Name ____________________ Class ____________ Date ________

# 15.1 Central Angles and Inscribed Angles

**Essential Question:** How can you determine the measures of central angles and inscribed angles of a circle?

Resource Locker

## Explore Investigating Central Angles and Inscribed Angles

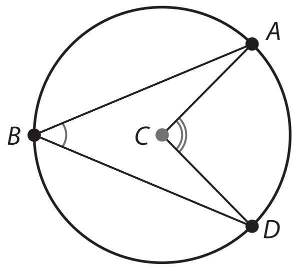
A **chord** is a segment whose endpoints lie on a circle.

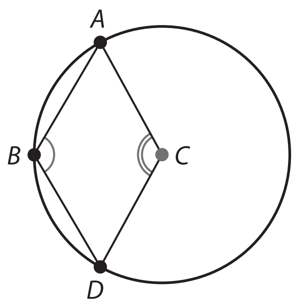
A **central angle** is an angle less than 180° whose vertex lies at the center of a circle.

An **inscribed angle** is an angle whose vertex lies on a circle and whose sides contain chords of the circle.

The diagram shows two examples of an inscribed angle and the corresponding central angle.

| Chords |
|---|
| $\overline{AB}$ and $\overline{BD}$ |
| **Central Angle** |
| $\angle ACD$ |
| **Inscribed Angle** |
| $\angle ABD$ |




Ⓐ Use a compass to draw a circle. Label the center *C*.

Ⓑ Use a straightedge to draw an acute inscribed angle on your circle from Step A. Label the angle as $\angle DEF$.

Ⓒ Use a straightedge to draw the corresponding central angle, $\angle DCF$.

Ⓓ Use a protractor to measure the inscribed angle and the central angle. Record the measure of the inscribed angle, the measure of the central angle, and the measure of 360° minus the central angle. List your results in the table.

| Angle Measure | Circle *C* | Circle 2 | Circle 3 | Circle 4 | Circle 5 | Circle 6 | Circle 7 |
|---|---|---|---|---|---|---|---|
| m$\angle DEF$ | | | | | | | |
| m$\angle DCF$ | | | | | | | |
| 360° − m$\angle DCF$ | | | | | | | |

(E) Repeat Steps A-D six more times. Examine a variety of inscribed angles (two more acute, one right, and three obtuse). Record your results in the table in Step D.

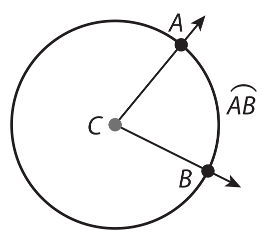
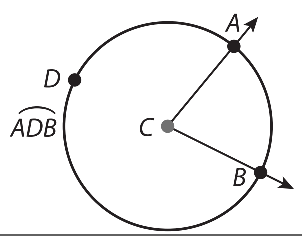
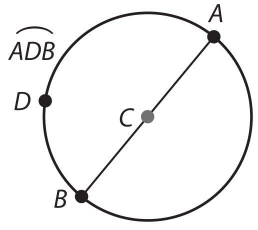
**Reflect**

**1.** Examine the values in the first and second rows of the table. Is there a mathematical relationship that exists for some or all of the values? Make a conjecture that summarizes your observation.

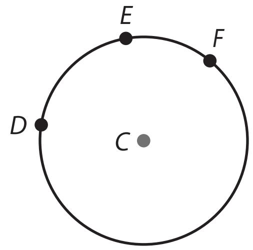
**2.** Examine the values in the first and third rows of the table. Is there a mathematical relationship that exists for some or all of the values? Make a conjecture that summarizes your observation.

## Explain 1 Understanding Arcs and Arc Measure

An **arc** is a continuous portion of a circle consisting of two points (called the endpoints of the arc) and all the points on the circle between them.

| Arc | Measure | Figure |
|---|---|---|
| A **minor arc** is an arc whose points are on or in the interior of a corresponding central angle. | The measure of a minor arc is equal to the measure of the central angle. $m\overset{\frown}{AB} = m\angle ACB$ |  |
| A **major arc** is an arc whose points are on or in the exterior of a corresponding central angle. | The measure of a major arc is equal to 360° minus the measure of the central angle. $m\overset{\frown}{ADB} = 360° - m\angle ACB$ |  |
| A **semicircle** is an arc whose endpoints are the endpoints of a diameter. | The measure of a semicircle is 180°. |  |

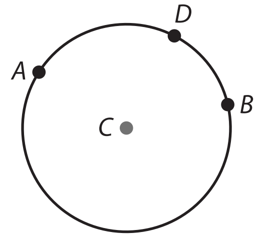
**Adjacent arcs** are arcs of the same circle that intersect in exactly one point. $\widehat{DE}$ and $\widehat{EF}$ are adjacent arcs.




