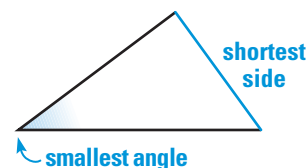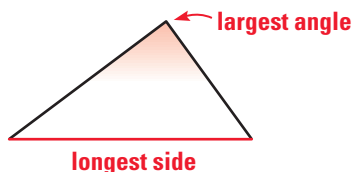## 4.7 Triangle Inequalities

Goal

Use triangle measurements to decide which side is longest and which angle is largest.

The diagrams below show a relationship between the longest and shortest sides of a triangle and the largest and smallest angles.

largest angle

longest side

shortest side

smallest angle

THEOREMS 4.10 and 4.11

Theorem 4.10

Words If one side of a triangle is longer than another side, then the angle opposite the longer side is larger than the angle opposite the shorter side.

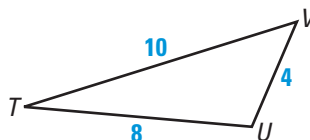**Symbols** If $BC > AB$, then $m\angle A > m\angle C$.
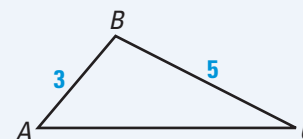
Theorem 4.11

Words If one angle of a triangle is larger than another angle, then the side opposite the larger angle is longer than the side opposite the smaller angle.

Symbols If $m\angle D > m\angle E$, then $EF > DF$.
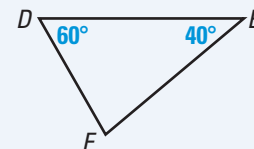
EXAMPLE 1 Order Angle Measures

Name the angles from largest to smallest.

Solution

$TV > TU$, so $m\angle U > m\angle V$. Also, $TU > UV$, so $m\angle V > m\angle T$.

ANSWER ▶ The order of the angles from largest to smallest is $\angle U, \angle V, \angle T$.
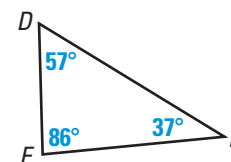
EXAMPLE 2 Order Side Lengths

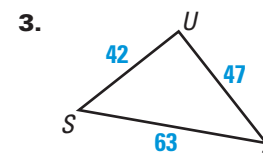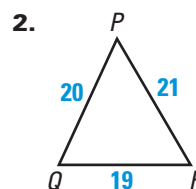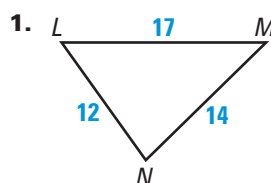Name the sides from longest to shortest.

Solution

$m\angle E > m\angle D$, so $DF > FE$. Also, $m\angle D > m\angle F$, so $FE > DE$.
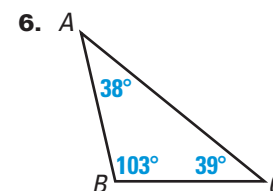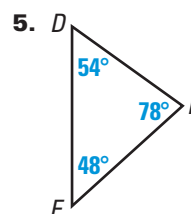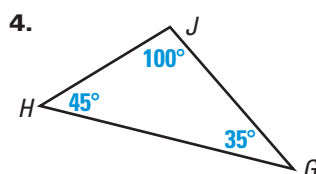
ANSWER ▶ The order of the sides from longest to shortest is $\overline{DF}, \overline{FE}, \overline{DE}$.

Checkpoint ✔ Order Angle Measures and Side Lengths

Name the angles from largest to smallest.

1.   2.   3.

Name the sides from longest to shortest.
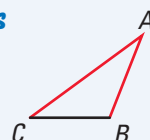
4.   5.   6.

Segments of a Triangle Not every group of three segments can be used to form a triangle. The lengths of the segments must have the following relationship.
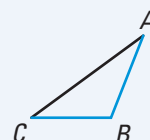
THEOREM 4.12

Triangle Inequality

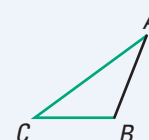**Words** The sum of the lengths of any two sides of a triangle is greater than the length of the third side.

Symbols
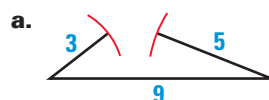
$CA + AB > BC$ $AB + BC > CA$ $BC + CA > AB$

4.7   *Triangle Inequalities*   **213**

EXAMPLE 3 *Use the Triangle Inequality*

Can the side lengths form a triangle? Explain.

a. 3, 5, 9   **b.** 3, 5, 8   **c.** 3, 5, 7

Solution

a.
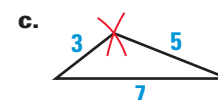
3   5
9

These lengths do
not form a triangle,
because $3 + 5 < 9$.

b.

