Name ____________ Class ____________ Date ____________

19.4 Mutually Exclusive and Overlapping Events

Essential Question: How are probabilities affected when events are mutually exclusive or overlapping?

Resource Locker

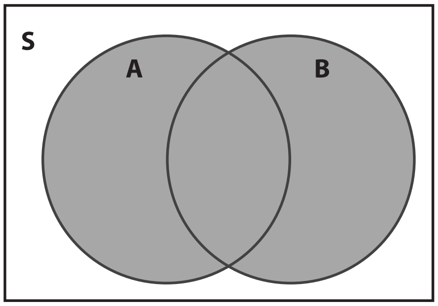
Explore 1 Finding the Probability of Mutually Exclusive Events

Two events are **mutually exclusive events** if they cannot both occur in the same trial of an experiment. For example, if you flip a coin it cannot land heads up and tails up in the same trial. Therefore, the events are mutually exclusive.

A number dodecahedron has 12 sides numbered 1 through 12. What is the probability that you roll the cube and the result is an even number or a 7?

(A) Let A be the event that you roll an even number. Let B be the event that you roll a 7. Let S be the sample space.

Complete the Venn diagram by writing all outcomes in the sample space in the appropriate region.



(B) Calculate $P(A)$.

$P(A) = \frac{\square}{\square} = \frac{\square}{\square}$

(C) Calculate $P(B)$.

$P(B) = \frac{\square}{\square}$

(D) Calculate $P(A \text{ or } B)$.

$n(S) = \square$

$n(A \text{ or } B) = n(A) + n(B)$

$= \square + \square = \square$

So, $P(A \text{ or } B) = \frac{n(A \text{ or } B)}{n(S)} = \frac{\square}{\square}$.

(E) Calculate $P(A) + P(B)$. Compare the answer to Step D.

$P(A) + P(B) = \frac{\square}{\square} + \frac{\square}{\square} = \frac{\square}{\square}$

$P(A) + P(B)$ ________ $P(A \text{ or } B)$.

Reflect

1. **Discussion** How would you describe mutually exclusive events to another student in your own words? How could you use a Venn diagram to assist in your explanation?

2. Look back over the steps. What can you conjecture about the probability of the union of events that are mutually exclusive?

Explore 2 Finding the Probability of Overlapping Events

The process used in the previous Explore can be generalized to give the formula for the probability of mutually exclusive events.

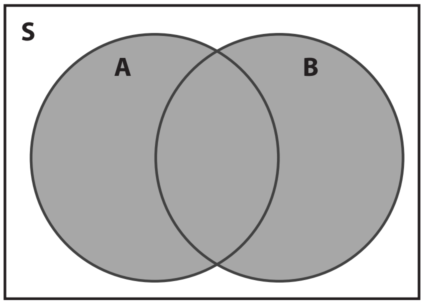
Mutually Exclusive Events
If A and B are mutually exclusive events, then $P(A \text{ or } B) = P(A) + P(B)$.

Two events are **overlapping events** (or inclusive events) if they have one or more outcomes in common.

What is the probability that you roll a number dodecahedron and the result is an even number or a number greater than 7?

(A) Let A be the event that you roll an even number. Let B be the event that you roll a number greater than 7. Let S be the sample space.

Complete the Venn diagram by writing all outcomes in the sample space in the appropriate region.



(B) Calculate $P(A)$.

$P(A) = \frac{\square}{\square} = \frac{\square}{\square}$

(C) Calculate $P(B)$.

$P(B) = \frac{\square}{\square}$

(D) Calculate $P(A \text{ and } B)$.

$P(A \text{ and } B) = \frac{\square}{\square} = \frac{\square}{\square}$

(E) Use the Venn diagram to find $P(A$ or $B)$.

$P(A \text{ or } B) = \frac{\square}{\square} = \frac{\square}{\square}$

(F) Now, use $P(A)$, $P(B)$, and $P(A \text{ and } B)$ to calculate $P(A \text{ or } B)$.

$P(A) = \square$ $\quad P(B) = \square$ $\quad P(A \text{ and } B) = \square$

$P(A) + P(B) - P(A \text{ and } B) = \square + \square - \square = \square$

Reflect

3. Why must you subtract $P(A \text{ and } B)$ from $P(A) + P(B)$ to determine $P(A \text{ or } B)$?

4. Look back over the steps. What can you conjecture about the probability of the union of two events that are overlapping?

Explain 1 Finding a Probability From a Two-Way Table of Data

The previous Explore leads to the following rule.

