Name _____ Class _____ Date _____

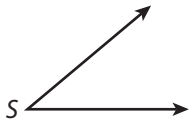
1.2 Angle Measures and Angle Bisectors

Essential Question: **How is measuring an angle similar to and different from measuring a line segment?**

Resource Locker

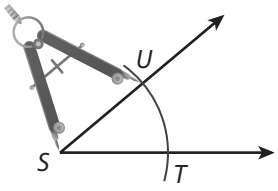
Explore Constructing a Copy of an Angle

Start with a point X and use a compass and straightedge to construct a copy of $\angle S$.



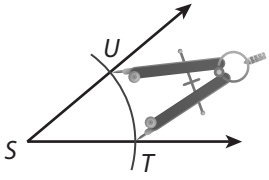
A Use a straightedge to draw a ray with endpoint X.

B Place the point of your compass on S and draw an arc that intersects both sides of the angle. Label the points of intersection T and U.



C Without adjusting the compass, place the point of the compass on X and draw an arc that intersects the ray. Label the intersection Y.

D Place the point of the compass on T and open it to the distance TU.



E Without adjusting the compass, place the point of the compass on Y and draw an arc. Label the intersection with the first arc Z.

F Use a straightedge to draw $\overrightarrow{XZ}$. $\angle X$ is a copy of $\angle S$.

Reflect

1. If you could place the angle you constructed on top of $\angle S$ so that $\overrightarrow{XY}$ coincides with $\overrightarrow{ST}$, what would be true about $\overrightarrow{XZ}$? Explain.

_____

_____

2. **Discussion** Is it possible to do the construction with a compass that is stuck open to a fixed distance? Why or why not?

_____

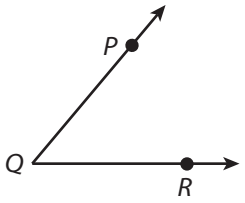
_____

Explain 1 Naming Angles and Parts of an Angle

An **angle** is a figure formed by two rays with the same endpoint.
The common endpoint is the **vertex** of the angle.
The rays are the **sides** of the angle.

Example 1 **Draw or name the given angle.**

Ⓐ $\angle PQR$

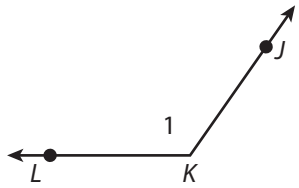


When an angle is named with three letters, the middle letter is the vertex. So, the vertex of angle $\angle PQR$ is point Q.

The sides of the angle are two rays with common endpoint Q. So, the sides of the angle are $\overrightarrow{QP}$ and $\overrightarrow{QR}$.

Draw and label the angle as shown.

Ⓑ



The vertex of the angle shown is point ☐. A name for the angle is $\angle$☐.

The vertex must be in the middle, so two more names for the angle are $\angle$☐☐☐ and $\angle$☐☐☐.

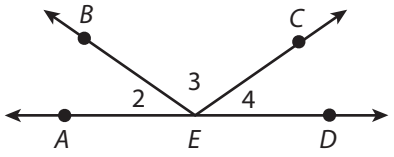
The angle is numbered, so another name is $\angle$☐.

Reflect

3. Without seeing a figure, is it possible to give another name for $\angle MKG$? If so, what is it? If not, why not?

Your Turn

Use the figure for 4–5.



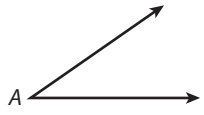
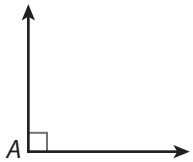
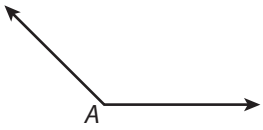
4. Name $\angle 2$ in as many different ways as possible.

5. Use a compass and straightedge to copy $\angle BEC$.

Explain 2 Measuring Angles

The distance around a circular arc is undefined until a measurement unit is chosen. **Degrees** (°) are a common measurement unit for circular arcs. There are 360° in a circle, so an angle that measures 1° is $\frac{1}{360}$ of a circle. The measure of an angle is written m∠A or m∠PQR.

You can classify angles by their measures.