3   5
8

These lengths do
not form a triangle,
because $3 + 5 = 8$.

c.

3   5
7

These lengths do
form a triangle,
because $3 + 5 > 7$,
$3 + 7 > 5$, and
$5 + 7 > 3$.

Checkpoint ✓ *Use the Triangle Inequality*

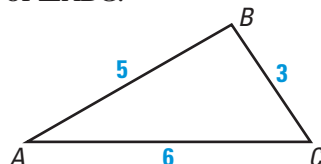**Can the side lengths form a triangle? Explain.**

7. 5, 7, 13   **8.** 6, 9, 12   **9.** 10, 15, 25

4.7 Exercises

Guided Practice

Vocabulary Check

1. Complete the statement: The symbol ">" means __?__ , and the
symbol "<" means __?__ .

Skill Check

2. Name the smallest angle
of △ABC.

B
5   3
A   6   C

3. Name the longest side
of △ABC.

B
80°
40°   60°
A   C
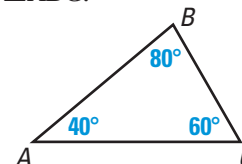
In Exercises 4 and 5, use the figure shown at the right.

4. Name the smallest and largest angles of △DEF.

5. Name the shortest and longest sides of △DEF.
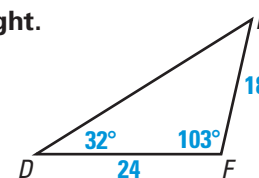
E
18
32°   103°
D   24   F

Can the side lengths form a triangle? Explain.

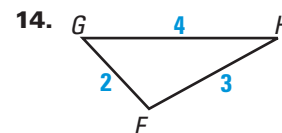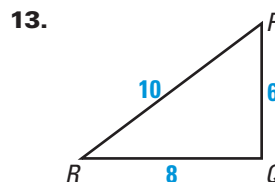**6.** 1, 2, 3   **7.** 6, 10, 15   **8.** 12, 16, 30

9. 7, 8, 13   **10.** 4, 9, 16   **11.** 5, 5, 10

Practice and Applications
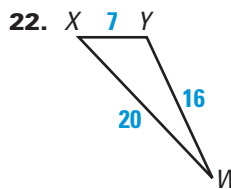
Comparing Angle Measures Name the smallest and largest angles of the triangle.

12.

13.

14.
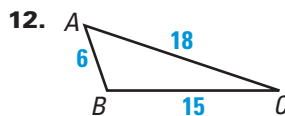
Comparing Side Lengths Name the shortest and longest sides of the triangle.

15.

16.

17.
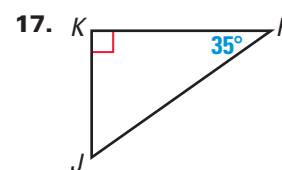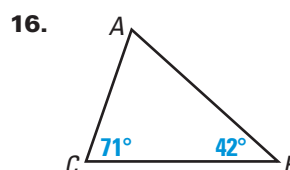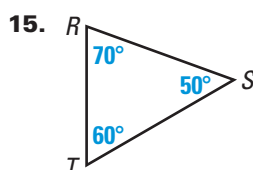
Ordering Angles Name the angles from largest to smallest.

18.

19.

20.

21.

22.

23.
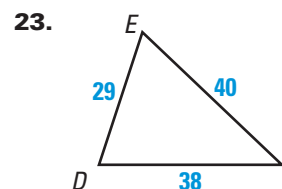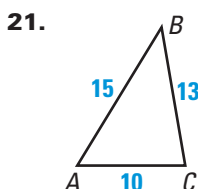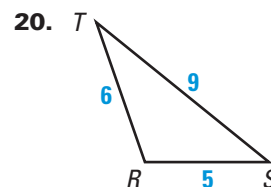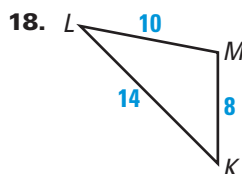
Link to
Kitchen Design

KITCHEN TRIANGLES
For ease of movement among appliances, the perimeter of an ideal kitchen triangle should be less than 22 feet and more than 15 feet.

