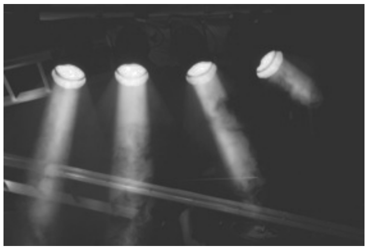
Name ______________________ Class ____________ Date ________

1.1 Segment Length and Midpoints

Essential Question: How do you draw a segment and measure its length?

Resource Locker

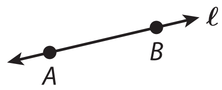
Explore Exploring Basic Geometric Terms



In geometry, some of the names of figures and other terms will already be familiar from everyday life. For example, a *ray* like a beam of light from a spotlight is both a familiar word and a geometric figure with a mathematical definition.

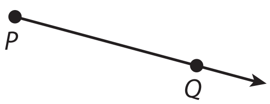
The most basic figures in geometry are *undefined terms*, which cannot be defined using other figures. The terms *point*, *line*, and *plane* are undefined terms. Although they do not have formal definitions, they can be described as shown in the table.

Undefined Terms

Term	Geometric Figure	Ways to Name the Figure
A **point** is a specific location. It has no dimension and is represented by a dot.		point P
A **line** is a connected straight path. It has no thickness and it continues forever in both directions.		line ℓ, line AB, line BA, $\overleftrightarrow{AB}$, or $\overleftrightarrow{BA}$
A **plane** is a flat surface. It has no thickness and it extends forever in all directions.		plane $\mathcal{R}$ or plane XYZ

In geometry, the word *between* is another undefined term, but its meaning is understood from its use in everyday language. You can use undefined terms as building blocks to write definitions for defined terms, as shown in the table.

Defined Terms

Term	Geometric Figure	Ways to Name the Figure
A **line segment** (or *segment*) is a portion of a line consisting of two points (called **endpoints**) and all points between them.		segment CD, segment DC, $\overline{CD}$, or $\overline{DC}$
A **ray** is a portion of a line that starts at a point (the *endpoint*) and continues forever in one direction.		ray PQ or $\overrightarrow{PQ}$

You can use points to sketch lines, segments, rays, and planes.

(A) Draw two points J and K. Then draw a line through them. (Remember that a line shows arrows at both ends.)

(B) Draw two points J and K again. This time, draw the line segment with endpoints J and K.

(C) Draw a point K again and draw a ray from endpoint K. Plot a point J along the ray.

(D) Draw three points J, K, and M so that they are not all on the same line. Then draw the plane that contains the three points. (You might also put a script letter such as $\mathcal{B}$ on your plane.)

(E) Give a name for each of the figures you drew. Then use a circle to choose whether the type of figure is an undefined term or a defined term.

Point	______________	undefined term/defined term
Line	______________	undefined term/defined term
Segment	______________	undefined term/defined term
Ray	______________	undefined term/defined term
Plane	______________	undefined term/defined term

Reflect

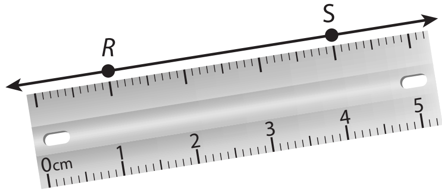
1. In Step C, would $\overrightarrow{JK}$ be the same ray as $\overrightarrow{KJ}$? Why or why not?

2. In Step D, when you name a plane using 3 letters, does the order of the letters matter?

3. **Discussion** If $\overleftrightarrow{PQ}$ and $\overleftrightarrow{RS}$ are different names for the same line, what must be true about points P, Q, R, and S?

Explain 1 Constructing a Copy of a Line Segment

The distance along a line is undefined until a unit distance, such as 1 inch or 1 centimeter, is chosen. You can use a ruler to find the distance between two points on a line. The distance is the absolute value of the difference of the numbers on the ruler that correspond to the two points. This distance is the length of the segment determined by the points.