## Arc Addition Postulate

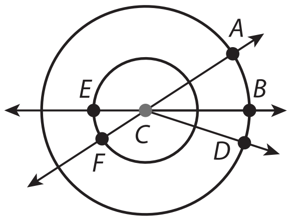
The measure of an arc formed by two adjacent arcs is the sum of the measures of the two arcs.

$m\widehat{ADB} = m\widehat{AD} + m\widehat{DB}$



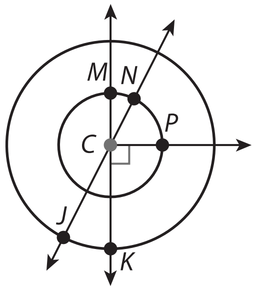

**Example 1**

(A) If $m\angle BCD = 18°$ and $m\widehat{EF} = 33°$, determine $m\widehat{ABD}$ using the appropriate theorems and postulates. $\overleftrightarrow{AF}$ and $\overleftrightarrow{BE}$ intersect at Point $C$.




If $m\widehat{EF} = 33°$, then $m\angle ECF = 33°$. If $m\angle ECF = 33°$, then $m\angle ACB = 33°$ by the Vertical Angles Theorem. If $m\angle ACB = 33°$ and $m\angle BCD = 18°$, then $m\widehat{AB} = 33°$ and $m\widehat{BD} = 18°$. By the Arc Addition Postulate, $m\widehat{ABD} = m\widehat{AB} + m\widehat{BD}$, and so $m\widehat{ABD} = 51°$.

(B) If $m\widehat{JK} = 27°$, determine $m\widehat{NP}$ using the appropriate theorems and postulates. $\overleftrightarrow{MK}$ and $\overleftrightarrow{NJ}$ intersect at Point $C$.




If $m\widehat{JK} = 27°$, then $m\angle JCK = 27°$. If $m\angle JCK = 27°$, then

$m\angle$ ☐ $= 27°$ by the ______________________.

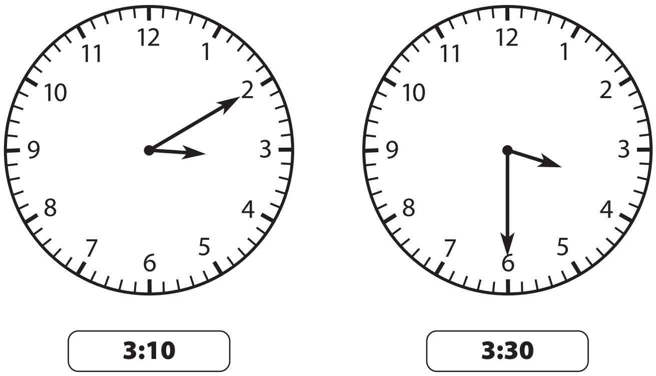
If $m\angle MCN = 27°$ and $m\angle MCP =$ ☐°, then $m\widehat{MN} = 27°$

and $m\widehat{MNP} =$ ☐°. By the ______________________,

$m\widehat{MNP} = m\widehat{MN} + m\widehat{NP}$, and so $m\widehat{NP} = m$ ☐ $- m\widehat{MN} =$ ☐°

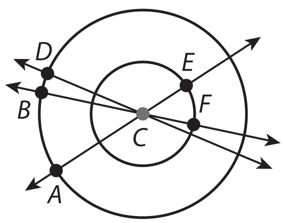
**Reflect**

**3.** The minute hand of a clock sweeps out an arc as time moves forward. From 3:10 p.m. to 3:30 p.m., what is the measure of this arc? Explain your reasoning.




**Your Turn**

**4.** If $m\widehat{EF} = 45°$ and $m\angle ACD = 56°$, determine $m\widehat{BD}$ using the appropriate theorems and postulates. $\overleftrightarrow{AE}$, $\overleftrightarrow{BF}$, and $\overleftrightarrow{DC}$ intersect at Point *C*.