Design In Exercises 24 and 25, use the following information.
The term "kitchen triangle" refers to the imaginary triangle formed by the refrigerator, the sink, and the stove. The distances shown are measured in feet.

24. What is wrong with the labels on the kitchen triangle?

25. Can a kitchen triangle have the following side lengths: 9 feet, 3 feet, and 5 feet? Explain why or why not.
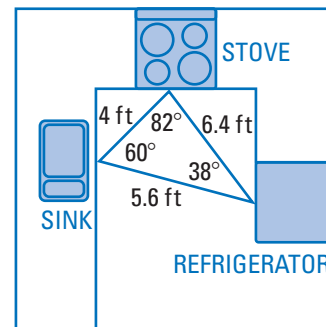
Ordering Sides Name the sides from longest to shortest.

26.

27.

28.

29.

30.

31.
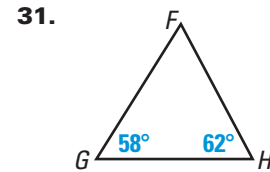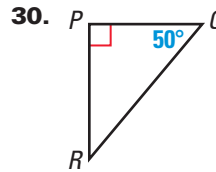
Error Analysis Explain why the side lengths given with the triangles are not correct.

32.

33.

EXAMPLE   *Use the Triangle Inequality*

Is it possible to draw a triangle that has side lengths of 4, 5, and 6? If so, draw the triangle.
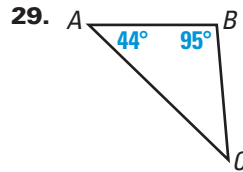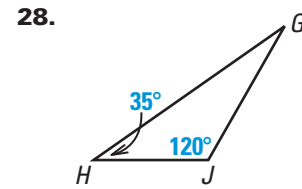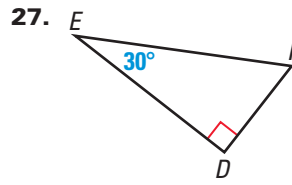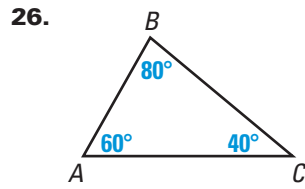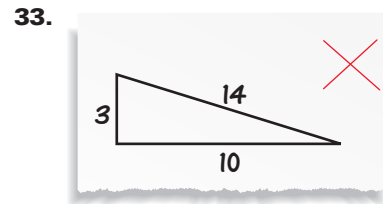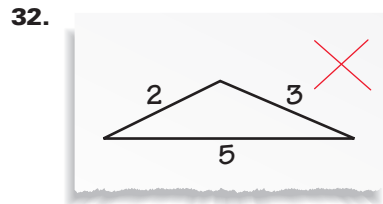
Solution

Yes, these side lengths satisfy the Triangle Inequality: $4 + 5 > 6$, $5 + 6 > 4$, and $4 + 6 > 5$. So, it is possible to draw the triangle, as shown below.

❶ Mark $\overline{AB}$ of length 4 cm on a line. Then draw an arc of radius 5 cm with center at B.
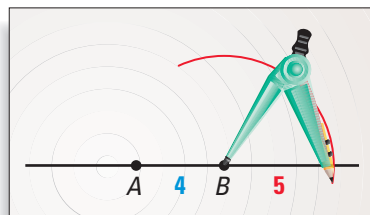
❷ Draw an arc of radius 6 cm with center at A. Mark the intersection of the two arcs as C. $\triangle ABC$ has side lengths of 4 cm, 5 cm, and 6 cm.
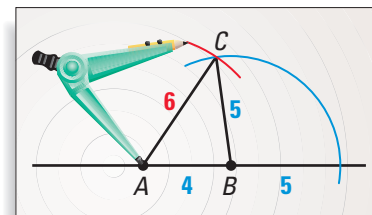
Using the Triangle Inequality Determine whether it is possible to draw a triangle with the given side lengths. If so, draw the triangle.