In the figure, the length of $\overline{RS}$, written RS (or SR), is the distance between R and S.

$$RS = |4 - 1| = |3| = 3 \text{ cm} \qquad \text{or} \qquad SR = |1 - 4| = |-3| = 3 \text{ cm}$$

Points that lie in the same plane are **coplanar**. Lines that lie in the same plane but do not intersect are **parallel**. Points that lie on the same line are **collinear**. The *Segment Addition Postulate* is a statement about collinear points. A **postulate** is a statement that is accepted as true without proof. Like undefined terms, postulates are building blocks of geometry.

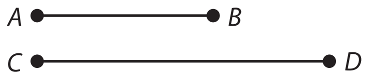
Postulate 1: Segment Addition Postulate

Let A, B, and C be collinear points. If B is between A and C, then $AB + BC = AC$.

A *construction* is a geometric drawing that produces an accurate representation without using numbers or measures. One type of construction uses only a compass and straightedge. You can construct a line segment whose length is equal to that of a given segment using these tools along with the Segment Addition Postulate.

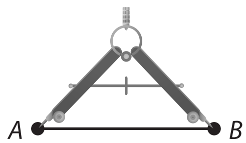
Example 1 **Use a compass and straightedge to construct a segment whose length is $AB + CD$.**

(A)



Step 1 Use the straightedge to draw a long line segment. Label an endpoint X. (See the art drawn in Step 4.)

Step 2 To copy segment AB, open the compass to the distance AB.



Step 3 Place the compass point on X, and draw an arc. Label the point Y where the arc and the segment intersect.

Step 4 To copy segment CD, open the compass to the distance CD. Place the compass point on Y, and draw an arc. Label the point Z where this second arc and the segment intersect.

$\overline{XZ}$ is the required segment.

(B) 

Step 1 Use the straightedge to draw a long line segment. Label an endpoint *X*.

Step 2 To copy segment *AB*, open the compass to the distance *AB*.

Step 3 Place the compass point on *X*, and draw an arc. Label the point *Y* where the arc and the segment intersect.

Step 4 To copy segment *CD*, open the compass to the distance *CD*. Place the compass point on Y, and draw an arc. Label the point *Z* where this second arc and the segment intersect.

Reflect

4. **Discussion** Look at the line and ruler above Example 1. Why does it not matter whether you find the distance from *R* to *S* or the distance from *S* to *R*?

5. In Part B, how can you check that the length of $\overline{YZ}$ is the same as the length of $\overline{CD}$?

Your Turn

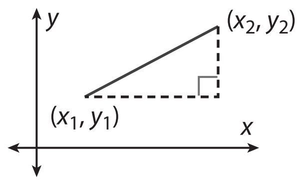
6. Use a ruler to draw a segment *PQ* that is 2 inches long. Then use your compass and straightedge to construct a segment *MN* with the same length as $\overline{PQ}$.

Explain 2 Using the Distance Formula on the Coordinate Plane

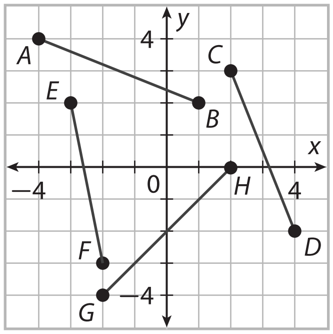
The Pythagorean Theorem states that $a^2 + b^2 = c^2$, where *a* and *b* are the lengths of the legs of a right triangle and *c* is the length of the hypotenuse. You can use the Distance Formula to apply the Pythagorean Theorem to find the distance between points on the coordinate plane.

The Distance Formula

The distance between two points (x_1, y_1) and (x_2, y_2) on the coordinate plane is $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.