The Addition Rule

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

Example 1 **Use the given two-way tables to determine the probabilities.**

(A) $P(\text{senior or girl})$

	Freshman	Sophomore	Junior	Senior	TOTAL
Boy	98	104	100	94	396
Girl	102	106	96	108	412
Total	200	210	196	202	808

To determine $P(\text{senior or girl})$, first calculate $P(\text{senior})$, $P(\text{girl})$, and $P(\text{senior and girl})$.

$P(\text{senior}) = \frac{202}{808} = \frac{1}{4}$; $P(\text{girl}) = \frac{412}{808} = \frac{103}{202}$ $\quad P(\text{senior and girl}) = \frac{108}{808} = \frac{27}{202}$

Use the addition rule to determine $P(\text{senior or girl})$.

$$\begin{aligned} P(\text{senior or girl}) &= P(\text{senior}) + P(\text{girl}) - P(\text{senior and girl}) \\ &= \frac{1}{4} + \frac{103}{202} - \frac{27}{202} \\ &= \frac{253}{404} \end{aligned}$$

Therefore, the probability that a student is a senior or a girl is $\frac{253}{404}$.

(B) $P\left((\text{domestic or late})^c\right)$

	Late	On Time	Total
Domestic Flights	12	108	120
International Flights	6	54	60
Total	18	162	180

To determine $P\left((\text{domestic or late})^c\right)$, first calculate $P(\text{domestic or late})$.

$P(\text{domestic}) = \frac{\square}{\square} = \frac{\square}{\square}$; $P(\text{late}) = \frac{\square}{\square} = \frac{\square}{\square}$; $P(\text{domestic and late}) = \frac{\square}{\square} = \frac{\square}{\square}$

Use the addition rule to determine $P(\text{domestic or late})$.

$$\begin{aligned} P(\text{domestic or late}) &= P(\text{domestic}) + P(\text{late}) - P(\text{domestic and late}) \\ &= \frac{\square}{\square} + \frac{\square}{\square} - \frac{\square}{\square} = \frac{\square}{\square} \end{aligned}$$

$$\begin{aligned} \text{Therefore, } P\left((\text{domestic or late})^c\right) &= 1 - P(\text{domestic or late}) \\ &= 1 - \frac{\square}{\square} \\ &= \frac{\square}{\square} \end{aligned}$$

Your Turn

5. Use the table to determine $P(\text{headache or no medicine})$.

	Took Medicine	No Medicine	TOTAL
Headache	12	15	27
No Headache	48	25	73
TOTAL	60	40	100

Elaborate

6. Give an example of mutually exclusive events and an example of overlapping events.

7. **Essential Question Check-In** How do you determine the probability of mutually exclusive events and overlapping events?

Evaluate: Homework and Practice

Personal Math Trainer

- Online Homework
- Hints and Help
- Extra Practice

1. A bag contains 3 blue marbles, 5 red marbles, and 4 green marbles. You choose one without looking. What is the probability that it is red or green?

2. A number icosahedron has 20 sides numbered 1 through 20. What is the probability that the result of a roll is a number less than 4 or greater than 11?

3. A bag contains 26 tiles, each with a different letter of the alphabet written on it. You choose a tile without looking. What is the probability that you choose a vowel (a, e, i, o, or u) or a letter in the word GEOMETRY?

4. **Persevere in Problem Solving** You roll two number cubes at the same time. Each cube has sides numbered 1 through 6. What is the probability that the sum of the numbers rolled is even or greater than 9? (*Hint:* Create and fill out a probability chart.)

The table shows the data for car insurance quotes for 125 drivers made by an insurance company in one week.

	Teen	Adult (20 or over)	Total
0 accidents	15	53	68
1 accident	4	32	36
2+ accidents	9	12	21
Total	28	97	125

You randomly choose one of the drivers. Find the probability of each event.

5. The driver is an adult.

6. The driver is a teen with 0 or 1 accident.

7. The driver is a teen.

8. The driver has 2+ accidents.

9. The driver is a teen and has 2+ accidents.

10. The driver is a teen or a driver with 2+ accidents.

Use the following information for Exercises 11–16. The table shown shows the results of a customer satisfaction survey for a cellular service provider, by location of the customer. In the survey, customers were asked whether they would recommend a plan with the provider to a friend.

	Arlington	Towson	Parkville	Total
Yes	40	35	41	116
No	18	10	6	34
Total	58	45	47	150

One of the customers that was surveyed was chosen at random. Find the probability of each event.