34. 4, 7, 10   **35.** 10, 12, 22   **36.** 17, 9, 30

216   Chapter 4   *Triangle Relationships*

**Visualize It!** Sketch a triangle and label it with the given angle measures and side lengths.

37. Angles: 59°, 46°, 75°
Sides: 13 cm, 9.7 cm, 11.5 cm

**38.** Angles: 135°, 15°, 30°
Sides: 7.1 cm, 2.6 cm, 5 cm

39. Taking a Shortcut Suppose you are walking south on the sidewalk of Pine Street. When you reach Pleasant Street, you cut across the empty lot to go to the corner of Oak Hill Avenue and Union Street. Explain why this route is shorter than staying on the sidewalks.
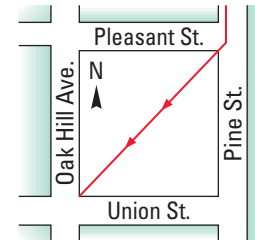
40. **You be the Judge** Suppose you are camping. You decide to hike 4.6 miles northwest and then turn and hike 1.8 miles east. Your friend tells you that you are about one and a half miles from camp. Is your friend right? Explain why or why not.

Logical Reasoning In Exercises 41–43, use the figure shown and the given information.
By adjusting the length of the boom lines from *A* to *B*, the operator of the crane shown can raise and lower the boom. Suppose the mast $\overline{AC}$ is 50 feet long and the boom $\overline{BC}$ is 100 feet long.
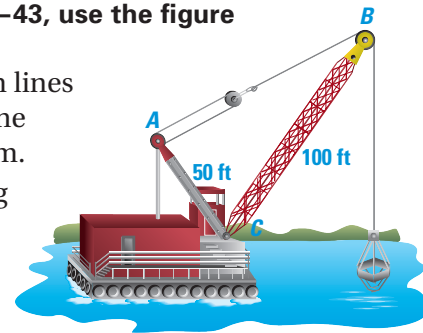
41. Is the boom *raised* or *lowered* when the boom lines are shortened?

42. *AB* must be less than __?__ feet.

43. As the boom is raised or lowered, is ∠*ACB* ever larger than ∠*BAC*? Explain.

Standardized Test Practice

44. Multi-Step Problem You are given an 18-inch piece of wire. You want to bend the wire to form a triangle so that the length of each side is a whole number.

a. Sketch four possible isosceles triangles and label each side length.

b. Sketch a possible acute scalene triangle.

c. Sketch a possible obtuse scalene triangle.

**d.** List three combinations of segment lengths with a sum of 18 that will not produce triangles.

Mixed Review

Identifying Parts of a Triangle In Exercises 45–48, use the figure shown to complete the statement. *(Lessons 4.1, 4.3, 4.4)*

45. __?__ is the hypotenuse of △*RST*.

46. In △*RST*, $\overline{RT}$ is the side __?__ ∠*RST*.

47. The legs of △*RST* are __?__ and __?__.

48. __?__ is the base of △*RST*.
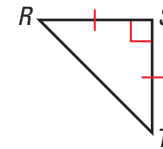
Finding Measures Find the measure of the numbered angle. *(Lesson 4.2)*

49.

50.

51.
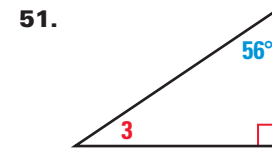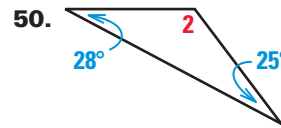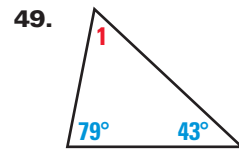
Algebra Skills

Solving Proportions Solve the proportion. *(Skills Review, p. 660)*

52. $\dfrac{x}{5} = \dfrac{6}{15}$

53. $\dfrac{18}{3} = \dfrac{6}{x}$

54. $\dfrac{x}{7} = \dfrac{6}{7}$

55. $\dfrac{27}{21} = \dfrac{9}{x}$

56. $\dfrac{5}{8} = \dfrac{x}{72}$

57. $\dfrac{7}{10} = \dfrac{49}{x}$

Quiz 3

Use the side lengths to classify the triangle as *acute, right,* or *obtuse*. *(Lesson 4.5)*
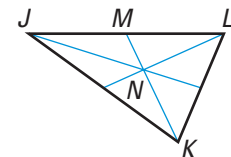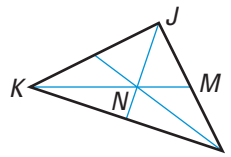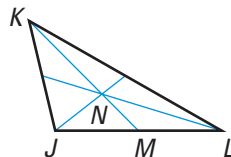
1. 6, 11, 14

2. 15, 7, 16

3. 18, 80, 82

N is the centroid of △*JKL*. Find *KN* and *MN*. *(Lesson 4.6)*

4. *KM* = 6

5. *KM* = 39

6. *KM* = 60

Name the sides from longest to shortest. *(Lesson 4.7)*

7.

8.

9.