Name ______________________ Class ______________ Date ________

2.1 Translations

Essential Question: How do you draw the image of a figure under a translation?

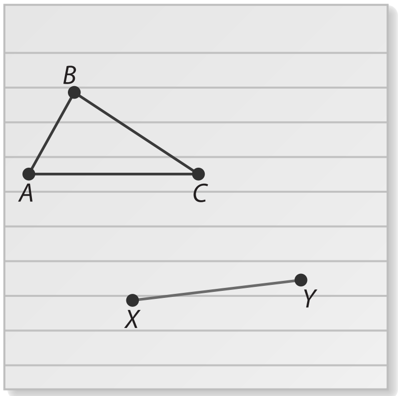
Resource Locker

Explore Exploring Translations

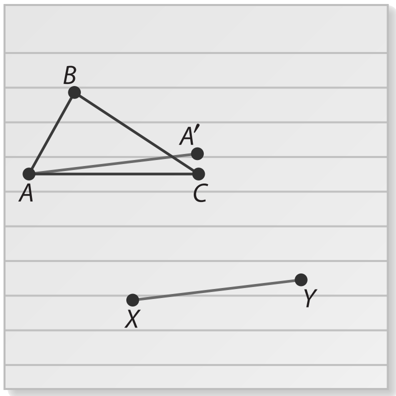
A translation slides all points of a figure the same distance in the same direction.

You can use tracing paper to model translating a triangle.

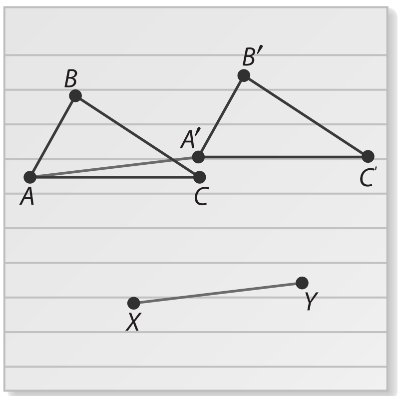
(A) First, draw a triangle on lined paper. Label the vertices *A*, *B*, and *C*. Then draw a line segment *XY*. An example of what your drawing may look like is shown.



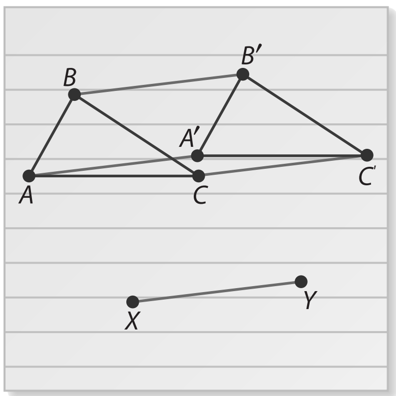
(B) Use tracing paper to draw a copy of triangle *ABC*. Then copy $\overline{XY}$ so that the point *X* is on top of point *A*. Label the point made from *Y* as *A*′.



(C) Using the same piece of tracing paper, place *A*' on *A* and draw a copy of △*ABC*. Label the corresponding vertices *B*′ and *C*′. An example of what your drawing may look like is shown.



(D) Use a ruler to draw line segments from each vertex of the preimage to the corresponding vertex on the new image.



(E) Measure the distances AA', BB', CC', and XY. Describe how AA', BB', and CC' compare to the length XY.

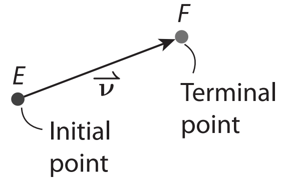
Reflect

1. Are BB', AA', and CC' parallel, perpendicular, or neither? Describe how you can check that your answer is reasonable.

2. How does the angle BAC relate to the angle $B'A'C'$? Explain.

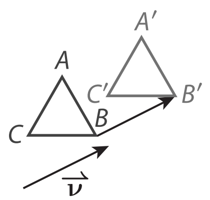
Explain 1 Translating Figures Using Vectors

A **vector** is a quantity that has both direction and magnitude.
The **initial point** of a vector is the starting point.
The **terminal point** of a vector is the ending point. The vector shown may be named $\overrightarrow{EF}$ or $\vec{v}$.