11. The customer was from Towson and said No.

12. The customer was from Parkville.

13. The customer said Yes.

14. The customer was from Parkville and said Yes.

15. The customer was from Parkville or said Yes.

16. Explain why you cannot use the rule $P(A \text{ or } B) = P(A) + P(B)$ in Exercise 15.

Use the following information for Exercises 17–21. Roberto is the owner of a car dealership. He is assessing the success rate of his top three salespeople in order to offer one of them a promotion. Over two months, for each attempted sale, he records whether the salesperson made a successful sale or not. The results are shown in the chart.

	Successful	Unsuccessful	Total
Becky	6	6	12
Raul	4	5	9
Darrell	6	9	15
Total	16	20	36

Roberto randomly chooses one of the attempted sales.

17. Find the probability that the sale was one of Becky's or Raul's successful sales.

18. Find the probability that the sale was one of the unsuccessful sales or one of Raul's successful sales.

19. Find the probability that the sale was one of Darrell's unsuccessful sales or one of Raul's unsuccessful sales.

20. Find the probability that the sale was an unsuccessful sale or one of Becky's attempted sales.

21. Find the probability that the sale was a successful sale or one of Raul's attempted sales.

22. You are going to draw one card at random from a standard deck of cards. A standard deck of cards has 13 cards (2, 3, 4, 5, 6, 7, 8, 9, 10, jack, queen, king, ace) in each of 4 suits (hearts, clubs, diamonds, spades). The hearts and diamonds cards are red. The clubs and spades cards are black. Which of the following have a probability of less than $\frac{1}{4}$? Choose all that apply.

a. Drawing a card that is a spade and an ace

b. Drawing a card that is a club or an ace

c. Drawing a card that is a face card or a club

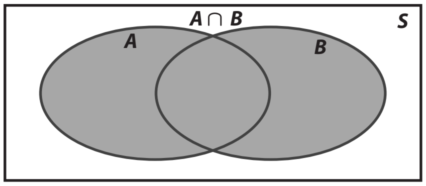
d. Drawing a card that is black and a heart

e. Drawing a red card and a number card from 2–9

H.O.T. Focus on Higher Order Thinking

23. Draw Conclusions A survey of 1108 employees at a software company finds that 621 employees take a bus to work and 445 employees take a train to work. Some employees take both a bus and a train, and 321 employees take only a train. To the nearest percent, find the probability that a randomly chosen employee takes a bus or a train to work. Explain.

24. Communicate Mathematical Ideas Explain how to use a Venn diagram to find the probability of randomly choosing a multiple of 3 or a multiple of 4 from the set of numbers from 1 to 25. Then find the probability.



25. Explain the Error Sanderson attempted to find the probability of randomly choosing a 10 or a diamond from a standard deck of playing cards. He used the following logic:

Let S be the sample space, A be the event that the card is a 10, and B be the event that the card is a diamond.

There are 52 cards in the deck, so $n(S) = 52$.

There are four 10s in the deck, so $n(A) = 4$.

There are 13 diamonds in the deck, so $n(B) = 13$.

One 10 is a diamond, so $n(A \cap B) = 1$.

$$P(A \cup B) = \frac{n(A \cup B)}{n(S)} = \frac{n(A) \cdot n(B) - n(A \cap B)}{n(S)} = \frac{4 \cdot 13 - 1}{52} = \frac{51}{52}$$

Describe and correct Sanderson's mistake.

Lesson Performance Task

What is the smallest number of randomly chosen people that are needed in order for there to be a better than 50% probability that at least two of them will have the same birthday? The astonishing answer is 23. Follow these steps to find why.

1. Can a person have a birthday on two different days? Use the vocabulary of this lesson to explain your answer.

Looking for the probability that two or more people in a group of 23 have matching birthdays is a challenge. Maybe there is one match but maybe there are five matches or seven or fourteen. A much easier way is to look for the probability that there are *no* matches in a group of 23. In other words, all 23 have different birthdays. Then use that number to find the answer.

2. There are 365 days in a non-leap year.

a. Write an expression for the number of ways can you assign different birthdays to 23 people. (Hint: Think of the people as standing in a line, and you are going to assign a different number from 1 to 365 to each person.)

b. Write an expression for the number ways can you assign any birthday to 23 people. (Hint: Now think about assigning any number from 1 to 365 to each of 23 people.)

c. How can you use your answers to (a) and (b) to find the probability that no people in a group of 23 have the same birthday? Use a calculator to find the probability to the nearest ten-thousandth.

d. What is the probability that at least two people in a group of 23 have the same birthday? Explain your reasoning.