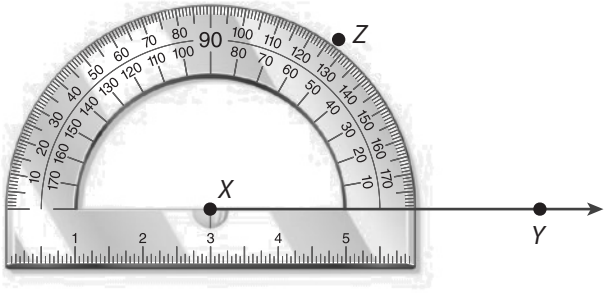
Classifying Angles			
Acute Angle	**Right Angle**	**Obtuse Angle**	**Straight Angle**
			
$0° < m\angle A < 90°$	$m\angle A = 90°$	$90° < m\angle A < 180°$	$m\angle A = 180°$

Example 2 **Use a protractor to draw an angle with the given measure.**

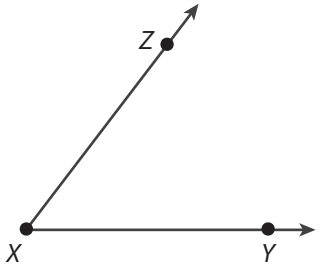
(A) 53°

Step 1 Use a straightedge to draw a ray, $\overrightarrow{XY}$.

Step 2 Place your protractor on point X as shown. Locate the point along the edge of the protractor that corresponds to 53°. Make a mark at this location and label it point Z.



Step 3 Draw $\overrightarrow{XZ}$. $m\angle ZXY = 53°$.



Ⓑ 138°

Step 1 Use a straightedge to draw a ray, $\overrightarrow{AB}$.

Step 2 Place your protractor on point A so that $\overrightarrow{AB}$ is at zero.

Step 3 Locate the point along the edge of the protractor that corresponds to 138°. Make a mark at this location and label it point C.

Step 4 Draw $\overrightarrow{AC}$. $m\angle CAB = 138°$.

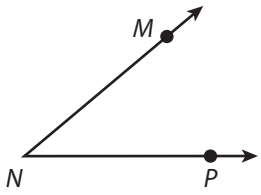
Reflect

6. Explain how you can use a protractor to check that the angle you constructed in the Explore is a copy of the given angle.

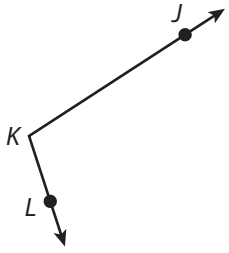
Your Turn

Each angle can be found in the rigid frame of the bicycle. Use a protractor to find each measure.

7.

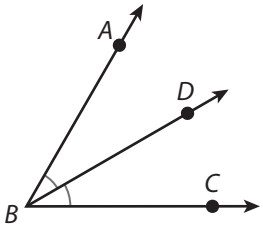


8.



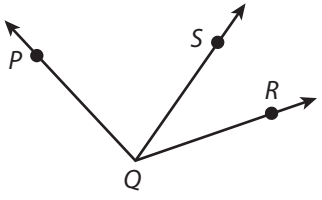
Explain 3 Constructing an Angle Bisector

An **angle bisector** is a ray that divides an angle into two angles that both have the same measure. In the figure, $\overrightarrow{BD}$ bisects $\angle ABC$, so $m\angle ABD = m\angle CBD$. The arcs in the figure show equal angle measures.



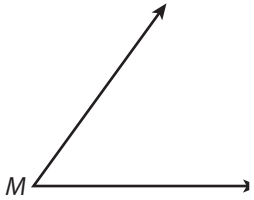
Postulate 2: Angle Addition Postulate

If S is in the interior of $\angle PQR$, then $m\angle PQR = m\angle PQS + m\angle SQR$.

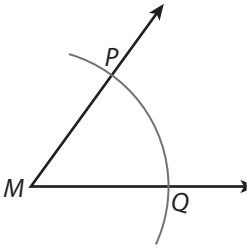


Example 3 **Use a compass and straightedge to construct the bisector of the given angle. Check that the measure of each of the new angles is one-half the measure of the given angle.**

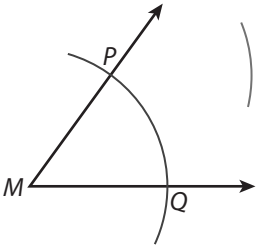
Ⓐ



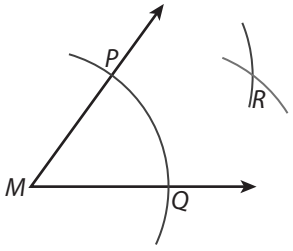
Step 1 Place the point of your compass on point *M*. Draw an arc that intersects both sides of the angle. Label the points of intersection *P* and *Q*.



