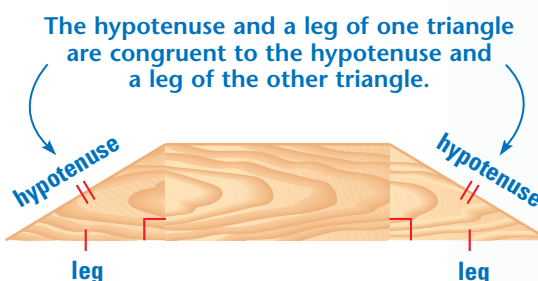# 5.4 Hypotenuse-Leg Congruence Theorem: HL

Goal
Use the HL Congruence Theorem and summarize congruence postulates and theorems.

Key Words
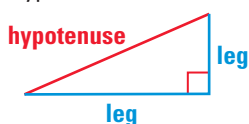• hypotenuse p. 192
• leg of a right triangle p. 192

The triangles that make up the skateboard ramp below are right triangles.

The hypotenuse and a leg of one triangle are congruent to the hypotenuse and a leg of the other triangle.

hypotenuse   hypotenuse

leg   leg

Student Help

VOCABULARY TIP
Remember that the longest side of a right triangle is called the hypotenuse.

hypotenuse   leg
leg

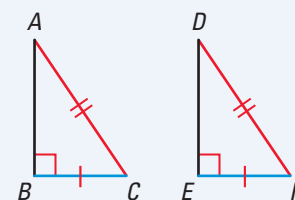### THEOREM 5.2

Hypotenuse-Leg Congruence Theorem (HL)

Words If the hypotenuse and a leg of a right triangle are congruent to the hypotenuse and a leg of a second right triangle, then the two triangles are congruent.

Symbols If △ABC and △DEF are right triangles, and

H $\overline{AC} \cong \overline{DF}$, and

L $\overline{BC} \cong \overline{EF}$,

then △ABC ≅ △DEF.

EXAMPLE 1 Determine When To Use HL

Is it possible to show that △JGH ≅ △HKJ using the HL Congruence Theorem? Explain your reasoning.
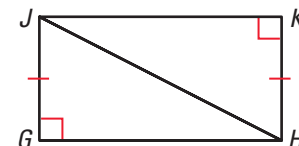
Solution

In the diagram, you are given that △JGH and △HKJ are right triangles.

By the Reflexive Property, you know $\overline{JH} \cong \overline{JH}$ (hypotenuse) and you are given that $\overline{JG} \cong \overline{HK}$ (leg). You can use the HL Congruence Theorem to show that △JGH ≅ △HKJ.
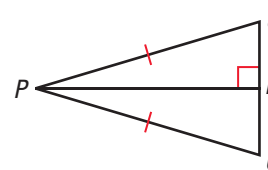
EXAMPLE **2** **Use the HL Congruence Theorem**

Use the diagram to prove that $\triangle PRQ \cong \triangle PRS$.

Solution

Given ▶ $\overline{PR} \perp \overline{SQ}$
$\overline{PQ} \cong \overline{PS}$

Prove ▶ $\triangle PRQ \cong \triangle PRS$

| Statements	Reasons
1. $\overline{PR} \perp \overline{SQ}$	1. Given
2. $\angle PRQ$ and $\angle PRS$ are right ∠s.	2. $\perp$ lines form right angles.
3. $\triangle PRQ$ and $\triangle PRS$ are right triangles.	3. Definition of right triangle
H 4. $\overline{PQ} \cong \overline{PS}$	4. Given
L 5. $\overline{PR} \cong \overline{PR}$	5. Reflexive Prop. of Congruence
6. $\triangle PRQ \cong \triangle PRS$	6. HL Congruence Theorem

SUMMARY **TRIANGLE CONGRUENCE POSTULATES AND THEOREMS**

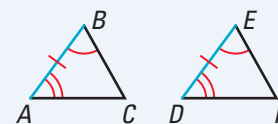You have studied five ways to prove that $\triangle ABC \cong \triangle DEF$.