Translation

It is convenient to describe translations using vectors. A **translation** is a transformation along a vector such that the segment joining a point and its image has the same length as the vector and is parallel to the vector.



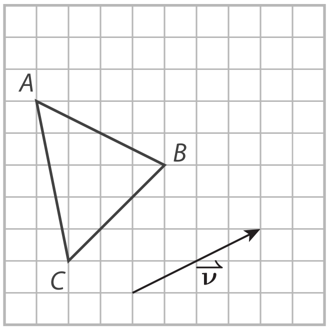
For example, BB' is a line segment that is the same length as and is parallel to vector $\vec{v}$.

You can use these facts about parallel lines to draw translations.

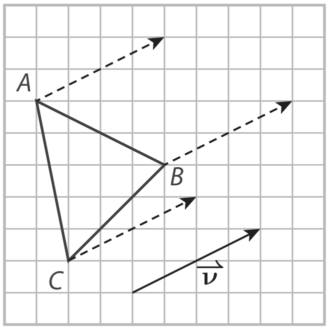
- Parallel lines are always the same distance apart and never intersect.
- Parallel lines have the same slope.

Example 1 **Draw the image of $\triangle ABC$ after a translation along $\vec{v}$.**

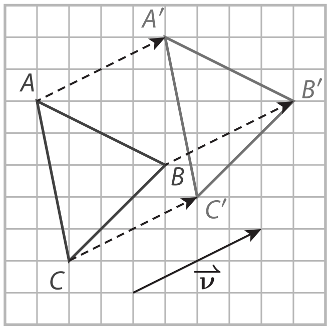
(A)



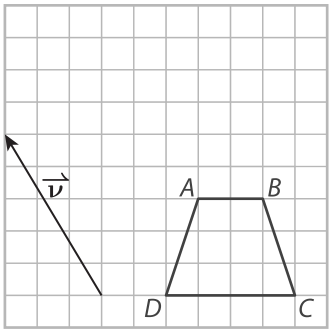
Draw a copy of $\vec{v}$ with its initial point at vertex A of $\triangle ABC$. The copy must be the same length as $\vec{v}$, and it must be parallel to $\vec{v}$. Repeat this process at vertices B and C.



Draw segments to connect the terminal points of the vectors. Label the points A', B', and C'. $\triangle A'B'C'$ is the image of $\triangle ABC$.



(B)



Draw a vector from the vertex A that is the same length as and __________ vector $\vec{v}$. The terminal point A' will be __________ units up and 3 units __________.

Draw three more vectors that are parallel from ______, ______, and ______ with terminal points B', C', and D'.

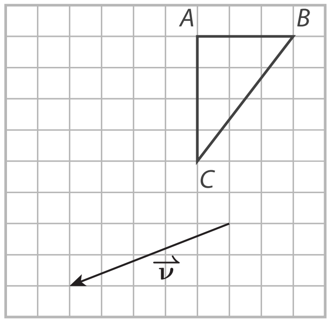
Draw segments connecting A', B', C', and D' to form ____________________.

Reflect

3. How is drawing an image of quadrilateral $ABCD$ like drawing an image of $\triangle ABC$? How is it different?

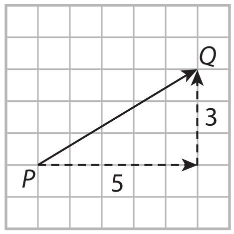
Your Turn

4. Draw the image of $\triangle ABC$ after a translation along $\vec{v}$.



Explain 2 Drawing Translations on a Coordinate Plane

A vector can also be named using component form, $\langle a, b\rangle$, which specifies the horizontal change a and the vertical change b from the initial point to the terminal point. The component form for $\overline{PQ}$ is $\langle 5, 3\rangle$.



You can use the component form of the vector to draw coordinates for a new image on a coordinate plane. By using this vector to move a figure, you are moving the x-coordinate 5 units to the right. So, the new x-coordinate would be 5 greater than the x-coordinate in the preimage. Using this vector you are also moving the y-coordinate up 3 units. So, the new y-coordinate would be 3 greater than the y-coordinate in the preimage.

