Name _____ Date _____ Class _____

LESSON 9-4

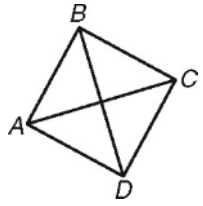
Conditions for Rectangles, Rhombuses, and Squares

Practice and Problem Solving: A/B

Fill in the blanks to complete each theorem.

1. If one pair of consecutive sides of a parallelogram are congruent, then the parallelogram is a _____.
2. If the diagonals of a parallelogram are _____, then the parallelogram is a rhombus.
3. If the _____ of a parallelogram are congruent, then the parallelogram is a rectangle.
4. If one diagonal of a parallelogram bisects a pair of opposite angles, then the parallelogram is a _____.
5. If one angle of a parallelogram is a right angle, then the parallelogram is a _____.

Use the figure for Problems 6–7. Determine whether each conclusion is valid. If not, tell what additional information is needed to make it valid.



6. **Given:** $\overline{AC}$ and $\overline{BD}$ bisect each other. $\overline{AC} \cong \overline{BD}$

   Conclusion: *ABCD* is a square.

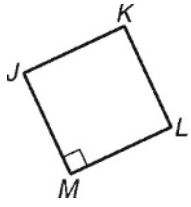
   _____

7. **Given:** $\overline{AC} \perp \overline{BD}$, $\overline{AB} \cong \overline{BC}$

   Conclusion: *ABCD* is a rhombus.

   _____

Complete Problems 8–11 to show that the conclusion is valid.



Given: $\overline{JK} \cong \overline{ML}$, $\overline{JM} \cong \overline{KL}$, and $\overline{JK} \cong \overline{KL}$. $\angle M$ is a right angle.

Conclusion: *JKLM* is a square.

8. Because $\overline{JK} \cong \overline{ML}$ and $\overline{JM} \cong \overline{KL}$, *JKLM* is a _____.
9. Because *JKLM* is a parallelogram and $\angle M$ is a right angle, *JKLM* is a _____.
10. Because *JKLM* is a parallelogram and $\overline{JK} \cong \overline{KL}$, *JKLM* is a _____.
11. Because *JKLM* is a _____ and a _____, *JKLM* is a square.

$\overline{EC} \cong \overline{ED}$, $\triangle ECD$ is an isosceles triangle. The base angles of an isosceles triangle are congruent, so $\angle EDC \cong \angle ECD$.

Practice and Problem Solving: C

1. $\sqrt{a^2 + b^2}$
2. $\sqrt{2}s$
3. $\frac{\sqrt{2}}{2}d$
4. $\frac{\sqrt{f^2 + g^2}}{2}$
5. $\sqrt{3}w$
6. 30°-60°-90°. The triangle formed by a diagonal shares an angle with the equilateral triangle (three sides of length *w*) formed by the side of the rectangle and half of each diagonal. Therefore, one angle of the triangle is 60°, another is the vertex of the rectangle (90°), and the third angle must be 30°.
7. $2\sqrt{13}$; $2\sqrt{13}$; $-\frac{3}{2}$; $\frac{2}{3}$; (0, 1); (0, 1)
8. 25.3 ft
9. The diagonal forms two isosceles triangles whose corresponding base angles are all congruent (each is half the measure of the bisected angles). The diagonal is a shared side, so the triangles are congruent by ASA. Therefore all four sides of the quadrilateral are congruent, and by SSS the other diagonal also forms two congruent isosceles triangles. Their base angles are all congruent, so each one measures half of the opposite angles of the quadrilateral, which means that the second diagonal bisects both of the other opposite angles.

Practice and Problem Solving: Modified

1. B
2. C
3. A
4. perpendicular
5. congruent
6. parallelogram
7. bisects
8. 3 in.
9. $3\frac{1}{4}$ in.
10. $1\frac{5}{8}$ in.
11. a. Given
    b. parallel
    c. Alternate Interior Angles Theorem
    d. $\angle 2 \cong \angle 3$
    e. Transitive Property of $\cong$

Reading Strategies

1. no
2. yes
3. yes
4. no
5. no
6. All quadrilaterals are polygons with 4 sides.
7. All 4 angles have to be right angles.
8. All 4 sides have to be congruent.
9. All 4 angles have to be right angles.

Success for English Learners

1. Yes; A square has 4 congruent sides.
2. All sides of a rhombus are congruent, so the length of one side of a rhombus can be substituted for the length of another side.

LESSON 9-4

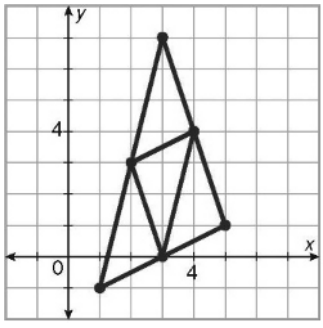
Practice and Problem Solving: A/B

1. rhombus
2. perpendicular
3. diagonals
4. rhombus
5. rectangle
6. Not valid; Possible explanation: You need to know that $\overline{AC} \perp \overline{BD}$.
7. Not valid; Possible explanation: You need to know that $\overline{AC}$ and $\overline{BD}$ bisect each other.
8. parallelogram
9. rectangle

10. rhombus
11. rectangle; rhombus

Practice and Problem Solving: C

1. Parallelogram and rhombus; Possible explanation: In a square or a rectangle, the interior angles must measure 90°. Therefore, the longest side of the triangle formed by any two sides and a diagonal must be the diagonal.
2. rhombus
3. $x\sqrt{3}$
4. 60° and 120°
5. 3
6. 1
7. 1
8. 1
9. an infinite number
10. 3
11. 4
12. 3
13. (1, −1), (5, 1), (3, 7)
14. midsegment triangle



Practice and Problem Solving: Modified

1. valid, perpendicular, opposite sides are congruent
2. valid, congruent, parallel, parallelogram
3. not valid, $\overline{BC}$, parallel
4. not valid, $\overline{BD}$, perpendicular
5. parallelogram
6. rectangle
7. rhombus
8. rectangle, rhombus

Reading Strategies

1. rectangle
2. rhombus
3. square
4. rectangle
5. square
6. rhombus
7. rhombus

Success for English Learners

1. You also need to know that one angle of *WXYZ* is a right angle, and any two adjacent sides are congruent.
2. No; you also need to know that $\angle WZQ \cong \angle YZQ$ because you need to know that $\overline{XZ}$ bisects a pair of opposite angles.

LESSON 9-5

Practice and Problem Solving: A/B

1. 55°
2. 22°
3. 123°
4. 60
5. 98°
6. 4.9
7. $n = 11.5$
8. $x = 12$ or -12
9. $a = 1.4$
10. $z = \sqrt{19}$ or $-\sqrt{19}$
11. trapezoid midsegment
12. 19.45 m

Practice and Problem Solving: C

1. Area $= \frac{1}{2}(AC)(BD)$
2. Yes; Possible answer: The length of *AE* is half the length of *AC*, and *BE* may be found from *BA* and *AE* by using the Pythagorean Theorem. *BD* is the sum of *BE* and *ED*. The area is $\frac{1}{2}(AC)(BD)$.
3. No; Possible answer: There is no way to use the Pythagorean Theorem to find the length of *AE*, and thus *AC*, with the information provided.