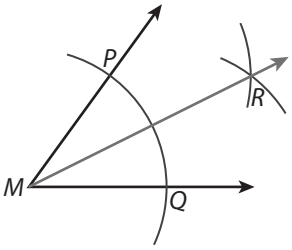
Step 2 Place the point of the compass on *P* and draw an arc in the interior of the angle.



Step 3 Without adjusting the compass, place the point of the compass on *Q* and draw an arc that intersects the last arc you drew. Label the intersection of the arcs *R*.

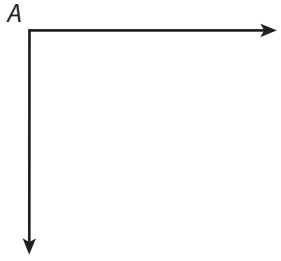


Step 4 Use a straightedge to draw $\overrightarrow{MR}$.



Step 5 Measure with a protractor to confirm that $m\angle PMR = m\angle QMR = \frac{1}{2}m\angle PMQ$.
$27° = 27° = \frac{1}{2}(54°)$✓

Ⓑ



Step 1 Draw an arc centered at *A* that intersects both sides of the angle. Label the points of intersection *B* and *C*.

Step 2 Draw an arc centered at *B* in the interior of the angle.

Step 3 Without adjusting the compass, draw an arc centered at *C* that intersects the last arc you drew. Label the intersection of the arcs *D*.

Step 4 Draw $\overrightarrow{AD}$.

Step 5 Check that $m\angle BAD = m\angle CAD = \frac{1}{2}m\angle BAC$.

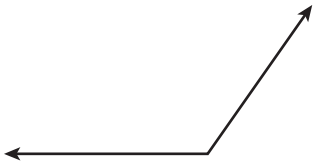
Reflect

9. **Discussion** Explain how you could use paper folding to construct the bisector of an angle.

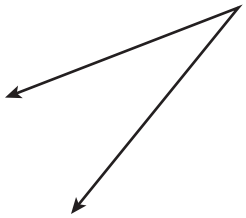
Your Turn

Use a compass and straightedge to construct the bisector of the given angle. Check that the measure of each of the new angles is one-half the measure of the given angle.

10.



11.



Elaborate

12. What is the relationship between a segment bisector and an angle bisector?

13. When you copy an angle, do the lengths of the segments you draw to represent the two rays affect whether the angles have the same measure? Explain.

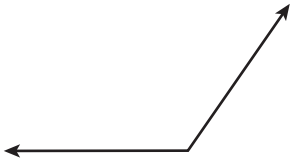
14. **Essential Question Check-In** Many protractors have two sets of degree measures around the edge. When you measure an angle, how do you know which of the two measures to use?

Evaluate: Homework and Practice

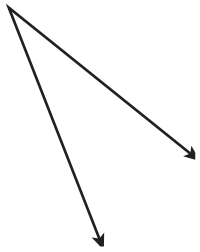
- Online Homework
- Hints and Help
- Extra Practice

Use a compass and straightedge to construct a copy of each angle.

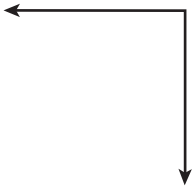
1.



2.



3.



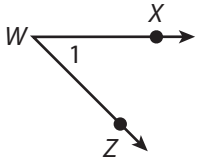
Draw an angle with the given name.

4. $\angle JWT$

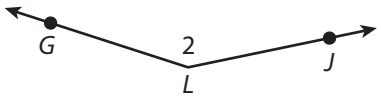
5. $\angle NBQ$

Name each angle in as many different ways as possible.

6.



7.



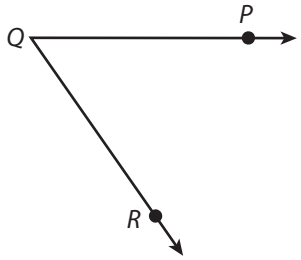
Use a protractor to draw an angle with the given measure.

8. 19°

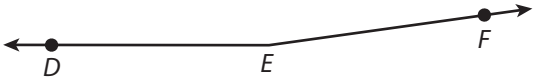
9. 100°

Use a protractor to find the measure of each angle.

10.

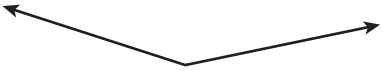


11.

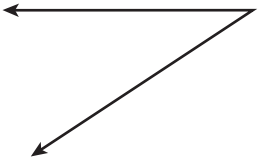


Use a compass and straightedge to construct the bisector of the given angle. Check that the measure of each of the new angles is one-half the measure of the given angle.