Rules for Translations on a Coordinate Plane	
Translation a units to the right	$\langle x, y\rangle \rightarrow \langle x + a, y\rangle$
Translation a units to the left	$\langle x, y\rangle \rightarrow \langle x - a, y\rangle$
Translation b units up	$\langle x, y\rangle \rightarrow \langle x, y + b\rangle$
Translation b units down	$\langle x, y\rangle \rightarrow \langle x, y - b\rangle$

So, when you move an image to the right a units and up b units, you use the rule $(x, y) \rightarrow (x + a, y + b)$ which is the same as moving the image along vector $\langle a, b\rangle$.

Example 2 **Calculate the vertices of the image figure. Graph the preimage and the image.**

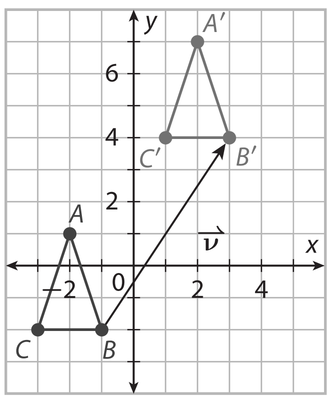
(A) Preimage coordinates: $(-2, 1)$, $(-3, -2)$, and $(-1, -2)$. Vector: $\langle 4, 6\rangle$

Predict which quadrant the new image will be drawn in: 1st quadrant.

Use a table to record the new coordinates. Use vector components to write the transformation rule.

Preimage coordinates (x, y)	Image $(x + 4, y + 6)$
(–2, 1)	(2, 7)
(–3, –2)	(1, 4)
(–1, –2)	(3, 4)

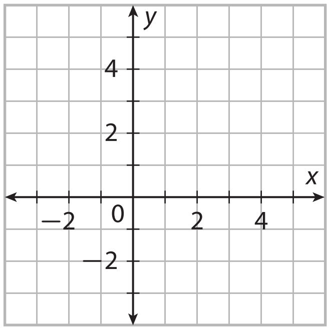
Then use the preimage coordinates to draw the preimage, and use the image coordinates to draw the new image.



(B) Preimage coordinates: $A(3, 0)$, $B(2, -2)$, and $C(4, -2)$. Vector $\langle -2, 3 \rangle$

Prediction: The image will be in Quadrant ____.

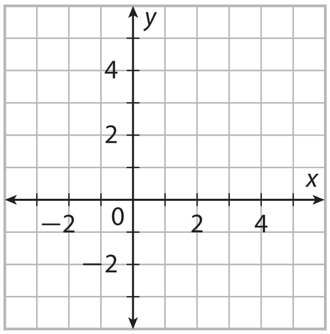
Preimage coordinates (x, y)	Image $(x - \square, y + \square)$
$(3, 0)$	$(\square, \square)$
$(2, -2)$	$(\square, \square)$
$(4, -2)$	$(\square, \square)$



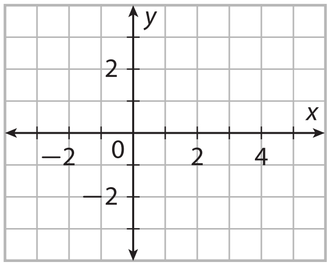
Your Turn

Draw the preimage and image of each triangle under a translation along $\langle -4, 1 \rangle$.

5. Triangle with coordinates: $A(2, 4)$, $B(1, 2)$, $C(4, 2)$.



6. Triangle with coordinates: $P(2, -1)$, $Q(2, -3)$, $R(4, -3)$.

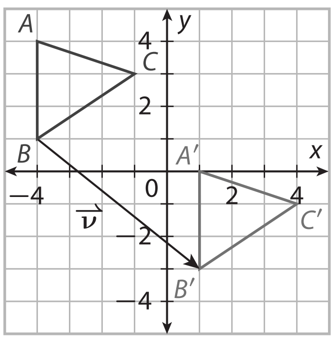


Explain 3 Specifying Translation Vectors

You may be asked to specify a translation that carries a given figure onto another figure. You can do this by drawing the translation vector and then writing it in component form.

Example 3 **Specify the component form of the vector that maps $\triangle ABC$ to $\triangle A'B'C'$.**

(A)



Determine the components of $\vec{v}$.