SSS
Side $\overline{AB} \cong \overline{DE}$
Side $\overline{AC} \cong \overline{DF}$
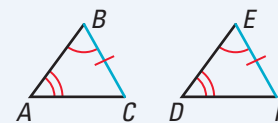**S**ide $\overline{BC} \cong \overline{EF}$
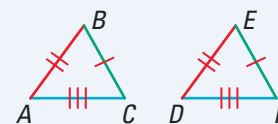
SAS
Side $\overline{AB} \cong \overline{DE}$
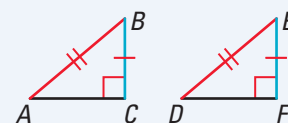**A**ngle $\angle B \cong \angle E$
Side $\overline{BC} \cong \overline{EF}$
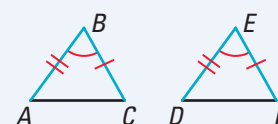
ASA
Angle $\angle A \cong \angle D$
Side $\overline{AB} \cong \overline{DE}$
Angle $\angle B \cong \angle E$

AAS
Angle $\angle A \cong \angle D$
Angle $\angle B \cong \angle E$
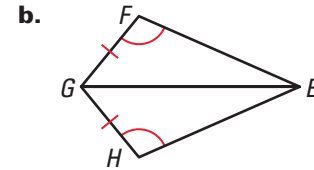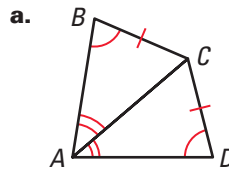**S**ide $\overline{BC} \cong \overline{EF}$

HL
$\triangle ABC$ and $\triangle DEF$ are right triangles.
Hypotenuse $\overline{AB} \cong \overline{DE}$
Leg $\overline{BC} \cong \overline{EF}$

EXAMPLE **3** **Decide Whether Triangles are Congruent**

Does the diagram give enough information to show that the triangles are congruent? If so, state the postulate or theorem you would use.

a.

b.

Solution

a. From the diagram, you know that $\angle BAC \cong \angle DAC$, $\angle B \cong \angle D$, and $\overline{BC} \cong \overline{DC}$. You can use the AAS Congruence Theorem to show that $\triangle BAC \cong \triangle DAC$.

b. From the diagram, you know that $\overline{FG} \cong \overline{HG}$, $\overline{EG} \cong \overline{EG}$, and $\angle EFG \cong \angle EHG$. Because the congruent angles are not included between the congruent sides, you cannot show that $\triangle FGE \cong \triangle HGE$.
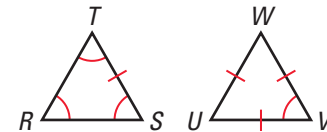
Student Help

STUDY TIP
There is no SSA Congruence Theorem or Postulate, so you cannot conclude that the triangles in Example 3(b) are congruent.

EXAMPLE **4** **Prove Triangles are Congruent**

Use the information in the diagram to prove that $\triangle RST \cong \triangle UVW$.

Solution

	Statements	Reasons
A	**1.** $\angle S \cong \angle V$	**1.** Given
S	**2.** $\overline{ST} \cong \overline{VW}$	**2.** Given
	3. $\triangle UVW$ is equilateral.	**3.** Definition of equilateral triangle
	4. $\angle V \cong \angle W$	**4.** Equilateral triangles are equiangular.
	5. $\angle T \cong \angle V$	**5.** Given
A	**6.** $\angle T \cong \angle W$	**6.** Transitive Prop. of Congruence
	7. $\triangle RST \cong \triangle UVW$	**7.** ASA Congruence Postulate

Checkpoint ✓ *Decide Whether Triangles are Congruent*

Does the diagram give enough information to show that the triangles are congruent? If so, state the postulate or theorem you would use.
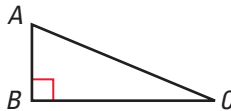
1.

2.

3.

5.4 *Hypotenuse-Leg Congruence Theorem: HL* **259**
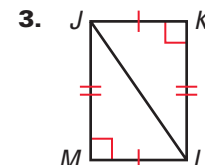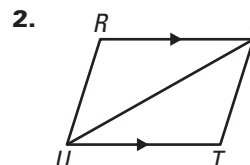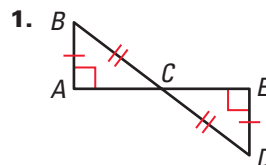
5.4 Exercises

Guided Practice

Vocabulary Check

Tell whether the segment is a *leg* or the *hypotenuse* of the right triangle.