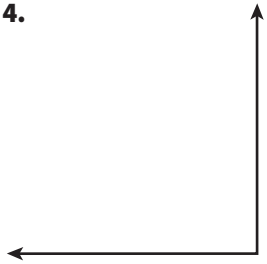
12.



13.



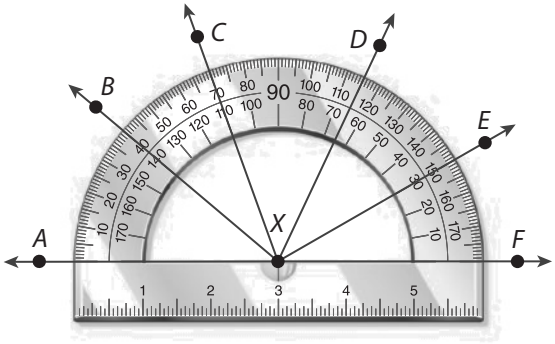
14.



Use the Angle Addition Postulate to find the measure of each angle.

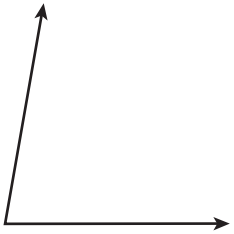
15. $\angle BXC$

16. $\angle BXE$

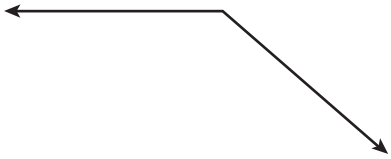


Use a compass and straightedge to copy each angle onto a separate piece of paper. Then use paper folding to construct the angle bisector.

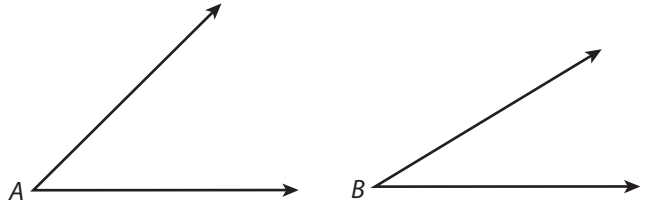
17.



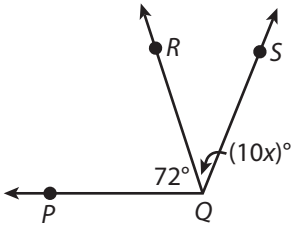
18.



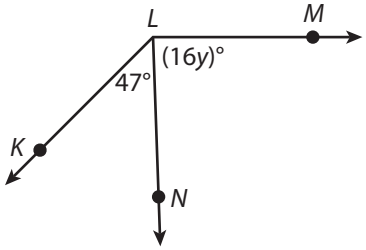
19. Use a compass and straightedge to construct an angle whose measure is $m\angle A + m\angle B$. Use a protractor to check your construction.



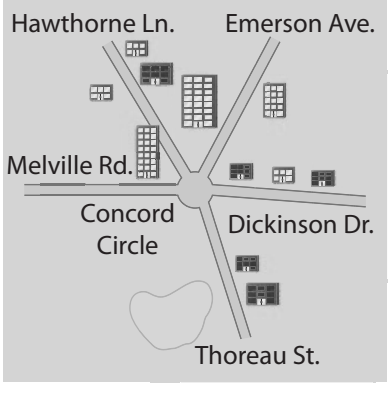
20. Find the value of x, given that $m\angle PQS = 112°$.



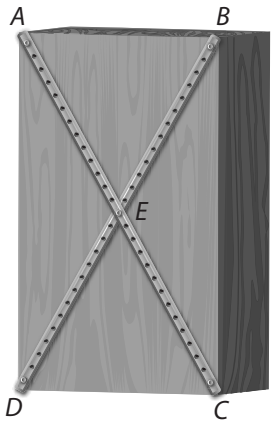
21. Find the value of y, given that $m\angle KLM = 135°$.



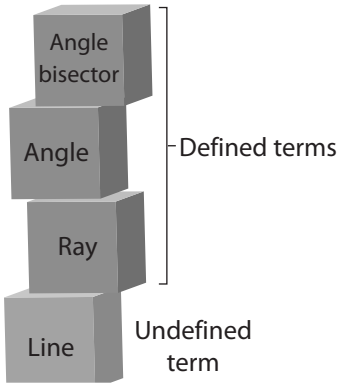
22. **Multi-Step** The figure shows a map of five streets that meet at Concord Circle. The measure of the angle formed by Melville Road and Emerson Avenue is 118°. The measure of the angle formed by Emerson Avenue and Thoreau Street is 134°. Hawthorne Lane bisects the angle formed by Melville Road and Emerson Avenue. Dickinson Drive bisects the angle formed by Emerson Avenue and Thoreau Street. What is the measure of the angle formed by Hawthorne Lane and Dickinson Drive? Explain your reasoning.