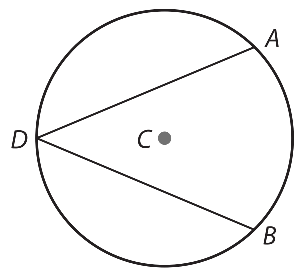

## Explain 2 Using the Inscribed Angle Theorem

In the Explore you looked at the relationship between central angles and inscribed angles. Those results, combined with the definitions of arc measure, lead to the following theorem about inscribed angles and their *intercepted arcs*. An **intercepted arc** consists of endpoints that lie on the sides of an inscribed angle and all the points of the circle between them.

**Inscribed Angle Theorem**

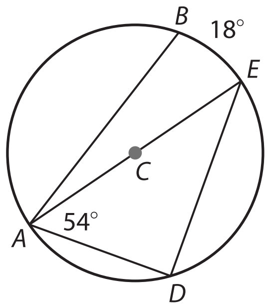
The measure of an inscribed angle is equal to half the measure of its intercepted arc.

$m\angle ADB = \frac{1}{2} m\widehat{AB}$



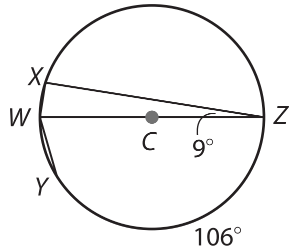
**Example 2** **Use the Inscribed Angle Theorem to find inscribed angle measures.**

(A) Determine $m\widehat{DE}$, $m\widehat{BD}$, $m\angle DAB$, and $m\angle ADE$ using the appropriate theorems and postulates.




By the Inscribed Angle Theorem, $m\angle DAE = \frac{1}{2}m\widehat{DE}$, and so $m\widehat{DE} = 2 \times 54° = 108°$. By the Arc Addition Postulate, $m\widehat{BD} = m\widehat{BE} + m\widehat{ED} = 18° + 108° = 126°$. By the Inscribed Angle Theorem, $m\angle DAB = \frac{1}{2}m\widehat{BD} = \frac{1}{2} \times 126° = 63°$. Note that $\widehat{ABE}$ is a semicircle, and so $m\widehat{ABE} = 180°$. By the Inscribed Angle Theorem, $m\angle ADE = \frac{1}{2}m\widehat{ABE} = \frac{1}{2} \times 180° = 90°$.

(B) Determine $m\widehat{WX}$, $m\widehat{XZ}$, $m\angle XWZ$, and $m\angle WXZ$ using the appropriate theorems and postulates.




By the Inscribed Angle Theorem, $m\angle WZX =$ ____ $m\widehat{WX}$,

and so $m\widehat{WX} = 2 \times 9° =$ ____. Note that $\widehat{WXZ}$ is a ____________ and,

therefore, $m\widehat{WXZ} = 180°$. By the ____________________,

$m\widehat{WXZ} = m\widehat{WX} + m\widehat{XZ}$ and then $m\widehat{XZ} = 180° - 18° =$ ____

By the ____________________, $m\angle XWZ = \frac{1}{2}m\widehat{XZ} = \frac{1}{2} \times 162° = 81°$.

Note that ____ is a semicircle, and so $m\widehat{WYZ} =$ ____. By the Inscribed Angle Theorem, $m\angle WXZ = \frac{1}{2}m$ ____ $= \frac{1}{2} \times$ ____ $=$ ____.

**Reflect**

5. **Discussion** Explain an alternative method for determining $m\angle\widehat{XZ}$ in Example 2B.

______________________________

______________________________

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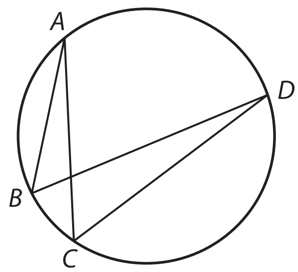
6. **Justify Reasoning** How does the measure of $\angle ABD$ compare to the measure of $\angle ACD$? Explain your reasoning.

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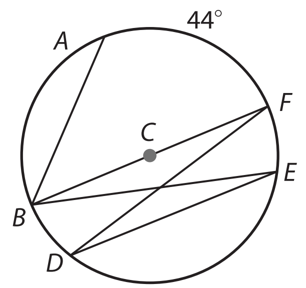
**Your Turn**

**7.** If m∠$EDF = 15°$, determine m∠$ABE$ using the appropriate theorems and postulates.




## Explain 3 Investigating Inscribed Angles on Diameters

You can examine angles that are inscribed in a semicircle. Example 3 Construct and analyze an angle inscribed in  a semicircle.

(A) Use a compass to draw a circle with center $C$. Use a straightedge to draw a diameter of the circle. Label the diameter $\overline{DF}$.