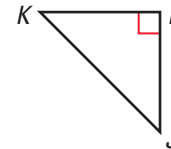**1.** $\overline{AC}$

2. $\overline{BC}$

3. $\overline{AB}$
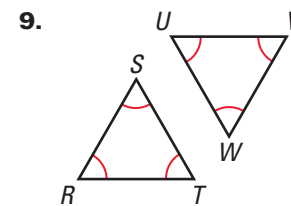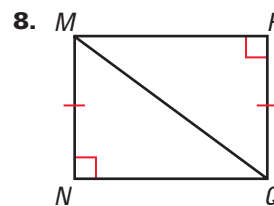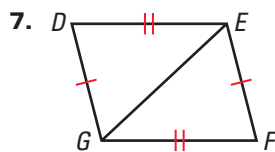
4. $\overline{KL}$

5. $\overline{KJ}$

6. $\overline{JL}$

Skill Check

Determine whether you are given enough information to show that the triangles are congruent. Explain your answer.
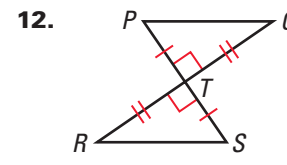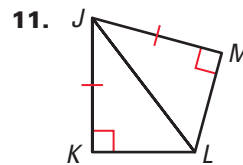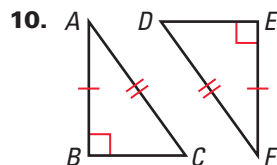
7.

8.

9.

Practice and Applications

**Extra Practice**

See p. 683.

HL Congruence Theorem Determine whether you can use the HL Congruence Theorem to show that the triangles are congruent. Explain your reasoning.

10.

11.

12.

Landscaping To support a tree, you attach wires from the trunk of the tree to stakes in the ground as shown below.

13. What information do you need to know in order to use the HL Congruence Theorem to show that $\triangle JKL \cong \triangle MKL$?

14. Suppose K is the midpoint of $\overline{JM}$. Name a theorem or postulate you could use to show that $\triangle JKL \cong \triangle MKL$. Explain your reasoning.
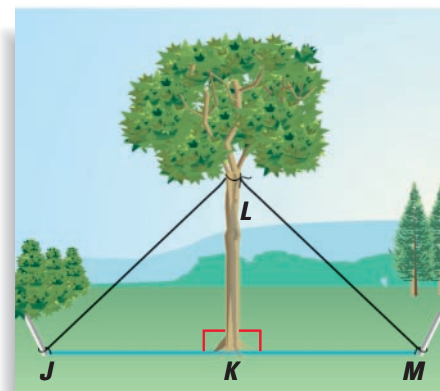
Homework Help

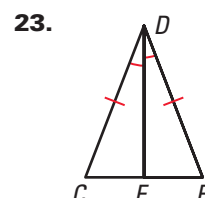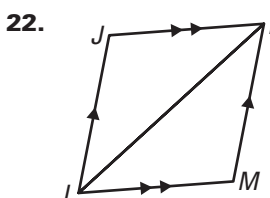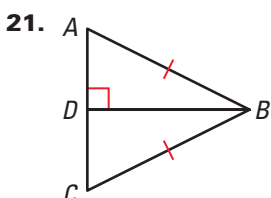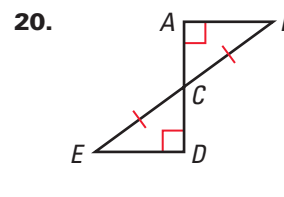**Example 1:** Exs. 10–13, 24, 29–31
Example 2: Ex. 32
Example 3: Exs. 13–24, 29–31
Example 4: Ex. 32

You be the Judge Decide whether enough information is given to show that the triangles are congruent. If so, state the theorem or postulate you would use. Explain your reasoning.

15.

16.

17.

18.

19.

20.

21.

22.