23. **Represent Real-World Problems** A carpenter is building a rectangular bookcase with diagonal braces across the back, as shown. The carpenter knows that $\angle ADC$ is a right angle and that $m\angle BDC$ is 32° greater than $m\angle ADB$. Write and solve an equation to find $m\angle BDC$ and $m\angle ADB$.

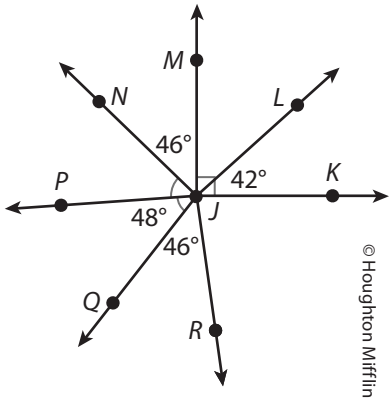


24. Describe the relationships among the four terms.

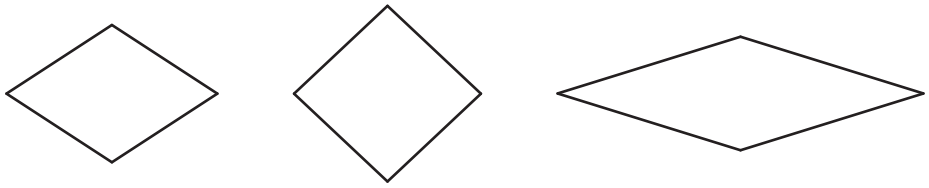


25. Determine whether each of the following pairs of angles have equal measures. Select the correct answer for each lettered part.

A. $\angle KJL$ and $\angle LJM$	◯ Yes	◯ No
B. $\angle MJP$ and $\angle PJR$	◯ Yes	◯ No
C. $\angle LJP$ and $\angle NJR$	◯ Yes	◯ No
D. $\angle MJK$ and $\angle PJR$	◯ Yes	◯ No
E. $\angle KJR$ and $\angle MJP$	◯ Yes	◯ No



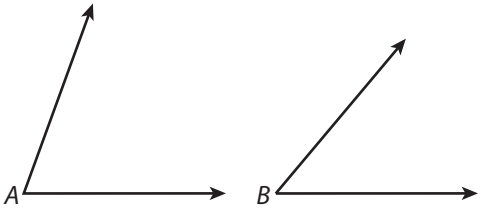
26. **Make a Conjecture** A rhombus is a quadrilateral with four sides of equal length. Use a compass and straightedge to bisect one of the angles in each of the rhombuses shown. Then use your results to state a conjecture.



H.O.T. Focus on Higher Order Thinking

27. **What If?** What happens if you perform the steps for constructing an angle bisector when the given angle is a straight angle? Does the construction still work? If so, explain why and show a sample construction. If not, explain why not.

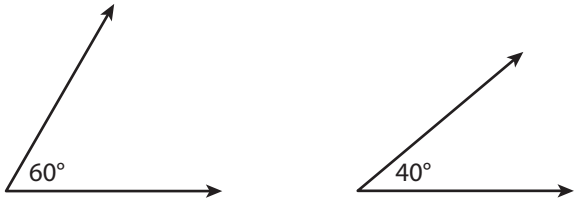
28. **Critical Thinking** Use a compass and straightedge to construct an angle whose measure is $m\angle A - m\angle B$. Use a protractor to check your construction.



29. **Communicate Mathematical Ideas** Explain the steps for using a compass and straightedge to construct an angle with $\frac{1}{4}$ the measure of a given angle. Then draw an angle and show the construction.

Lesson Performance Task

A store sells custom-made stands for tablet computers. When an order comes in, the customer specifies the angle at which the stand should hold the tablet. Then an employee bends a piece of aluminum to the correct angle to make the stand. The figure shows the templates that the employee uses to make a 60° stand and a 40° stand.



The store receives an order for a 50° stand. The employee does not have a template for a 50° stand and does not have a protractor. Can the employee use the existing templates and a compass and straightedge to make a template for a 50° stand? If so, explain how and show the steps the employee should use. If not, explain why not.