(B) Use a straightedge to draw an inscribed angle ∠$DEF$ on your circle from Step A whose sides contain the endpoints of the diameter.

(C) Use a protractor to determine the measure of ∠$DEF$ (to the nearest degree). Record the results in the table.

| Angle Measure | Circle *C* | Circle 2 | Circle 3 | Circle 4 |
|---|---|---|---|---|
| m∠*DEF* | | | | |

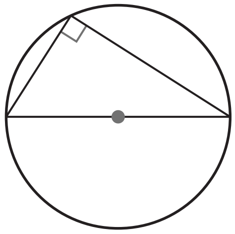
(D) Repeat the process three more times. Make sure to vary the size of the circle, and the location of the vertex of the inscribed angle. Record the results in the table in Part C.

(E) Examine the results, and make a conjecture about the measure of an angle inscribed in a semicircle.

(F) How can does the Inscribed Angle Theorem justify your conjecture?

**Inscribed Angle of a Diameter Theorem**

The endpoints of a diameter lie on an inscribed angle if and only if the inscribed angle is a right angle.

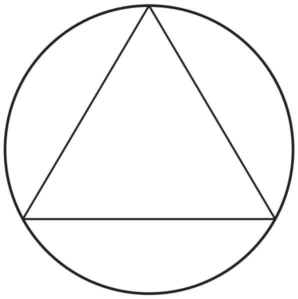


**Reflect**

**8.** A right angle is inscribed in a circle. If the endpoints of its intercepted arc are connected by a segment, must the segment pass through the center of the circle?

## Elaborate

**9.** An equilateral triangle is inscribed in a circle. How does the relationship between the measures of the inscribed angles and intercepted arcs help determine the measure of each angle of the triangle?



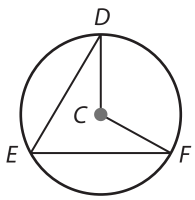
**10.** **Essential Question Check-In** What is the relationship between inscribed angles and central angles in a circle?

## Evaluate: Homework and Practice

Personal Math Trainer
- Online Homework
- Hints and Help
- Extra Practice

**Identify the chord(s), inscribed angle(s), and central angle(s) in the figure. The center of the circles in Exercises 1, 2, and 4 is *C*.**

**1.**

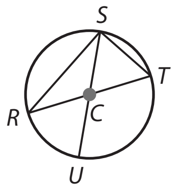



Chord(s): ________

InscribedAngle(s): ________

Central Angle(s): ________

**2.**

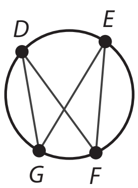



Chord(s): ________________

Inscribed Angle(s): ______________________________

Central Angle(s): ____________________________

**3.**



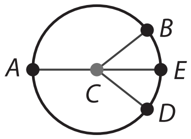

Chord(s): ______________

Inscribed Angle(s):

__________________________

Central Angle(s): ______

**4.**



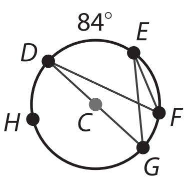

Chord(s): ________

Inscribed Angle(s): ______

Central Angle(s): ____________________

__________________________

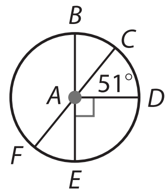
**In circle C, m$\overset{\frown}{DE} = 84°$. Find each measure.**




**5.** m∠*DGE*

**6.** m∠*EFD*

**The center of the circle is $A$. Find each measure using the appropriate theorems and postulates.**

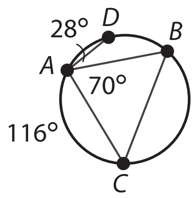



**7.** $m\overset{\frown}{CE}$

**8.** $m\overset{\frown}{DF}$

**9.** $m\overset{\frown}{BEC}$

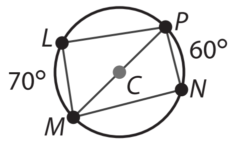
**Find each measure using the appropriate theorems and postulates. $m\overset{\frown}{AC} = 116°$**




**10.** $m\overset{\frown}{BC}$

**11.** $m\overset{\frown}{AD}$

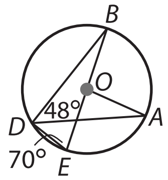
**The center of the circle is $C$. Find each measure using the appropriate theorems and postulates. $m\overset{\frown}{LM} = 70°$ and $m\overset{\frown}{NP} = 60°$.**