Example 2 **Determine whether the given segments have the same length. Justify your answer.**



Ⓐ $\overline{AB}$ and $\overline{CD}$

Write the coordinates of the endpoints. $A(-4, 4)$, $B(1, 2)$, $C(2, 3)$, $D(4, -2)$

Find the length of $\overline{AB}$. $AB = \sqrt{(1 - (-4))^2 + (2 - 4)^2}$

Simplify the expression. $= \sqrt{5^2 + (-2)^2} = \sqrt{29}$

Find the length of $\overline{CD}$. $CD = \sqrt{(4 - 2)^2 + (-2 - 3)^2}$

Simplify the expression. $= \sqrt{2^2 + (-5)^2} = \sqrt{29}$

So, $AB = CD = \sqrt{29}$. Therefore, $\overline{AB}$ and $\overline{CD}$ have the same length.

Ⓑ $\overline{EF}$ and $\overline{GH}$

Write the coordinates of the endpoints. $E(-3, 2)$, $F(\square, \square)$, $G(-2, -4)$, $H(\square, \square)$

Find the length of $\overline{EF}$. $EF = \sqrt{(\square - (-3))^2 + (\square - 2)^2}$

Simplify the expression. $= \sqrt{(\square)^2 + (\square)^2} = \sqrt{\square}$

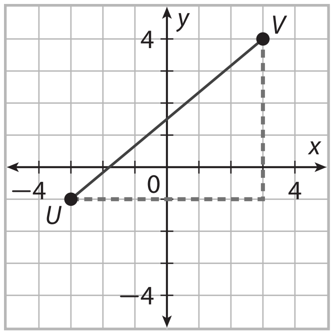
Find the length of $\overline{GH}$. $GH = \sqrt{(\square - (-2))^2 + (\square - (-4))^2}$

Simplify the expression. $= \sqrt{(\square)^2 + (\square)^2} = \sqrt{\square}$

So, ________. Therefore, ____

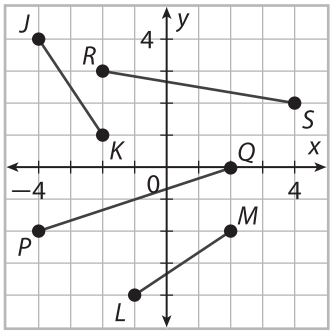
Reflect

7. Consider how the Distance Formula is related to the Pythagorean Theorem. To use the Distance Formula to find the distance from $U(-3, -1)$ to $V(3, 4)$, you write $UV = \sqrt{\left(3 - (-3)\right)^2 + \left(4 - (-1)\right)^2}$. Explain how $\left(3 - (-3)\right)$ in the Distance Formula is related to a in the Pythagorean Theorem and how $\left(4 - (-1)\right)$ in the Distance Formula is related to b in the Pythagorean Theorem.



Your Turn

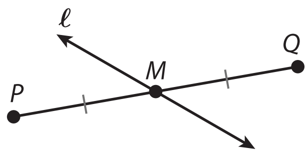
8. Determine whether $\overline{JK}$ and $\overline{LM}$ have the same length. Justify your answer.



Explain 3 Finding a Midpoint

The **midpoint** of a line segment is the point that divides the segment into two segments that have the same length. A line, ray, or other figure that passes through the midpoint of a segment is a **segment bisector**.

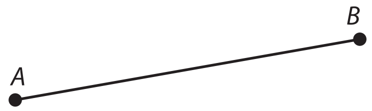
In the figure, the tick marks show that $PM = MQ$. Therefore, M is the midpoint of $\overline{PQ}$ and line ℓ bisects $\overline{PQ}$.



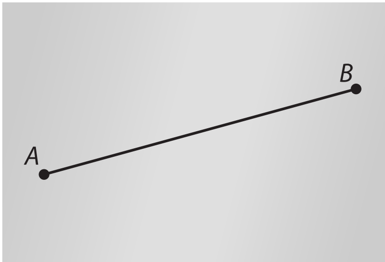
You can use paper folding as a method to construct a bisector of a given segment and locate the midpoint of the segment.