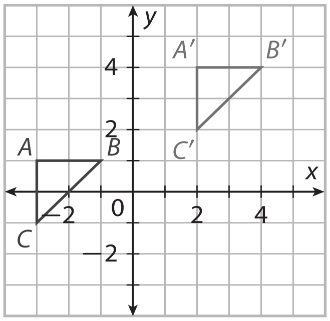
The horizontal change from the initial point $(-4, 1)$ to the terminal point $(1, -3)$ is $1 - (-4) = 5$.

The vertical change from the initial point $(-4, 1)$ to the terminal point $(1, -3)$ is $-3 - 1 = -4$

Write the vector in component form.

$$\vec{v} = \langle 5, -4 \rangle$$

(B)



Draw the vector $\vec{v}$ from a vertex of $\triangle ABC$ to its image in $\triangle A'B'C'$.

Determine the components of $\vec{v}$.

The horizontal change from the initial point $(-3, 1)$ to the terminal point $(2, 4)$ is ____ − ____ = ____.

The vertical change from the initial point to the terminal point is ____ − ____ = ____

Write the vector in component form. $\vec{v} = \langle \square, \square \rangle$

Reflect

7. What is the component form of a vector that translates figures horizontally? Explain.

Your Turn

8. In Example 3A, suppose $\triangle A'B'C'$ is the preimage and $\triangle ABC$ is the image after translation. What is the component form of the translation vector in this case? How is this vector related to the vector you wrote in Example 3A?

Elaborate

9. How are translations along the vectors $\langle a, -b\rangle$ and $\langle -a, b\rangle$ similar and how are they different?

10. A translation along the vector $\langle -2, 7\rangle$ maps point P to point Q. The coordinates of point Q are $(4, -1)$. What are the coordinates of point P? Explain your reasoning.

11. A translation along the vector $\langle a, b\rangle$ maps points in Quadrant I to points in Quadrant III. What can you conclude about a and b? Justify your response.

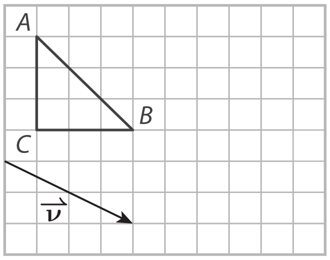
12. **Essential Question Check-In** How does translating a figure using the formal definition of a translation compare to the previous method of translating a figure?

Evaluate: Homework and Practice

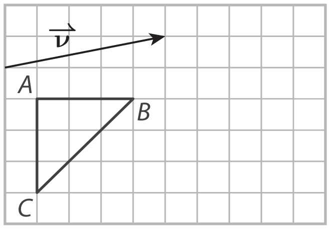
- Online Homework
- Hints and Help
- Extra Practice

Draw the image of $\triangle ABC$ after a translation along $\vec{v}$.

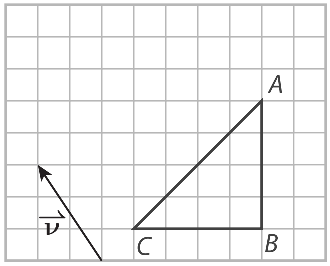
1.



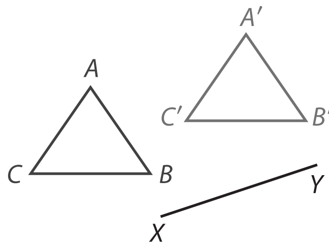
2.



3.

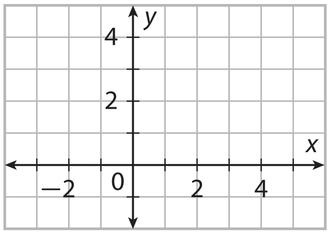


4. Line segment $\overline{XY}$ was used to draw a copy of $\triangle ABC$. $\overline{XY}$ is 3.5 centimeters long. What is the length of $AA' + BB' + CC'$?

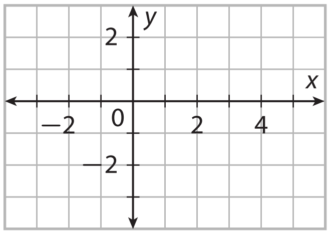


Draw the preimage and image of each triangle under the given translation.

