Name ____________________ Class ____________ Date ________

1.4 Reasoning and Proof

Essential Question: How do you go about proving a statement?

Resource Locker

Explore Exploring Inductive and Deductive Reasoning

A **conjecture** is a statement that is believed to be true. You can use *inductive* or *deductive* reasoning to show, or *prove*, that a conjecture is true. **Inductive reasoning** is the process of reasoning that a rule or statement is true because specific cases are true. **Deductive reasoning** is the process of using logic to draw conclusions.

Complete the steps to make a conjecture about the sum of three consecutive counting numbers.

(A) Write a sum to represent the first three consecutive counting numbers, starting with 1. ____________________

(B) Is the sum divisible by 3? ____________________

(C) Write the sum of the next three consecutive counting numbers, starting with 2. ____________________

(D) Is the sum divisible by 3? ____________________

(E) Complete the conjecture:

The __________ of three consecutive counting numbers is divisible by __________.

Recall that postulates are statements you accept are true. A **theorem** is a statement that you can prove is true using a series of logical steps. The steps of deductive reasoning involve using appropriate undefined words, defined words, mathematical relationships, postulates, or other previously-proven theorems to prove that the theorem is true.

Use deductive reasoning to prove that the sum of three consecutive counting numbers is divisible by 3.

(F) Let the three consecutive counting numbers be represented by n, $n + 1$, and ______.

(G) The sum of the three consecutive counting numbers can be written as $3n +$ ______.

(H) The expression $3n + 3$ can be factored as $3(\boxed{})$.

(I) The expression $3(n + 1)$ is divisible by $\boxed{}$ for all values of n.

(J) Recall the conjecture in Step E: The sum of three consecutive counting numbers is divisible by 3.

Look at the steps in your deductive reasoning. Is the conjecture true or false? ______________

Reflect

1. **Discussion** A **counterexample** is an example that shows a conjecture to be false. Do you think that counterexamples are used mainly in inductive reasoning or in deductive reasoning?

2. Suppose you use deductive reasoning to show that an angle is not acute. Can you conclude that the angle is obtuse? Explain.

Explain 1 Introducing Proofs

A **conditional statement** is a statement that can be written in the form "If p, then q" where p is the *hypothesis* and q is the *conclusion*. For example, in the conditional statement "If $3x - 5 = 13$, then $x = 6$," the hypothesis is "$3x - 5 = 13$" and the conclusion is "$x = 6$."

Most of the Properties of Equality can be written as conditional statements. You can use these properties to solve an equation like "$3x - 5 = 13$" to prove that "$x = 6$."

Properties of Equality	
Addition Property of Equality	If $a = b$, then $a + c = b + c$.
Subtraction Property of Equality	If $a = b$, then $a - c = b - c$.
Multiplication Property of Equality	If $a = b$, then $ac = bc$.
Division Property of Equality	If $a = b$ and $c \neq 0$, then $\frac{a}{c} = \frac{b}{c}$.
Reflexive Property of Equality	$a = a$
Symmetric Property of Equality	If $a = b$, then $b = a$.
Transitive Property of Equality	If $a = b$ and $b = c$, then $a = c$.
Substitution Property of Equality	If $a = b$, then b can be substituted for a in any expression.

Example 1 **Use deductive reasoning to solve the equation. Use the Properties of Equality to justify each step.**

Ⓐ $\mathbf{14 = 3x - 4}$

$14 = 3x - 4$	
$18 = 3x$	Addition Property of Equality
$6 = x$	Division Property of Equality
$x = 6$	Symmetric Property of Equality

Ⓑ $\mathbf{9 = 17 - 4x}$

$9 = 17 - 4x$	
$\boxed{\quad} = -4x$	________ Property of Equality
$\boxed{\quad} = -4x$	
$\boxed{\quad} = x$	________ Property of Equality
$x = \boxed{\quad}$	________ Property of Equality

Your Turn

Write each statement as a conditional.

3. All zebras belong to the genus *Equus*.

4. The bill will pass if it gets two-thirds of the vote in the Senate.

5. Use deductive reasoning to solve the equation $3 - 4x = -5$.

6. Identify the Property of Equality that is used in each statement.

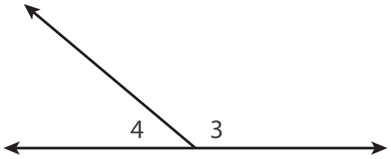
If $x = 2$, then $2x = 4$.	
$5 = 3a$; therefore, $3a = 5$.	
If $T = 4$, then $5T + 7$ equals 27.	
If $9 = 4x$ and $4x = m$, then $9 = m$.	

Explain 2 Using Postulates about Segments and Angles

Recall that two angles whose measures add up to 180° are called *supplementary angles*. The following theorem shows one type of supplementary angle pair, called a *linear pair*. A **linear pair** is a pair of adjacent angles whose non-common sides are opposite rays. You will prove this theorem in an exercise in this lesson.

The Linear Pair Theorem

If two angles form a linear pair, then they are supplementary.