Example 3 **Use paper folding to construct a bisector of each segment.**

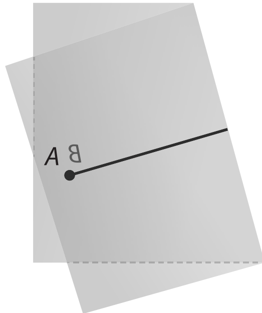
(A)



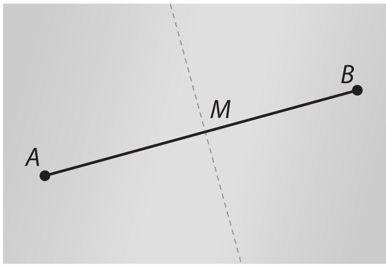
Step 1 Use a compass and straightedge to copy $\overline{AB}$ on a piece of paper.



Step 2 Fold the paper so that point *B* is on top of point *A*.

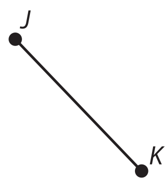


Step 3 Open the paper. Label the point where the crease intersects the segment as point *M*.



Point *M* is the midpoint of $\overline{AB}$ and the crease is a bisector of $\overline{AB}$.

(B) **Step 1** Use a compass and straightedge to copy $\overline{JK}$ on a piece of paper.



Step 2 Fold the paper so that point *K* is on top of point ____________.

Step 3 Open the paper. Label the point where the crease intersects the segment as point *N*.

Point *N* is the ____________ of $\overline{JK}$ and the crease is a ____________ of $\overline{JK}$.

Step 4 Make a sketch of your paper folding construction or attach your folded piece of paper.

Reflect

9. Explain how you could use paper folding to divide a line segment into four segments of equal length.

Your Turn

10. Explain how to use a ruler to check your construction in Part B.

____

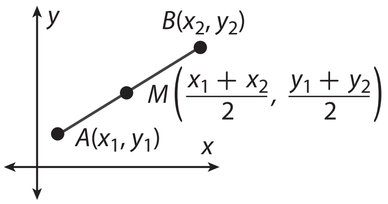
____

Explain 4 Finding Midpoints on the Coordinate Plane

You can use the *Midpoint Formula* to find the midpoint of a segment on the coordinate plane.

The Midpoint Formula

The midpoint M of $\overline{AB}$ with endpoints $A(x_1, y_1)$ and $B(x_2, y_2)$ is given by $M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.



Example 4 **Show that each statement is true.**

(A) If $\overline{PQ}$ has endpoints $P(-4, 1)$ and $Q(2, -3)$, then the midpoint M of $\overline{PQ}$ lies in Quadrant III.

Use the Midpoint Formula to find the midpoint of $\overline{PQ}$. Substitute the coordinates, then simplify.

$$M\left(\frac{-4 + 2}{2}, \frac{1 + (-3)}{2}\right) = M(-1, -1)$$

So M lies in Quadrant III, since the x- and y-coordinates are both negative.

(B) If $\overline{RS}$ has endpoints $R(3, 5)$ and $S(-3, -1)$, then the midpoint M of $\overline{RS}$ lies on the y-axis.

Use the Midpoint Formula to find the midpoint of $\overline{RS}$. Substitute the coordinates, then simplify.

$$M\left(\frac{3 + \square}{2}, \frac{5 + \square}{2}\right) = M(\square, \square)$$

So M lies on the y-axis, since ______________________.

Your Turn

Show that each statement is true.

11. If $\overline{AB}$ has endpoints $A(6, -3)$ and $B(-6, 3)$, then the midpoint M of $\overline{AB}$ is the origin.

12. If $\overline{JK}$ has endpoints $J(7, 0)$ and $K(-5, -4)$, then the midpoint M of $\overline{JK}$ lies in Quadrant IV.

Elaborate

13. Explain why the Distance Formula is not needed to find the distance between two points that lie on a horizontal or vertical line.