**12.** $m\angle MNP$

**13.** $m\angle LMN$

**The center of the circle is *O*. Find each arc or angle measure using the appropriate theorems and postulates.**



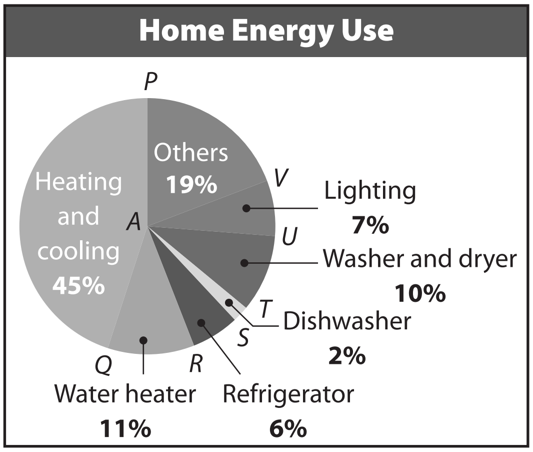

**14.** m∠*BDE*

**15.** $m\overarc{ABD}$

**16.** $m\overarc{ED}$

**17.** m∠*DBE*

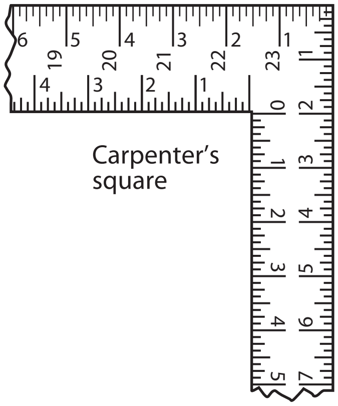
**Represent Real-World Problems** **The circle graph shows how a typical household spends money on energy. Use the graph to find the measure of each arc.**



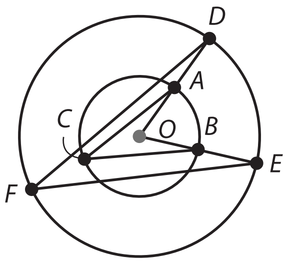

**18.** $m\overarc{PQ}$

**19.** $m\overarc{UPT}$

**20.** **Communicate Mathematical Ideas** A carpenter's square is a tool that is used to draw right angles. Suppose you are building a toy car and you have four small circles of wood that will serve as the wheels. You need to drill a hole in the center of each wheel for the axle. Explain how you can use the carpenter's square to find the center of each wheel.




**21.** Choose the expressions that are equivalent to m∠$AOB$. Select all that apply.




**A.** $\frac{1}{2}$m∠$ACB$ **E.** m∠$DOE$

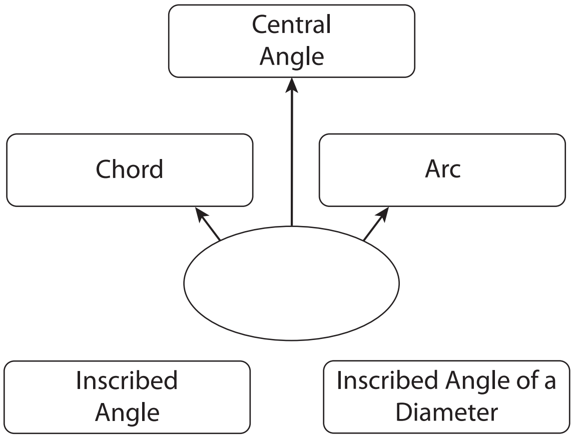
**B.** m∠$ACB$ **F.** m∠$DFE$

**C.** 2m∠$ACB$ **G.** 2m∠$DFE$

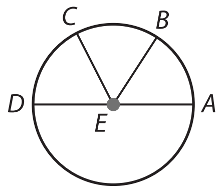
**D.** m$\widehat{AB}$ **H.** m$\widehat{DE}$



**22.** **Analyze Relationships** Draw arrows to connect the concepts shown in the boxes. Then explain how the terms shown in the concept map are related.

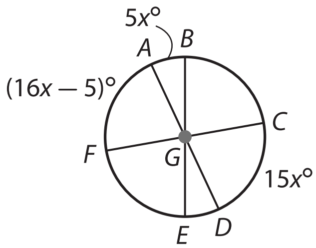


**23.** In circle $E$, the measures of $\angle DEC$, $\angle CEB$, and $\angle BEA$ are in the ratio 3:4:5. Find $m\overset{\frown}{AC}$.