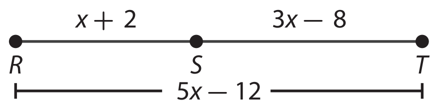


$m\angle 3 + m\angle 4 = 180°$

You can use the Linear Pair Theorem, as well as the Segment Addition Postulate and Angle Addition Postulate, to find missing values in expressions for segment lengths and angle measures.

Example 2 **Use a postulate or theorem to find the value of *x* in each figure.**

(A) Given: $RT = 5x - 12$



Use the Segment Addition Postulate.

$$RS + ST = RT$$

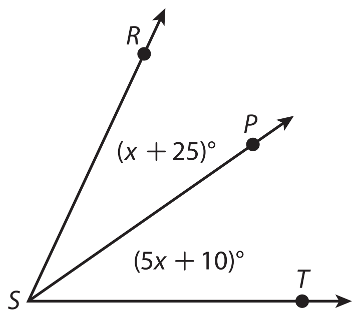
$$(x + 2) + (3x - 8) = 5x - 12$$

$$4x - 6 = 5x - 12$$

$$6 = x$$

$$x = 6$$

(B) Given: $m\angle RST = (15x - 10)°$



Use the ______________ Postulate.

$m\angle RST = m\angle \boxed{} + m\angle \boxed{}$

$(15x - 10)° = \boxed{}° + \boxed{}°$

$15x - 10 = \boxed{}$

$\boxed{}\, x = \boxed{}$

$x = \boxed{}$

Reflect

7. **Discussion** The Linear Pair Theorem uses the terms *opposite rays* as well as *adjacent angles*. Write a definition for each of these terms. Compare your definitions with your classmates.

Your Turn

8. Two angles *LMN* and *NMP* form a linear pair. The measure of $\angle LMN$ is twice the measure of $\angle NMP$. Find $m\angle LMN$.

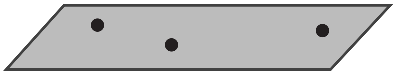
Explain 3 Using Postulates about Lines and Planes

Postulates about points, lines, and planes help describe geometric figures.

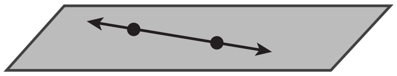
Postulates about Points, Lines, and Planes

Through any two points, there is exactly one line.

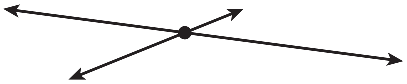
Through any three noncollinear points, there is exactly one plane containing them.



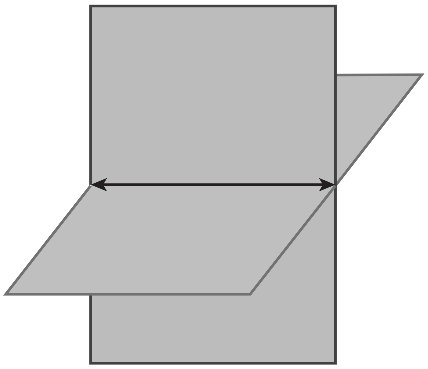
If two points lie in a plane, then the line containing those points lies in the plane.



If two lines intersect, then they intersect in exactly one point.

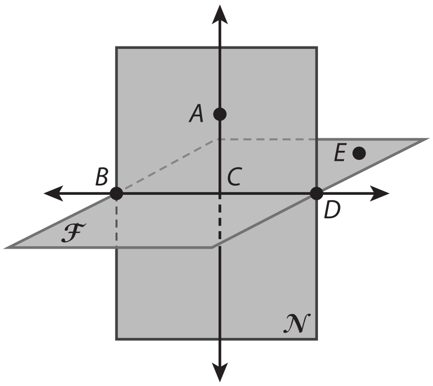


If two planes intersect, then they intersect in exactly one line.



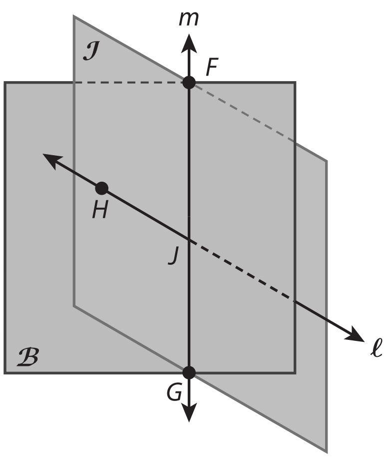
Example 3 **Use each figure to name the results described.**

(A)



Description	Example from the figure
the line of intersection of two planes	Possible answer: The two planes intersect in line *BD*.
the point of intersection of two lines	The line through point *A* and the line through point *B* intersect at point *C*.
three coplanar points	Possible answer: The points *B*, *D*, and *E* are coplanar.
three collinear points	The points *B*, *C*, and *D* are collinear.

(B)



Description	Example from the figure
the line of intersection of two planes	
the point of intersection of two lines	
three coplanar points	
three collinear points	

Reflect

9. Find examples in your classroom that illustrate the postulates of lines, planes, and points.

10. Draw a diagram of a plane with three collinear points and three points that are noncollinear.

Elaborate

11. What is the difference between a postulate and a definition? Give an example of each.

12. Give an example of a diagram illustrating the Segment Addition Postulate. Write the Segment Addition Postulate as a conditional statement.