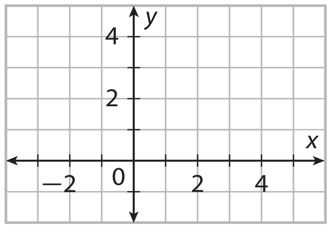
5. Triangle: $A(-3, -1)$; $B(-2, 2)$; $C(0, -1)$; Vector: $\langle 3, 2 \rangle$



6. Triangle: $P(1, -3)$; $Q(3, -1)$; $R(4, -3)$; Vector: $\langle -1, 3 \rangle$



7. Triangle: $X(0, 3)$; $Y(-1, 1)$; $Z(-3, 4)$; Vector: $\langle 4, -2 \rangle$

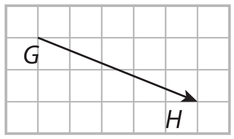


8. Find the coordinates of the image under the transformation $\langle 6, -11 \rangle$.

$(x, y) \rightarrow$ $\qquad$ $(2, -3) \rightarrow$

$(3, 1) \rightarrow$ $\qquad$ $(4, -3) \rightarrow$

9. Name the vector. Write it in component form.



10. Match each set of coordinates for a preimage with the coordinates of its image after applying the vector $\langle 3, -8 \rangle$. Indicate a match by writing a letter for a preimage on the line in front of the corresponding image.

A. $(1, 1)$; $(10, 1)$; $(6, 5)$ ________ $(6, -10)$; $(6, -4)$; $(9, -3)$

B. $(0, 0)$; $(3, 8)$; $(4, 0)$; $(7, 8)$ ________ $(1, -6)$; $(5, -6)$; $(-1, -8)$; $(7, -8)$

C. $(3, -2)$; $(3, 4)$; $(6, 5)$ ________ $(4, -7)$; $(13, -7)$; $(9, -3)$

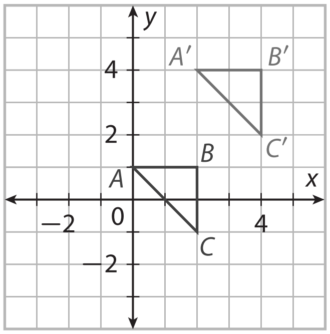
D. $(-2, 2)$; $(2, 2)$; $(-4, 0)$; $(4, 0)$ ________ $(3, -8)$; $(6, 0)$; $(7, -8)$; $(10, 0)$

11. Persevere in Problem Solving Emma and Tony are playing a game. Each draws a triangle on a coordinate grid. For each turn, Emma chooses either the horizontal or vertical value for a vector in component form. Tony chooses the other value, alternating each turn. They each have to draw a new image of their triangle using the vector with the components they chose and using the image from the prior turn as the preimage. Whoever has drawn an image in each of the four quadrants first wins the game.

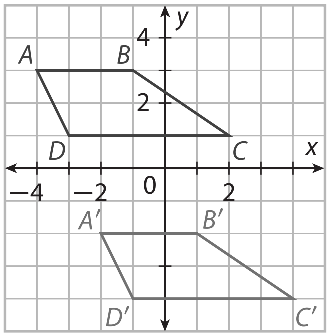
Emma's initial triangle has the coordinates $(-3, 0)$, $(-4, -2)$, $(-2, -2)$ and Tony's initial triangle has the coordinates $(2, 4)$, $(2, 2)$, $(4, 3)$. On the first turn the vector $\langle 6, -5 \rangle$ is used and on the second turn the vector $\langle -10, 8 \rangle$ is used. What quadrant does Emma need to translate her triangle to in order to win? What quadrant does Tony need to translate his triangle to in order to win?

Specify the component form of the vector that maps each figure to its image.

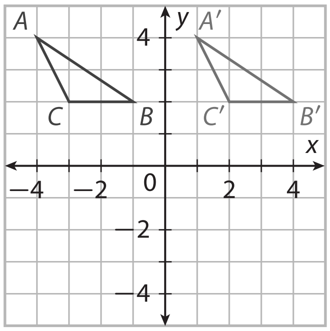
12.