23.
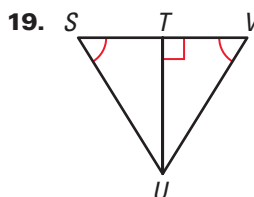
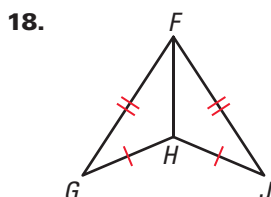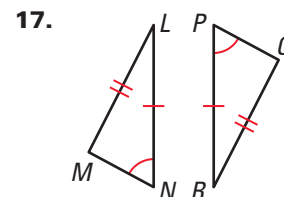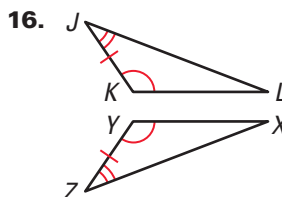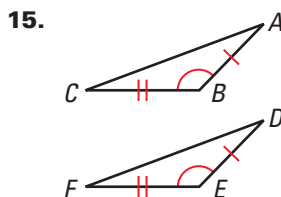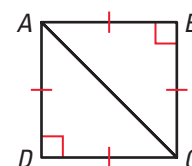**24. Logical Reasoning** Three students are given the diagram shown at the right and asked which congruence postulate or theorem can be used to show that $\triangle ABC \cong \triangle CDA$. Explain why all three answers are correct.

> **Meghan**
>
> $\triangle ABC \cong \triangle CDA$ by the SSS Congruence Postulate.

> **Keith**
>
> $\triangle ABC \cong \triangle CDA$ by the SAS Congruence Postulate.

> **Angie**
>
> $\triangle ABC \cong \triangle CDA$ by the Hypotenuse-Leg Congruence Theorem.

Student Help
CLASSZONE.COM

HOMEWORK HELP
Extra help with problem solving in Exs. 25–28 is at classzone.com

Visualize It! Use the given information to sketch $\triangle LMN$ and $\triangle STU$. Mark the triangles with the given information.
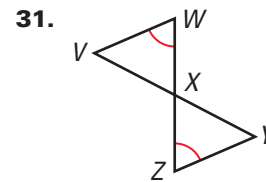
25. $\angle LNM$ and $\angle TUS$ are right angles. $\overline{LM} \cong \overline{TS}$, $\overline{TU} \cong \overline{LN}$

26. $\overline{LM} \perp \overline{MN}$, $\overline{ST} \perp \overline{TU}$, $\overline{LM} \cong \overline{ST}$, $\overline{LN} \cong \overline{SU}$

27. $\overline{LM} \perp \overline{MN}$, $\overline{ST} \perp \overline{TU}$, $\overline{LM} \cong \overline{NM} \cong \overline{UT} \cong \overline{ST}$

28. $\overline{ML} \perp \overline{LN}$, $\overline{TS} \perp \overline{SU}$, $\overline{LN} \cong \overline{SU}$, $\overline{MN} \cong \overline{TU}$
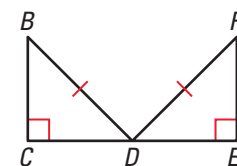
Missing Information What congruence is needed to show that the triangles are congruent? Using that congruence, tell which theorem or postulate you would use to show that the triangles are congruent.

29.

30.

31.
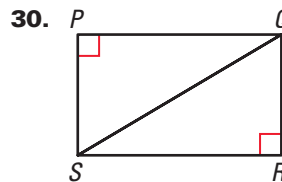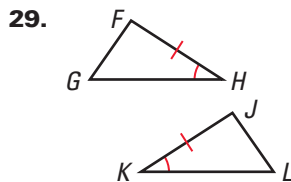
32. Logical Reasoning Fill in the missing statements and reasons.

Given ▶ $\overline{BD} \cong \overline{FD}$
D is the midpoint of $\overline{CE}$.
$\angle BCD$ and $\angle FED$ are right angles.