13. Explain why photographers often use a tripod when taking pictures.

14. **Essential Question Check-In** What are some of the reasons you can give in proving a statement using deductive reasoning?

Evaluate: Homework and Practice

Personal Math Trainer

- Online Homework
- Hints and Help
- Extra Practice

Explain why the given conclusion uses inductive reasoning.

1. Find the next term in the pattern: 3, 6, 9.
The next term is 12 because the previous terms are multiples of 3.

2. $3 + 5 = 8$ and $13 + 5 = 18$, therefore the sum of two odd numbers is an even number.

3. My neighbor has two cats and both cats have yellow eyes.
Therefore when two cats live together, they will both have yellow eyes.

4. It always seems to rain the day after July 4th.

Give a counterexample for each conclusion.

5. If x is a prime number, then $x + 1$ is not a prime number.

6. The difference between two even numbers is positive.

7. Points A, B, and C are noncollinear, so therefore they are noncoplanar.

8. The square of a number is always greater than the number.

In Exercises 9–12 use deductive reasoning to write a conclusion.

9. If a number is divisible by 2, then it is even.
The number 14 is divisible by 2.

Use deductive reasoning to write a conclusion.

10. If two planes intersect, then they intersect in exactly one line.
Planes $\mathfrak{R}$ and $\mathfrak{I}$ intersect.

11. Through any three noncollinear points, there is exactly one plane containing them.
Points W, X, and Y are noncollinear.

12. If the sum of the digits of an integer is divisible by 3, then the number is divisible by 3.
The sum of the digits of 46,125 is 18, which is divisible by 3.

Identify the hypothesis and conclusion of each statement.

13. If the ball is red, then it will bounce higher.

14. If a plane contains two lines, then they are coplanar.

15. If the light does not come on, then the circuit is broken.

16. You must wear your jacket if it is cold outside.

Use a definition, postulate, or theorem to find the value of *x* in the figure described.

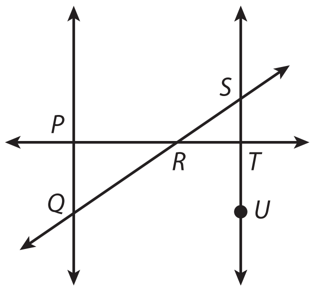
17. Point E is between points D and F. If $DE = x - 4$, $EF = 2x + 5$, and $DF = 4x - 8$, find x.

18. Y is the midpoint of $\overline{XZ}$. If $XZ = 8x - 2$ and $YZ = 2x + 1$, find x.

19. $\overrightarrow{SV}$ is an angle bisector of $\angle RST$. If $m\angle RSV = (3x + 5)°$ and $m\angle RST = (8x - 14)°$, find x.

20. $\angle ABC$ and $\angle CBD$ are a linear pair. If $m\angle ABC = m\angle CBD = 3x - 6$, find x.

Use the figure for Exercises 21 and 22.



21. Name three collinear points.

22. Name two linear pairs.

Explain the error in each statement.

23. Two planes can intersect in a single point.

24. Three points have to be collinear.

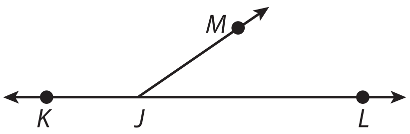
25. A line is contained in exactly one plane

26. If $x^2 = 25$, then $x = 5$.

H.O.T. Focus on Higher Order Thinking

27. **Analyze Relationships** What is the greatest number of intersection points 4 coplanar lines can have? What is the greatest number of planes determined by 4 noncollinear points? Draw diagrams to illustrate your answers.

28. **Justify Reasoning** Prove the Linear Pair Theorem.
Given: $\angle MJK$ and $\angle MJL$ are a linear pair of angles.
Prove: $\angle MJK$ and $\angle MJL$ are supplementary.



Complete the proof by writing the missing reasons. Choose from the following reasons.

Angle Addition Postulate

Definition of linear pair

Substitution Property of Equality

Given

Statements	Reasons
1. $\angle MJK$ and $\angle MJL$ are a linear pair.	**1.**
2. $\overrightarrow{JL}$ and $\overrightarrow{JK}$ are opposite rays.	**2.**
3. $\overrightarrow{JL}$ and $\overrightarrow{JK}$ form a straight line.	**3.** Definition of opposite rays
4. $m\angle LJK = 180°$	**4.** Definition of straight angle
5. $m\angle MJK + m\angle MJL = m\angle LJK$	**5.**
6. $m\angle MJK + m\angle MJL = 180°$	**6.**
7. $\angle MJK$ and $\angle MJL$ are supplementary.	**7.** Definition of supplementary angles

Lesson Performance Task

If two planes intersect, then they intersect in exactly one line.

Find a real-world example that illustrates the postulate above. Then formulate a conjecture by completing the following statement:

If three planes intersect, then ______________________________.

Justify your conjecture with real-world examples or a drawing.