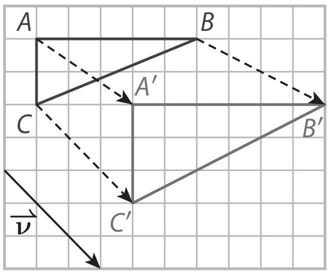
13.



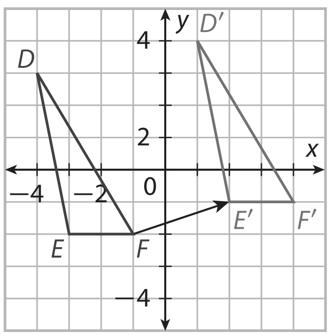
14.



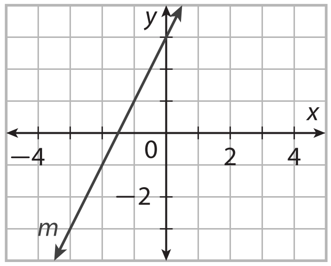
15. Explain the Error Andrew is using vector $\vec{v}$ to draw a copy of $\triangle ABC$. Explain his error.



16. Explain the Error Marcus was asked to identify the vector that maps $\triangle DEF$ to $\triangle D'E'F'$. He drew a vector as shown and determined that the component form of the vector is $\langle 3, 1 \rangle$. Explain his error.



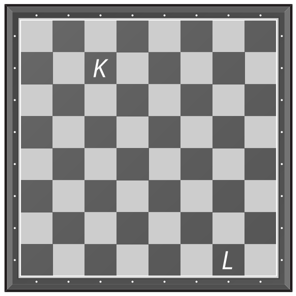
17. **Algebra** A cartographer is making a city map. Line *m* represents Murphy Street. The cartographer translates points on line *m* along the vector $\langle 2, -2 \rangle$ to draw Nolan Street. Draw the line for Nolan Street on the coordinate plane and write its equation. What is the image of the point $(0, 3)$ in this situation?



H.O.T. Focus on Higher Order Thinking

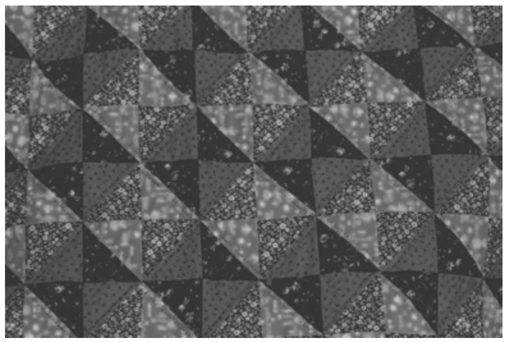
18. **Represent Real-World Problems** A builder is trying to level out some ground with a front-end loader. He picks up some excess dirt at (9, 16) and then maneuvers through the job site along the vectors $\langle -6, 0 \rangle$, $\langle 2, 5 \rangle$, $\langle 8, 10 \rangle$ to get to the spot to unload the dirt. Find the coordinates of the unloading point. Find a single vector from the loading point to the unloading point.

19. **Look for a Pattern** A checker player's piece begins at *K* and, through a series of moves, lands on *L*. What translation vector represents the path from *K* to *L*?



20. **Represent Real-World Problems** A group of hikers walks 2 miles east and then 1 mile north. After taking a break, they then hike 4 miles east to their final destination. What vector describes their hike from their starting position to their final destination? Let 1 unit represent 1 mile.

21. **Communicate Mathematical Ideas** In a quilt pattern, a polygon with vertices $(-4, -2)$, $(-3, -1)$, $(-2, -2)$, and $(-3, -3)$ is translated repeatedly along the vector $\langle 2, 2 \rangle$. What are the coordinates of the third polygon in the pattern? Explain how you solved the problem.



Lesson Performance Task

A contractor is designing a pattern for tiles in an entryway, using a sun design called Image *A* for the center of the space. The contractor wants to duplicate this design three times, labeled Image *B*, Image *C*, and Image *D*, above Image *A* so that they do not overlap. Identify the three vectors, labeled $\vec{m}$, $\vec{n}$, and $\vec{p}$ that could be used to draw the design, and write them in component form. Draw the images on grid paper using the vectors you wrote.