Prove ▶ $\triangle BCD \cong \triangle FED$

| Statements	Reasons
1. $\overline{BD} \cong \overline{FD}$	**1.** _____?_____
2. _____?_____	**2.** Given
3. _____?_____	**3.** Definition of midpoint
4. $\angle BCD$ and $\angle FED$ are right angles.	**4.** _____?_____
5. _____?_____ are right triangles.	**5.** Definition of right triangle
6. $\triangle BCD \cong \triangle FED$	**6.** _____?_____

Standardized Test Practice

33. Multi-Step Problem The diagram below is a plan showing the light created by two spotlights. Both spotlights are the same distance from the stage.
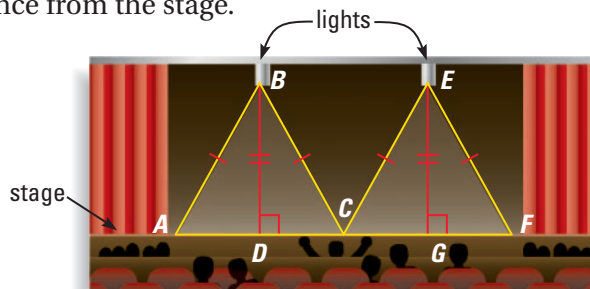
a. Show that $\triangle ABD \cong \triangle CBD$. Tell what theorem or postulate you use and explain your reasoning.

b. Is there another way to show that $\triangle ABD \cong \triangle CBD$? If so, tell how. Explain your reasoning.

c. Are all four right triangles in the diagram congruent? Explain your reasoning.

Mixed Review

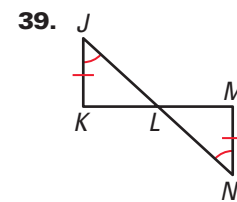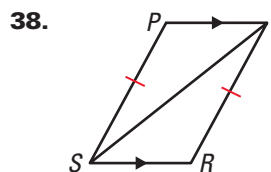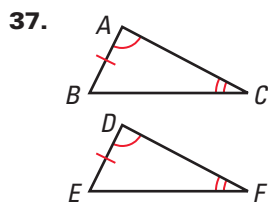**Parallel Lines** Find $m\angle 1$ and $m\angle 2$. Explain your reasoning. *(Lesson 3.4)*

34.

110°

1 2

35.

82°   **1**

2

36.

1 57°

2
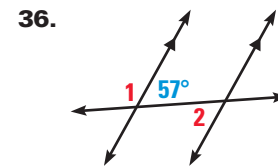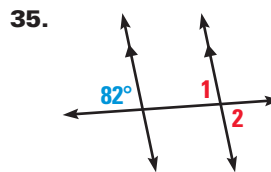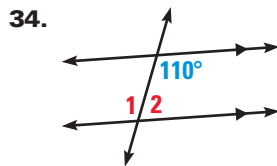
Showing Congruence Decide whether enough information is given to show that the triangles are congruent. If so, state the theorem or postulate you would use. Explain your reasoning. *(Lessons 5.2, 5.3)*

37.

A

B   C

D

E   F

38.

P   Q

S   R

39.

J

K   L   M

N

Algebra Skills

Evaluating Expressions Evaluate. *(Skills Review, p. 670)*

40. $2 \cdot 4 + 5$

41. $10 - 5 \cdot 2$

42. $3 + 4^2 - 11$

43. $7 \cdot 2 + 6 \cdot 3$
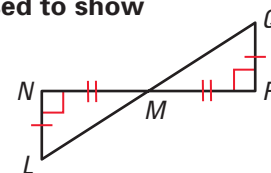
44. $3 \cdot 5 - 2 \cdot 7$

45. $5^2 - 10 \cdot 2$

Quiz 2

Tell whether the theorem or postulate can be used to show that $\triangle LMN \cong \triangle QMP$. *(Lessons 5.3, 5.4)*

1. ASA

2. AAS

3. HL

4. SSS

Q

N   P
M

L

Tell whether enough information is given to show that the triangles are congruent. If so, tell which theorem or postulate you would use. Explain your reasoning. *(Lessons 5.3, 5.4)*

5.

A   B

D   C

6.

J

Q   L   K

R   P

7.

S

T   U

V

8.

E   F

H   G

9.

R   S

T

V   U

10.

J   K

M   L

5.4 *Hypotenuse-Leg Congruence Theorem: HL*   **263**