14. When you use the Distance Formula, does the order in which you subtract the x- and y-coordinates matter? Explain.

15. When you use the Midpoint Formula, can you take either point as (x_1, y_1) or (x_2, y_2)? Why or why not?

16. **Essential Question Check-In** What is the difference between finding the length of a segment that is drawn on a sheet of blank paper and a segment that is drawn on a coordinate plane?

Evaluate: Homework and Practice

Personal Math Trainer

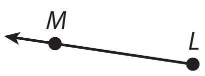
- Online Homework
- Hints and Help
- Extra Practice

Write the term that is suggested by each figure or description. Then state whether the term is an undefined term or a defined term.

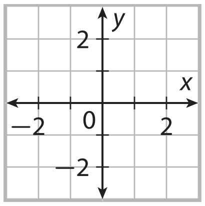
1.

2.

3.



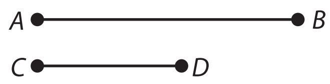
4.



Use a compass and straightedge to construct a segment whose length is $AB + CD$.

5.

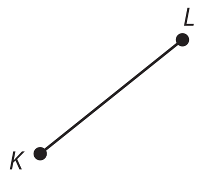
6.



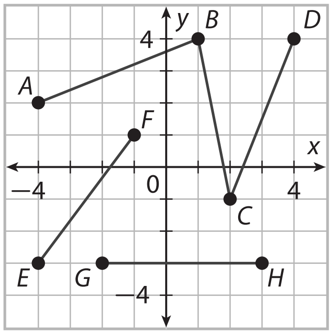
Copy each segment onto a sheet of paper. Then use paper folding to construct a bisector of the segment.

7.

8.



Determine whether the given segments have the same length. Justify your answer.



9. $\overline{AB}$ and $\overline{BC}$

10. $\overline{EF}$ and $\overline{GH}$

11. $\overline{AB}$ and $\overline{CD}$

12. $\overline{BC}$ and $\overline{EF}$

Show that each statement is true.

13. If $\overline{DE}$ has endpoints $D(-1, 6)$ and $E(3, -2)$, then the midpoint M of $\overline{DE}$ lies in Quadrant I.

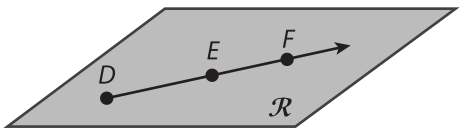
14. If $\overline{ST}$ has endpoints $S(-6, -1)$ and $T(0, 1)$, then the midpoint M of $\overline{ST}$ lies in on the x-axis.

Show that each statement is true.

15. If $\overline{JK}$ has endpoints $J(-2, 3)$ and $K(6, 5)$, and $\overline{LN}$ has endpoints $L(0, 7)$ and $N(4, 1)$, then $\overline{JK}$ and $\overline{LN}$ have the same midpoint.

16. If $\overline{GH}$ has endpoints $G(-8, 1)$ and $H(4, 5)$, then the midpoint M of $\overline{GH}$ lies on the line $y = -x + 1$.

Use the figure for Exercises 17 and 18.



17. Name two different rays in the figure.

18. Name three different segments in the figure.

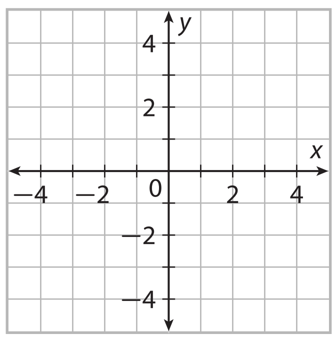
Sketch each figure.

19. two rays that form a straight line and that intersect at point P

20. two line segments that both have a midpoint at point M

21. Draw and label a line segment, $\overline{JK}$, that is 3 inches long. Use a ruler to draw and label the midpoint M of the segment.

22. Draw the segment PQ with endpoints $P(-2, -1)$ and $Q(2, 4)$ on the coordinate plane. Then find the length and midpoint of $\overline{PQ}$.



