body in the mirror.