**H.O.T. Focus on Higher Order Thinking**

**24. Explain the Error** The center of the circle is $G$. Below is a student's work to find the value of $x$. Explain the error and find the correct value of $x$.




$\overline{AD}$ is a diameter, so $m\overset{\frown}{ACD} = 180°$.

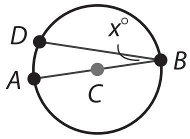
Since $m\overset{\frown}{ACD} = m\overset{\frown}{AB} + m\overset{\frown}{BC} + m\overset{\frown}{CD}$, $m\overset{\frown}{AB} + m\overset{\frown}{BC} + m\overset{\frown}{CD} = 180°$.

$5x + 90 + 15x = 180$

$20x = 90$

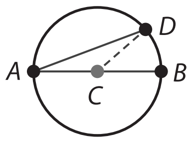
$x = 4.5$

**25. Multi-Step** An inscribed angle with a diameter as a side has measure $x°$. If the ratio of $m\overset{\frown}{AD}$ to $m\overset{\frown}{DB}$ is 1:4, what is $m\overset{\frown}{DB}$?




**26. Justify Reasoning** To prove the Inscribed Angle Theorem you need to prove three cases. In Case 1, the center of the circle is on a side of the inscribed angle. In Case 2, the center the circle is in the interior of the inscribed angle. In Case 3, the center the circle is in the exterior of the inscribed angle.

**a.** Fill in the blanks in the proof for Case 1 to show that $m\angle DAB = \frac{1}{2} m\overset{\frown}{DB}$.




**Given:** $\angle DAB$ is inscribed in circle $C$.

**Prove:** $m\angle DAB = \frac{1}{2} m\overset{\frown}{DB}$

**Proof:** Let $m\angle A = x°$. Draw $\overline{DC}$.

$\triangle ADC$ is ____________. So $m\angle A = m\angle$ [ ] by the Isosceles Triangle Theorem.

Then [ ] $= 2x°$ by the Exterior Angle Theorem. So, $m\overset{\frown}{DB} =$ [ ] by the definition of the measure of an arc of a circle.

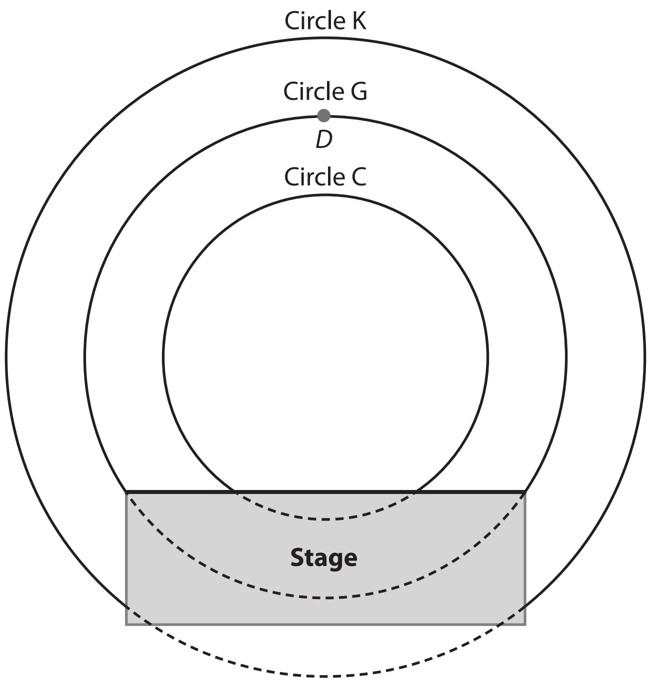
Since $m\overset{\frown}{DB} =$ [ ] and $m\angle DAB =$ [ ], $m\angle DAB = \frac{1}{2}\overset{\frown}{DB}$.

**b.** Draw and label a diagram for Case 2. Then use a paragraph proof to prove that the inscribed angle is one-half the intercepted arc.

**c.** Draw and label a diagram for Case 3. Then use a paragraph proof to prove that the inscribed angle is one-half the intercepted arc.

# Lesson Performance Task

Diana arrives late at the theater for a play. Her ticket entitles her to sit anywhere in Circle G. She had hoped to sit in Seat *D*, which she thought would give her the widest viewing angle of the stage. But Seat *D* is taken, as are all the other nearby seats in Circle G. The seating chart for the theater is shown.




Identify two other spots where Diana can sit that will give her the same viewing angle she would have had in Seat *D*. Explain how you know how your points would provide the same viewing angle, and support your claim by showing the viewing angles on the drawing.