23. Multi-Step The sign shows distances from a rest stop to the exits for different towns along a straight section of highway. The state department of transportation is planning to build a new exit to Freestone at the midpoint of the exits for Roseville and Edgewood. When the new exit is built, what will be the distance from the exit for Midtown to the exit for Freestone?

24. On a town map, each unit of the coordinate plane represents 1 mile. Three branches of a bank are located at $A(-3, 1)$, $B(2, 3)$, and $C(4, -1)$. A bank employee drives from Branch A to Branch B and then drives halfway to Branch C before getting stuck in traffic. What is the minimum total distance the employee may have driven before getting stuck in traffic? Round to the nearest tenth of a mile.

25. A city planner designs a park that is a quadrilateral with vertices at $J(-3, 1)$, $K(1, 3)$, $L(5, -1)$, and $M(-1, -3)$. There is an entrance to the park at the midpoint of each side of the park. A straight path connects each entrance to the entrance on the opposite side. Assuming each unit of the coordinate plane represents 10 meters, what is the total length of the paths to the nearest meter?

26. Communicate Mathematical Ideas A video game designer places an anthill at the origin of a coordinate plane. A red ant leaves the anthill and moves along a straight line to $(1, 1)$, while a black ant leaves the anthill and moves along a straight line to $(-1, -1)$. Next, the red ant moves to $(2, 2)$, while the black ant moves to $(-2, -2)$. Then the red ant moves to $(3, 3)$, while the black ant moves to $(-3, -3)$, and so on. Explain why the red ant and the black ant are always the same distance from the anthill.

27. Which of the following points are more than 5 units from the point $P(-2, -2)$? Select all that apply.

A. $A(1, 2)$

B. $B(3, -1)$

C. $C(2, -4)$

D. $D(-6, -6)$

E. $E(-5, 1)$

H.O.T. Focus on Higher Order Thinking

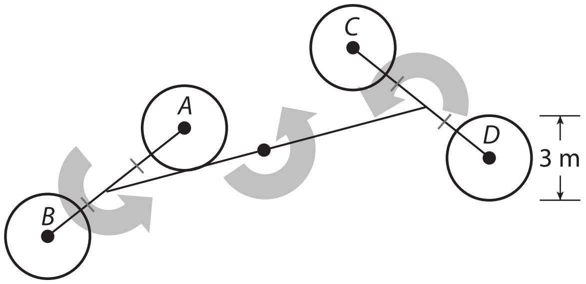
28. Analyze Relationships Use a compass and straightedge to construct a segment whose length is $AB - CD$. Use a ruler to check your construction.

29. Critical Thinking Point M is the midpoint of $\overline{AB}$. The coordinates of point A are $(-8, 3)$ and the coordinates of M are $(-2, 1)$. What are the coordinates of point B?

30. Make a Conjecture Use a compass and straightedge to copy $\overline{AB}$ so that one endpoint of the copy is at point X. Then repeat the process three more times, making three different copies of $\overline{AB}$ that have an endpoint at point X. Make a conjecture about the set of all possible copies of $\overline{AB}$ that have an endpoint at point X.

Lesson Performance Task

A carnival ride consists of four circular cars—*A*, *B*, *C*, and *D*—each of which spins about a point at its center. The center points of cars *A* and *B* are attached by a straight beam, as are the center points of cars *C* and *D*. The two beams are attached at their midpoints by a rotating arm. The figure shows how the beams and arm can rotate.



A plan for the ride uses a coordinate plane in which each unit represents one meter. In the plan, the center of car *A* is $(-6, -1)$, the center of car *B* is $(-2, -3)$, the center of car *C* is $(3, 4)$, and the center of car *D* is $(5, 0)$. Each car has a diameter of 3 meters.

The manager of the carnival wants to place a fence around the ride. Describe the shape and dimensions of a fence that will be appropriate to enclose the ride. Justify your answer.